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APPLICATION OF THE METHOD OF WEIGHTED RESIDUALS TO THE SCATTERING AND DIFFRACTION OF ELASTIC SH WAVES BY SURFACE AND SUB-SURFACE TOPOGRAPHY OF ARBITRARY SHAPE

by

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ABSTRACT

The weighted residual method was applied to the problem of scattering and diffraction of plane SH waves by surface and sub-surface discontinuities of arbitrary shape below or on the surface of a two-dimensional half-space. In order to demonstrate the versatility of the method, it was applied to shallow and deep circular, elliptical, and rectangular cavities, inclusions, canyons and alluvial valleys. Results obtained match those obtained using available closed form solutions. It was shown that significant ground motion amplifications, with respect to the amplitude of incident waves, occurred on the ground surface near the cavity or inclusion, near or in the canyon or alluvial valley. Amplifications were dependent upon the shape and depth of the cavity or inclusion, the relative properties of the material in the inclusion and the surrounding medium, and the frequency and angle of incidence of incoming waves. With respect to the canyons and valleys, amplifications were dependent upon the shape and depth of the canyon or valley, the relative properties of the material in the valley and the surrounding medium, and the frequency and angle of incidence of incoming waves. Amplification profiles for the lower frequency incident waves were simple near the discontinuity on the surface of the half-space with peak amplifications that did not vary much from 2 except on the surface of the alluvium, the value expected on the surface of the half-space in the absence of a sub-surface discontinuity. As the frequency of the incident waves is increased, the amplification profiles near the cavity and inclusion became more complicated with peak values exceeding 5 for elliptically shaped cavities located near the ground surface and reaching 10 on the surface of the valley alluvium.

Keywords: Weighted residuals, SH-wave, diffraction, canyon, cavity, elastic inclusion, alluvial valley, half-space

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INTRODUCTION

One of the many areas of earthquake engineering and seismological research has been the effect of local site conditions on ground motion. Among the local site topographies of interest are cavities and elastic inclusions located below the ground surface, and canyons and alluvial valleys located on the ground surface. In this paper, the problem of the scattering and diffraction of incident SH waves by an arbitrarily shaped discontinuity on or below the surface of a two-dimensional half-space is studied. A numerical solution for the problem is appropriate as the boundary between the material of the discontinuity and the half-space may be irregular, making it difficult to describe the solution in closed form. The method of weighted residuals is implemented in order to study possible amplifications and de-amplifications of displacements on the surface of the half-space above and near a sub-surface discontinuity or adjacent to and within an alluvial valley or canyon.

Currently, with respect to sub-surface discontinuities, closed form solutions exist only for a cavity in an infinite space (Mow and Pao, 1973), a circular cavity in a half-space (Lee, 1977) and an elastic tunnel and inclusion in a half-space (Lee and Trifunac, 1979). Other analytic solutions have been developed by Gregory

(1967), Gregory (1970), Datta (1978), Dravinski (1983). Currently, with respect to surface discontinuities, closed form solutions exist for a semi-cylindrical valley (Trifunac, 1971), a semi-elliptical valley (Wong and Trifunac, 1974a), a semi-cylindrical canyon (Trifunac, 1973) and a semi-elliptical canyon (Wong and Trifunac, 1974b). Analytical solutions for a shallow circular cylindrical alluvial valley (Todorovska and Lee, 1991) and a shallow circular cylindrical canyon Cao and Lee (1989) have also been developed. A closed form solution for the scattering and diffraction of P, SV, and SH-waves by a three-dimensional alluvial valley has also been developed (Lee, 1984). Several approximate numerical approaches and boundary integral methods have been applied by researchers. These include Manoogian (1992), and Lee and Wu (1994a, 1994b), Manoogian and Lee (1995), Lee and Manoogian (1995), and Manoogian (1995).

This paper presents the application of the weighted residual approach to the scattering and diffraction of incident SH waves by arbitrarily shaped discontinuities on and near the surface of the half-space. This approach is used to evaluate boundary conditions and is a special case of the method of moments (Harrington; 1967, 1968). This approach has been applied to electromagnetic wave fields (Harrington, 1967), acoustic radiation fields (Fenlon, 1969), elastic inclusions, canyons, cavities, and alluvial valleys of irregular shape (Manoogian, 1992), Lee and Wu (1994a, 1994b), Manoogian and Lee (1995), and Lee and Manoogian (1995). Use of this method results in a matrix equation from which the unknown coefficients are determined and used to develop a series solution for the scattering, diffraction, and transmission of waves by the inclusion or cavity. This paper presents a new application of the weighted residual approach used to determine the scattering and diffraction of plane waves by discontinuities of various types on or within an elastic half-space. The method is applied to elastic inclusions, cavities, canyons and alluvial valleys of many shapes and the character of amplifications above and near the discontinuity are studied.

MODEL, EXCITATION AND SOLUTION--Cavity and Elastic Inclusion

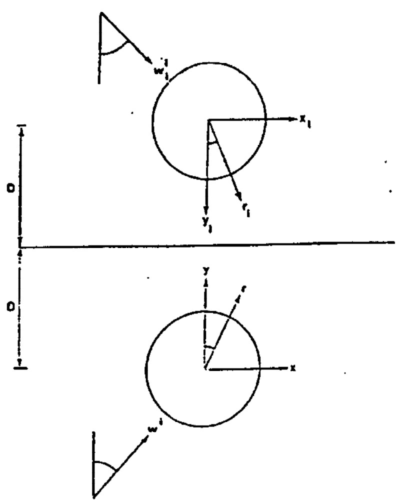

Figure 1. Elastic Inclusion Model

The cross section of the model to be studied is shown in Figure 1. It represents a circular elastic inclusion of arbitrary shape situated below the surface of the half-space. Although the approach is derived using a circular elastic inclusion, the resulting equations may be used for an elastic inclusion of any size or shape. The origin is at the center of the elastic inclusion. The half-space is assumed to consist of an elastic, homogeneous, isotropic, material with rigidity μ and shear wave velocity c_β. The material in the inclusion is assumed to consist of an elastic, homogeneous isotropic material with rigidity μ_v and shear wave velocity $c_{\beta v}$. For a cavity, the rigidity and density would be zero. Coordinate systems are shown in Figure 1. The z-axis may be assumed to be perpendicular to tthese coordinate systems.

Initially, define the excitation, $w^{(i)}$, as shown below:

$$w^{(i)}=\exp(-i\omega t)\exp(ikr\cos(\theta-\delta)) \quad (1)$$

This corresponds to a wave with incidence angle δ, amplitude of 1, excitation frequency ω, and wavelength $\lambda=2\pi/k$, where $k=\omega/c_\beta$, c_x and c_y are the components of the phase velocity in the direction of the coordinate axes. In order to develop a solution to the problem, consider an unbounded medium with an identical inclusion located at y=2D. Consider two additional coordinate systems defined within the cavity at O_1 with (x,y)=(0,2D); a Cartesian coordinate system (x_1,y_1) and a polar coordinate system (r_1,θ_1) as shown in Figure 1. Assume another incident plane SH-wave as defined below:

$$w_1^{(i)}=\exp(-i\omega t)\exp(ikr_1\cos(\theta_1-\delta)) \quad (2)$$

Omit exp(-iωt) from later expressions. Transform $w_1^{(i)}$ into the original coordinate system and combine with $w^{(i)}$

as shown below:

$$w^{(i)}+w_1^{(i)}=\exp(ikr\cos(\theta+\delta))+\exp(i2kD\cos\delta)\exp(-ikr\cos(\theta-\delta)) \tag{3}$$

Due to the presence of the inclusion, the waves are scattered and diffracted within the half-space and transmitted into the inclusion. Within the half-space, the result is a sum of the incident, reflected, scattered, and diffracted waves. Within the inclusion, the result consists of the transmitted waves. These must satisfy the wave equation as defined below:

$$\frac{\partial^2 w}{\partial r^2}+\frac{1}{r}\frac{\partial w}{\partial r}+\frac{1}{r^2}\frac{\partial^2 w}{\partial\theta^2}=\frac{1}{c_\beta^2}\frac{\partial^2 w}{\partial t^2} \tag{4}$$

Assume waves scattered by the inclusion and its image in the form shown below:

$$w^{(s)}=\sum_{n=0}^{\infty}H_n^{(1)}(kr(\theta))(A_n\cos n\theta+B_n\sin n\theta)) \qquad n=0,1,2,\ldots \tag{5}$$

$$w_1^{(s)}=\sum_{n=0}^{\infty}H_n^{(1)}(kr_1)(A_n\cos n\theta_1+B_n\sin n\theta_1) \qquad n=0,1,2,\ldots \tag{6}$$

Equation (6) is transformed into the (r,θ) coordinate system using the Graf addition theorem (Abramowitz and Stegun, 1964). The transmitted wave is defined as shown below:

$$w^{(v)}=\sum_{n=0}^{\infty}J_n(k_v r(\theta))(C_n\cos n\theta+D_n\sin n\theta) \qquad n=0,1,2,\ldots \tag{7}$$

Define $k_v=\omega/c_{\beta v}$ as the wave number in the inclusion. Boundary conditions of interest include the stress free boundary conditions at the free surface of the half-space and displacement and stress continuity conditions between the half-space and the inclusion must be used. The stress free condition at the surface of the half-space is shown below:

$$\tau_{yz}=\mu\frac{\partial w}{\partial y}\Big|_{y=D}=0 \tag{8}$$

Continuity conditions at the interface between the half-space and the inclusion are shown below:

$$0=w^{(i)}+w_1^{(i)}+w^{(s)}-w^{(v)} \tag{9}$$

$$\tau=\tau_{nz}^{(i)}+\tau_{1\,nz}^{(i)}+\tau_{nz}^{(s)}+\tau_{nz}^{(v)} \tag{10}$$

The free surface boundary condition is automatically satisfied by $w^{(s)}$ and $w^{(v)}$. The displacement continuity condition, equation (9), is satisfied by substituting equations (3), (5), (6), and (7). Assemble (3), (5), (6), and (7) into equation (10) in order to satisfy the stress boundary condition. Assemble the resulting equations into the weighted residual forms shown below:

$$0=\int_{\theta_1}^{\theta_N} W_m(r(\theta),\theta)\,(w^{(i)}+w_1^{(i)}+w^{(s)}-w^{(v)})\,d\theta \qquad m=0,1,2,\ldots \tag{11}$$

$$0=\int_{\theta_1}^{\theta_N} W_m(r(\theta),\theta)\,(\tau^{(i)}+\tau_1^{(i)}+\tau^{(s)}-\tau^{(v)})\,d\theta \qquad m=0,1,2,\ldots \tag{12}$$

Define $W_m(r(\theta),\theta)$ as the weight function and $\tau^{(i+r)}$, $\tau^{(s)}$, and $\tau^{(v)}$ as stresses due to the incident and reflected waves, scattered waves, and transmitted waves. The weight functions used in this case were $\cos m\theta$ and $\sin m\theta$. Since sufficient convergence using n,m=0,1,2,...,12 or less was achieved and solutions matched closed form solutions, others were not used. Solutions resulting from the use of Hankel functions were successful. It was found that the use of Bessel functions resulted in an ill conditioned coefficient matrix. The weighted residual forms are assembled into matrix form $[C_{mn}]\{\alpha_n\}=\{b_m\}$. Denote the matrix $[C_{mn}]$ as the matrix of coefficients from the weighted residual expressions, equations (11) and (12). Vectors $\{\alpha_n\}$ and $\{b_m\}$ consist of the unknown constants and coefficients from incident and reflected wave weighted residual expressions. Constants A_n, B_n, C_n, and D_n are determined and substituted into equations (5), (6), and (7). The transmitted wave amplitudes are defined by equation (7). The scattered wave amplitudes are added to equation (3) to obtain amplitudes in the half-space.

MODEL, EXCITATION AND SOLUTION--Alluvial Valley and Canyon--Incident SH-waves

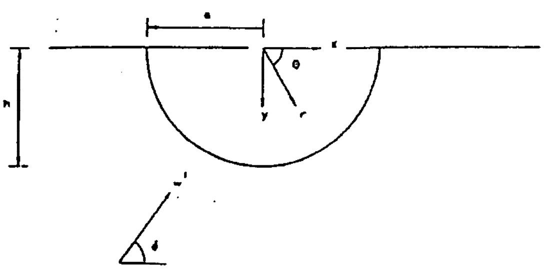

Figure 2. Alluvial Valley Model

The cross section of the model to be studied is shown in Figure 2. It represents a shallow alluvial valley of circular shape situated on the surface of the half-space. Although the approach is derived using a shallow circular alluvial valley, the resulting equations may be used for a valley of any size or shape. The origin is on the surface of the half-space, centered with respect to the canyon edges. The half-space is assumed to consist of an elastic, homogeneous, isotropic, material with rigidity μ and shear wave velocity c_β. The valley is assumed to consist of an elastic, homogeneous isotropic material with rigidity μ_v and shear wave velocity $c_{\beta v}$. For a canyon, the rigidity and density would be zero. Two coordinate systems are required and shown in Figure 2. In the absence of the valley, the incident SH-wave is reflected by the free surface (y=0) and defined as shown below. The incident and reflected waves are combined into the expression shown below:

$$w^{(i+r)}=\exp(ikr\cos(\theta+\delta))+\exp(ikr\cos(\theta-\delta)) \tag{13}$$

This corresponds to an incident wave with incidence angle δ, amplitude of 1, excitation frequency ω, and wavelength $\lambda=2\pi/k$, where $k=\omega/c_\beta$, c_x and c_y are the components of the phase velocity in the direction of the coordinate axes. Due to the presence of the valley, the waves are scattered and diffracted within the half-space and transmitted into the valley. Within the half-space, the result is a sum of the incident, reflected, scattered, and diffracted waves. Within the valley, the result consists of the transmitted waves. These must satisfy the wave equation as defined previously (4). Assume a scattered wave in the form shown below:

$$w^{(s)}=\sum_{n=0}^{\infty} A_n H_n^{(1)}(kr(\theta))\cos n\theta \qquad n=0,1,2,\ldots \tag{14}$$

The transmitted wave is defined as shown below:

$$w^{(v)}=\sum_{n=0}^{\infty} C_n J_n(k_v r(\theta))\cos n\theta \qquad (15)$$
$$n=0,1,2,\ldots$$

Define $k_v=\omega/c_{\beta v}$ as the wave number in the valley. In addition to the stress free boundary conditions at the free surface of the half-space, displacement and stress continuity conditions between the half-space and the valley must be used. These may be defined as shown below:

$$0=\frac{\mu}{r}\frac{\partial w}{\partial\theta}$$
$$0=\frac{\mu_v}{r}\frac{\partial w^{(v)}}{\partial\theta} \qquad (16)$$
$$\theta=0,\pi$$

at the surface of the half-space, and at the interface

$$0=w^{(i+r)}+w^{(s)}-w^{(v)} \qquad (17)$$

$$0=\tau^{(i+r)}+\tau^{(s)}-\tau^{(v)}=\mu\left(\frac{\partial w^{(i+r)}}{\partial r}n_r+\frac{1}{r}\frac{\partial w^{(i+r)}}{\partial\theta}n_\theta+\frac{\partial w^{(s)}}{\partial r}n_r+\frac{1}{r}\frac{\partial w^{(s)}}{\partial\theta}n_\theta\right.$$
$$\left.-\mu_v\left(\frac{\partial w^{(v)}}{\partial r}n_r+\frac{1}{r}\frac{\partial w^{(v)}}{\partial\theta}n_\theta\right)\right) \qquad (18)$$

Here at the interface between the valley and half-space, n_r and n_θ are the unit normals. The free surface boundary condition is automatically satisfied by $w^{(s)}$ and $w^{(v)}$. Assemble equations (17) and (18) into a weighted residual form shown as used in the case of the elastic inclusion The weight function used in this case was $\cos m\theta$ with m,n=0,1,2,...,18 or less. Since convergence was achieved and solutions matched closed form solutions, others were not used. The weighted residual forms are assembled into matrix form $[C_{mn}]\{\alpha_n\}=\{b_m\}$. Constants A_n, and C_n, are determined and substituted into equations (14), and (15). The transmitted wave amplitudes are defined by equation (15). Equation 14 is added to equation (13) to obtain amplitudes in the half-space.

SURFACE DISPLACEMENTS

Of particular interest are the displacement amplitudes on the surface of the half-space above the cavity or inclusion. If the amplitude of the incident plane SH-waves is 1, the responses shown define amplification and de-amplification factors. The resultant motion is defined by the modulus, $amplitude=(Re^2(w)+Im^2(w))^{1/2}$. In the absence of the cavity or inclusion, for a uniform half-space, the modulus of the ground displacement is 2. Due to the existence of the inclusion, incident waves are scattered and diffracted into the half-space and transmitted into the inclusion and moduli differ significantly from 2. Displacements were calculated for a discrete set of dimensionless frequencies, η, at intervals of 0.25 ranging from 0.25 to 2. The dimensionless frequency is $\eta=2a/\lambda=ka/\pi=\omega a/\pi\beta$ with a representing the radius of the inclusion. Figures that follow show the displacement amplitudes on the surface of the half-space. All displacements are plotted with respect to the dimensionless distance x/a.

Figure 3 shows the surface displacement amplitudes above an elliptical cavity with b/a=0.75, D=1.5a, $\delta=30°$, $\mu_f/\mu=0$, and $\rho_f/\rho=0$. Lower frequency incident waves produce simple surface amplification profiles. Higher frequency incident waves result in amplification profiles which are more complex with higher peak values approaching 8. On the surface where x/a<0, amplification profiles are complicated with higher, sharper peaks. At x/a>0 a shadow zone exists with simpler amplification profiles with values closer to 2.

Figure 4 shows the surface displacement amplitudes for a square inclusion with, D=1.5a, $\delta=60°$, $\mu_f/\mu=1/6$, $\rho_f/\rho=2/3$. Amplification profiles on the half-space are more prominent and complex for x/a<0 with peak

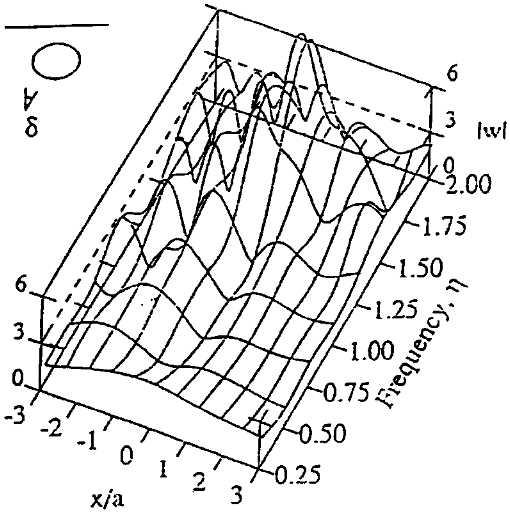

Figure 3. Underground Elliptical Cavity
Incident SH-Wave, D=1.5a, b/a=0.75 δ=30°

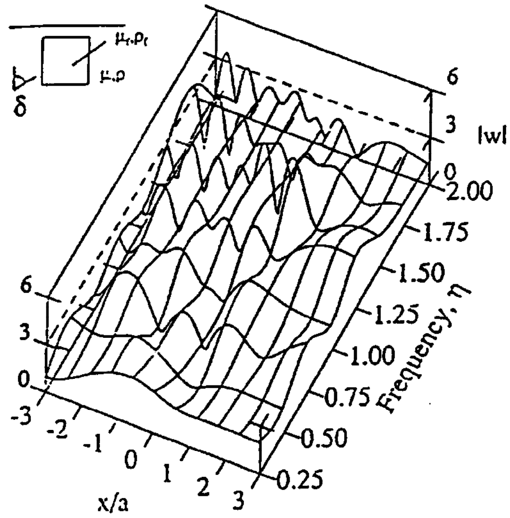

Figure 4. Underground Square Inclusion
D=1.5a, δ=60°, μ_f/μ=1/6, ρ_f/ρ=2/3

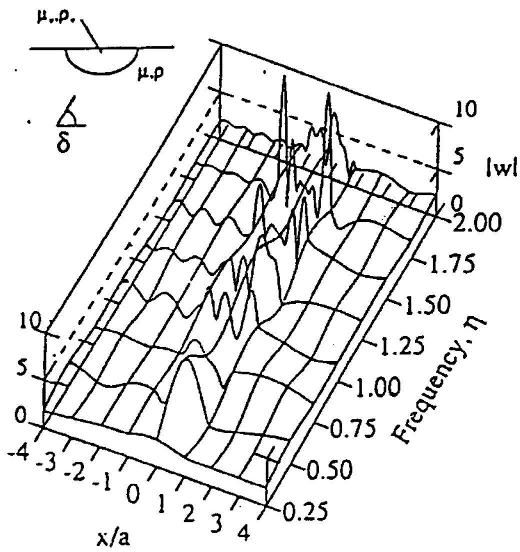

Figure 5. Shallow Elliptical Valley
D/W=0.35, δ=60°, μ_v/μ=1/6, and ρ_v/ρ=2/3

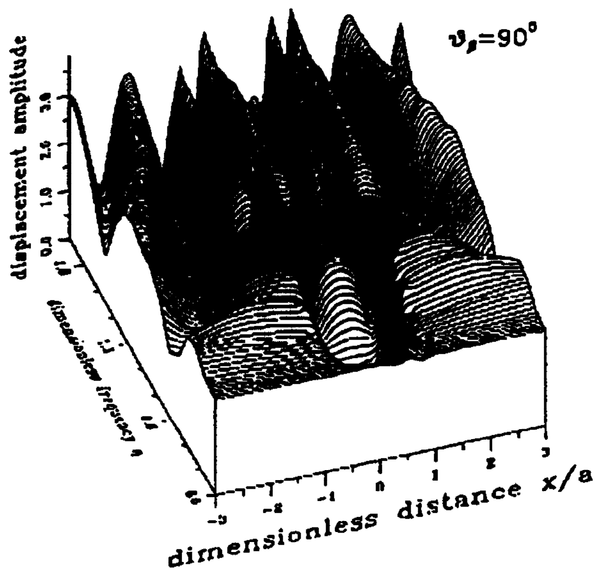

Figure 6. Nurek Canyon
h/a=0.833, δ=90°

amplifications approaching 6. On the other side of the inclusion, x/a>0, the amplitudes are smoother and tend towards 2.

In the absence of the valley, for a uniform half-space, the modulus of the ground displacement is 2. Due to the existence of the valley, incident waves are scattered and diffracted into the half-space and transmitted into the valley. Displacements were calculated for a discrete set of dimensionless frequencies, η, at intervals of 0.25 ranging from 0.25 to 2. Let a be the half-width of the valley, the distance between the edges of the valley at the surface. Figures that follow show the displacement amplitudes on the surface of the half-space and the valley. All displacements are plotted with respect to the dimensionless distance x/a.

Figure 5 shows the surface displacement amplitudes for a semi-elliptical valley with a depth to width ratio of 0.35 for a frequency range from 0.25 to 2 for an incident wave with an angle of incidence of 60°, $\mu_v/\mu=1/6$, $\rho_v/\rho=2/3$. These solutions match those of the closed form solution (Wong and Trifunac, 1974). Amplitude profiles on the half-space become more prominent and complex for x/a<-1. On the other side of the valley, x/a>1, the amplitudes are smoother and tend towards 2. Within the soft valley, responses are significantly amplified and have greater complexity with respect to the half-space with peaks approaching 10. It was difficult to obtain adequate convergence for the shallowest canyons.

Figure 6 shows displacement amplitudes in and near Nurek canyon for incident SH-waves with an orientation of 90°, b/a is about 0.8333 (Lee and Wu, 1994a). Displacement amplitude patterns shown are similar to those encountered in canyons with other shapes as discussed above. It shows the versatility of the approach as it was used on a real case. Amplitude patterns are more complex with magnitudes that tend toward 4 near the canyon edge.

CONCLUSIONS

1. The weighted residual approach approximation for the scattering, diffraction, and where appropriate, transmission of SH-waves by a discontinuity yields solutions which match the known closed form solutions.
2. Ground surface amplitudes on the half-space within and outside the valleys and canyons may be significantly larger than 2 and depend on the angle of incidence, dimensionless frequency, shape of the valley, and the relative properties of the alluvium.
3. Ground surface amplitudes on the half-space within the valleys may also be significantly larger than 2 and may differ from those on the half-space outside the valley and depend on the same factors and the properties of the valley relative to the half-space.

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