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ESTIMATION OF INCIDENT SEISMIC WAVES BASED ON SPATIAL VARIATION OF SURFACE GROUND MOTIONS

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ABSTRACT

This paper presents an analytical method to estimate the amplitude distribution of the amplitude of incident waves that corresponds to the cross power spectral density function defined on the ground surface. The analysis is made for a simplified earthquake model which is composed of the superposition of plane stochastic SH-waves traveling with varying angles. The procedure developed in this paper reverses the process studied by Kausel and Pais (1987) in which cross-correlation on the ground surface was computed by supposing a priori the sectorial distribution of the incident SH-waves. On the basis of the numerical results, an approximate but simple expression for the amplitude distribution function of the incident waves was presented.

KEYWORDS

earthquake ground motion, seismic waves, spatial variation of ground motion, coherence, SH-waves, cross power spectral density function, soil-structure interaction, kinematic interaction

INTRODUCTION

In the seismic response analyses of structures, it has been usually assumed that every point of foundation is subjected to the same ground motions during earthquakes. Due to the dense observation of the ground motions, however, the assumption of the same amplitude and phase of earthquake ground motion is not valid any more. It has been revealed also that the spatial variation of ground motions can not be expressed by the simple assumption of obliquely incident seismic waves from one direction. These facts indicate that the seismic waves impinge to the ground surface from various directions and with different amplitudes.

Some empirical models of the spatial variation of the ground motions at the ground surface have been proposed on the basis of the data provided by arrays of seismometers (e.g., Abrahamson and Bolt, 1985; Loh, 1985; Harichandran and Vanmarcke, 1986). Some other coherence models expressing the variability of the ground motions have been suggested by Matsushima (1977), Hoshiya and Ishii (1983), Luco and Wong (1986) and others based on different approaches. These models have been used to study the effects of spatially varying ground motions on the seismic response of structures with foundations supported on the soil surface (e.g., Matsushima, 1977; Luco and Wong, 1986; Luco and Mita, 1987; Harichandran, 1987; Veletsos and Prasad, 1988). The stochastic property of motions in the soil is of important for investigation of the seismic response of structures with embedded foundations or underground facilities such as buried pipelines. Kausel and Pais (1987) presented a method to determine the cross correlation function between the surface and a point at a depth in the soil on the basis of the knowledge of the cross correlation spectra on the surface. Additionally, they studied numerically the temporal and spatial variation of the surface ground motions assuming a priori the sectorial distribution of incident SH-waves. The stochastic responses of embedded foundations for varied patterns of SH-wave incidence have been discussed by Pais and Kausel (1990).

The objective of this paper is to estimate analytically the amplitude distribution of incoming waves with respect to the angle of incidence that corresponds to an empirical model of spatial variation of the ground motions defined at the soil surface. The analysis is made for a simplified earthquake model which is composed of the superposition of plane stochastic SH-waves traveling with varying angles. The procedure developed in this paper reverses the process presented by Kausel and Pais (1987) in which cross-correlation on the ground surface was computed by supposing a priori the sectorial distribution of the incident SH-waves. The results obtained in this study may be used for the seismic response analyses of embedded foundations and underground facilities subjected to spatially varying ground motions.

STATEMENT OF PROBLEM

The soil model considered in this study is illustrated in Fig. 1 which consists of layered soil with irregularly shaped interfaces. The ground motions in the top layer may be considered to be composed of waves traveling from various directions and with different amplitudes. This might be one of the factors that causes the incoherent motions on the ground surface. We consider in this study, for simplicity, the ground motions due to incidence of plane SH-waves propagating with varying angles. Thus, the free-field motions considered in this paper are composed of anti-plane motions which is perpendicular to the x-z plane shown in Fig. 1.

Most of the models of the spatial variation have been proposed in the form of cross power spectral density function and a model presented by Harichandran and Vanmarcke (1986) is used in this paper:

$$S(r,\omega) = \Gamma(r,\omega)\exp(-i\omega\frac{d_2 - d_1}{c})S_g(\omega), \tag{1}$$

in which r is the distance between the two points, $r = |\mathbf{r}_2 - \mathbf{r}_1|$ in which $\mathbf{r}_1$ and $\mathbf{r}_2$ are position vectors on the surface; ω is the circular frequency of motion; d_1 and d_2 are the components of $\mathbf{r}_1$ and $\mathbf{r}_2$ in the direction of propagation of the wave front, respectively; and c is the apparent horizontal velocity observed on the surface, which can be expressed with elastic wave velocity V_s and an incident angle α as given by

$$c = V_s / \cos\alpha. \tag{2}$$

The right hand side of equation (1) consists of three terms. The third term $S_g(\omega)$ is the power spectral density function which is assumed to be invariant within a local area, the second represents the wave passage

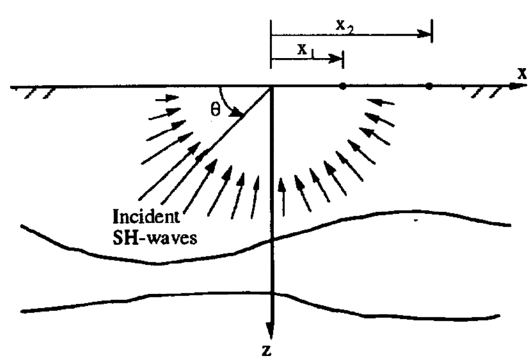


Fig. 1 Description of Model and Coordinate System.

effect of the ground motion and the third term $\Gamma(r,\omega)$ represents a coherence depending on the distance r between the two points and circular frequency ω. Following Veletsos and Prasad (1989), a coherence presented by Luco and Wong (1986) is used in this study which is given as follows

$$\Gamma(r,\omega)=\exp\left[-\left(\frac{\gamma\omega r}{V_s}\right)^2\right], \tag{3}$$

in which γ is a dimensionless index of incoherence. Under the conditions above described, the cross power spectral density functions at the two points on the x-axis shown in Fig. 1 can be expressed as follows:

$$S(x_1,x_2,\omega)=\exp\left[-\left(\frac{\gamma\omega|x_2-x_1|}{V_s}\right)^2\right]\exp\left[-i\frac{\omega\cos\alpha}{V_s}(x_2-x_1)\right]S_g(\omega). \tag{4}$$

Thus, the problem in this paper is to determine an amplitude distribution function of incoming waves with respect to the angle of incidence that corresponds to the cross power spectral density function defined in equation (4).

ANALYSIS OF AMPLITUDE DISTRIBUTION FUNCTION

Setting the coordinate system x, z as shown in Fig. 1, the free-field ground motion in the elastic top layer due to the incidence of SH-waves traveling with angle of incidence θ and the amplitude $A(\omega,\theta)/2$ can be expressed as follows:

$$v(x,z,t,\theta)=\frac{1}{2\pi}\int_{-\infty}^{+\infty}A(\omega,\theta)\cos\left(\frac{\omega\sin\theta}{V_s}z\right)\exp\left(-i\frac{\omega\cos\theta}{V_s}x\right)e^{i\omega t}d\omega, \tag{5}$$

in which V_s is the shear wave velocity in the homogeneous top layer.

Assuming that the free-field motion is composed of the superposition of multi-directional waves, the total motions can be expressed as in the form of

$$u(x,z,t)=\int_0^{\pi}v(x,z,t,\theta)d\theta=\frac{1}{2\pi}\int_{-\infty}^{+\infty}U(x,z,\omega)e^{i\omega t}d\omega, \tag{6}$$

in which

$$U(x,z,\omega)=\int_0^{\pi}A(\omega,\theta)\cos\left(\frac{\omega\sin\theta}{V_s}z\right)\exp\left(-i\frac{\omega\cos\theta}{V_s}x\right)d\theta. \tag{7}$$

Supposing that the motion expressed by equation (6) is stochastic with random value of $A(\omega,\theta)$, the cross power spectral of two processes $u(x_1,z_1,t)$ and $u(x_2,z_2,t)$ at the points (x_1,z_1) and (x_2,z_2) can be expressed as follows:

$$S_{12}(x_1,x_2,z_1,z_2,\omega)=E\left[U^*(x_1,z_1,\omega)U(x_2,z_2,\omega)\right], \tag{8}$$

in which the asterisk denotes complex conjugate and $E[\cdot]$ is the expectation operator.

Substituting from equation (7) into (8) leads to

$$S_{12}(x_1,x_2,z_1,z_2,\omega)=\int_0^{\pi}\int_0^{\pi}S_v(\theta',\theta'',\omega)\cos\left(\frac{\omega\sin\theta'}{V_s}z_1\right)\cos\left(\frac{\omega\sin\theta''}{V_s}z_2\right)\exp\left[i\frac{\omega}{V_s}(x_1\cos\theta'-x_2\cos\theta'')\right]d\theta'd\theta'', \tag{9}$$

in which

$$S_v(\theta',\theta'',\omega)=E\left[A^*(\omega,\theta')A(\omega,\theta'')\right]. \tag{10}$$

Following the assumptions employed by Kausel and Pais (1987), we express $S_v(\theta',\theta'',\omega)$ as in the form of

$$S_v(\theta',\theta'',\omega)=S_g(\omega)g(\theta')g(\theta'')f(|\theta'-\theta''|), \tag{11}$$

in which $S_g(\omega)$ is the power spectral density function defined on the soil surface; $g(\theta)$ is a wave amplitude distribution function with respect to θ; and $f(|\theta'-\theta''|)$ is the cross-correlation function for the wave component. Further simplification is made by introducing an assumption that the waves traveling from the distinct directions θ', θ'' are uncorrelated, and it leads to ,

$$f(|\theta' - \theta''|) = \delta(\theta' - \theta''), \tag{12}$$

in which $\delta(\,)$ is a Dirac-delta function (Kausel and Pais, 1987). Substitution from equation (11) with (12) into (9) and integrating with respect to θ' leads to

$$S_{12}(x_1, x_2, z_1, z_2, \omega) = S_g(\omega)\int_0^\pi g^2(\theta)\cos\left(\frac{\omega\sin\theta}{V_s}z_1\right)\cos\left(\frac{\omega\sin\theta}{V_s}z_2\right)\exp\left[i\frac{\omega\cos\theta}{V_s}(x_1 - x_2)\right]d\theta. \tag{13}$$

The function $g^2(\theta)$ may be interpreted as the intensity distribution function with respect to θ of the incident waves. In the case that $z_1 = z_2 = 0$ and $x_1 = x_2$ in equation (13), $S_{12}(\omega)$ must be equal to the power spectral density function at the soil surface $S_g(\omega)$. Hence, $g^2(\theta)$ must satisfy

$$\int_0^\pi g^2(\theta)d\theta = 1. \tag{14}$$

Letting $z_1 = z_2 = 0$ in equation (13), then it gives the cross power spectral density function over two points on the soil surface and equating the equations (4) and (13) yields

$$\int_0^\pi g^2(\theta)\exp\left(-i\frac{\omega\cos\theta}{V_s}\lambda\right)d\theta = \exp\left[-\left(\frac{\gamma\omega|\lambda|}{V_s}\right)^2\right]\exp\left(-i\frac{\omega\cos\alpha}{V_s}\lambda\right), \tag{15}$$

in which $\lambda = x_2 - x_1$ and $g^2(\theta)$ is the unknown function to be determined. It is quite natural to express $g^2(\theta)$ in the form of Fourier cosine series:

$$g^2(\theta) = \sum_{n=0}^{\infty} d_n \cos n\theta, \tag{16}$$

in which d_n are the Fourier coefficients. From equation (14), it may immediately be shown $d_0 = 1/\pi$. Substituting from equation (16) into (15) and making use of a well known formula (Gradshteyn and Ryzhik, 1980)

$$\int_0^\pi e^{iz\cos\theta}\cos n\theta d\theta = \pi i^n J_n(z), \tag{17}$$

equation (15) can be written in the form

$$\pi\sum_{n=0}^{\infty}(-i)^n d_n J_n(\eta) = \exp(-\gamma^2\eta^2)\exp(-i\eta\cos\alpha), \tag{18}$$

in which $J_n(\,)$ is Bessel function of the first kind and order n, and $\eta = \omega\lambda/V_s$ is the dimensionless frequency. Noting that d_n is real numbers, we obtain following equations:

$$\sum_{n=0}^{\infty}(-1)^n d_{2n}J_{2n}(\eta) = \frac{1}{\pi}\exp(-\gamma^2\eta^2)\cos(\eta\cos\alpha), \tag{19}$$

$$\sum_{n=0}^{\infty}(-1)^n d_{2n+1}J_{2n+1}(\eta) = \frac{1}{\pi}\exp(-\gamma^2\eta^2)\sin(\eta\cos\alpha). \tag{20}$$

In analyzing the unknown coefficients d_{2n} and d_{2n+1}, it is convenient to expand the right hand sides of equations (19) and (20) in terms of power series of η as follows:

$$\exp(-\gamma^2\eta^2)\cos(\eta\cos\alpha) = \sum_{n=0}^{\infty} C_{2n}\eta^{2n}, \qquad \exp(-\gamma^2\eta^2)\sin(\eta\cos\alpha) = \sum_{n=0}^{\infty} C_{2n+1}\eta^{2n+1}, \tag{21}$$

in which the coefficients C_{2n} and C_{2n+1} are given in the form of finite series:

$$C_{2n} = \sum_{k=0}^{n}(-1)^k\frac{(-\gamma^2)^{(n-k)}\cos^{2k}\alpha}{(n-k)!(2k)!}, \qquad C_{2n+1} = \sum_{k=0}^{n}(-1)^k\frac{(-\gamma^2)^{(n-k)}\cos^{2k+1}\alpha}{(n-k)!(2k+1)!}. \tag{22}$$

Substituting from equation (21) into (19) and (20) yields

$$\sum_{n=0}^{\infty}(-1)^n d_{2n}J_{2n}(\eta) = \frac{1}{\pi}\sum_{n=0}^{\infty} C_{2n}\eta^{2n}, \tag{23}$$

$$\sum_{n=0}^{\infty}(-1)^n d_{2n+1}J_{2n+1}(\eta) = \frac{1}{\pi}\sum_{n=0}^{\infty} C_{2n+1}\eta^{2n+1}. \tag{24}$$

Equations (23) and (24) are having the forms of Neumann series and the unknown coefficients d_{2n} and d_{2n+1} may be shown to be given as follows (Watson, 1966)

$$d_0 = \frac{1}{\pi} C_0 = \frac{1}{\pi} \tag{25a}$$

$$d_{2n} = (-1)^n \frac{2n}{\pi} \sum_{r=0}^{n} 2^{2(n-r)} \frac{(2n-r-1)!}{r!} C_{2(n-r)}, \quad n = 1,2,\cdots \tag{25b}$$

$$d_{2n+1} = (-1)^n \frac{2n+1}{\pi} \sum_{r=0}^{n} 2^{2(n-r)+1} \frac{(2n-r)!}{r!} C_{2(n-r)+1}, \quad n = 1,2,\cdots. \tag{25c}$$

Thus, the closed form solutions were obtained for equations (23) and (24). First several terms of d_{2n} and d_{2n+1} are shown in the Appendix. Once the coefficients d_{2n} and d_{2n+1} are obtained, the absolute value of the wave amplitude function may be evaluated by following equation:

$$|g(\theta)| = \sqrt{\sum_{n=0}^{\infty} d_n \cos n\theta}. \tag{26}$$

ANALYSIS OF COHERENCE

As the data on the coherence in the vertical direction of soil is very scarce, it is interesting to predict the coherence based on the earthquake model considered in this study. Nevertheless Kausel and Pais (1987) has presented the complete formulation for the analysis, a similar study is attempted here by use of a different approach to obtain the closed form solution. The coherence over two points in the soil may be evaluated by following equation:

$$\Gamma_{12}(\lambda, z_1, z_2, \omega) = \frac{|S_{12}(\lambda, z_1, z_2, \omega)|^2}{S_1(x_1, z_1, \omega) S_2(x_2, z_2, \omega)}, \tag{27}$$

in which λ is the horizontal distance, $\lambda = x_2 - x_1$; S_{12} is the cross spectral density function defined in equation (13); and S_1 and S_2 are the power spectral density functions at the points (x_1, z_1) and (x_2, z_2), respectively, which may be given as follows by substituting $z_2 = z_1$ and $x_2 = x_1$, or vice versa, into equation (13):

$$S_i(x_i, z_i, \omega) = S_g(\omega) \int_0^{\pi} g^2(\theta) \cos^2\left(\frac{\omega \sin\theta}{V_s} z_i \right) d\theta, \quad i = 1,2. \tag{28}$$

Substituting from equations (13) and (28) together with (16) into (27) yields

$$\Gamma_{12}(\lambda, z_1, z_2, \omega) = \frac{\gamma_{12}^2(\lambda, z_1, z_2, \omega)}{\gamma_1(z_1, \omega)\gamma_2(z_2, \omega)}, \tag{29}$$

in which

$$\gamma_{12}^2(\lambda, z_1, z_2, \omega) = \left| \sum_{n=0}^{\infty} d_n \int_0^{\pi} \cos n\theta \cos\left(\frac{\omega \sin\theta}{V_s} z_1 \right) \cos\left(\frac{\omega \sin\theta}{V_s} z_2 \right) \exp\left(i \frac{\omega \cos\theta}{V_s} \lambda \right) d\theta \right|^2, \tag{30}$$

and

$$\gamma_i(z_i, \omega) = \left| \sum_{n=0}^{\infty} d_n \int_0^{\pi} \cos n\theta \cos^2\left(\frac{\omega \sin\theta}{V_s} z_i \right) d\theta \right|, \quad i = 1,2. \tag{31}$$

The integration with respect to θ of equations (30) and (31) may be shown to be expressed in the closed forms:

$$\gamma_{12}^2(\lambda, z_1, z_2, \omega) = \left| \frac{\pi}{2} \sum_{n=0}^{\infty} d_{2n} \left\{ J_{2n}(\frac{\omega D_1}{V_s}) \cos 2n\alpha_1 + J_{2n}(\frac{\omega D_2}{V_s}) \cos 2n\alpha_2 \right\} \right.$$
$$\left. -i \frac{\pi}{2} \sum_{n=0}^{\infty} d_{2n+1} \left\{ J_{2n+1}(\frac{\omega D_1}{V_s}) \sin(2n+1)\alpha_1 + J_{2n+1}(\frac{\omega D_2}{V_s}) \sin(2n+1)\alpha_2 \right\} \right|^2, \tag{32}$$

and

$$\gamma_i(z_i, \omega) = \left| \pi \sum_{n=0}^{\infty} d_{2n} J_{2n}\left(\frac{\omega z_i}{V_s} \right) \right|, \quad i = 1,2 \tag{33}$$

in which

$$D_1 = \sqrt{(z_1 + z_2)^2 + \lambda^2}, \qquad D_2 = \sqrt{(z_1 - z_2)^2 + \lambda^2} \tag{34}$$

and α_1 and α_2 are angles to be determined from following equations:

$$\tan\alpha_1 = \frac{\lambda}{z_1 + z_2}, \qquad \tan\alpha_2 = \frac{\lambda}{z_1 - z_2}. \tag{35}$$

For the special case that $\lambda = z_2 = 0$ in equation (30), it reduces to

$$\gamma_{12}{}^2(0, z_1, 0, \omega) = \left| \pi \sum_{n=0}^{\infty} d_{2n} J_{2n}\left(\frac{\omega z_1}{V_s}\right) \right|^2. \tag{36}$$

Substituting from equations (36) and (33) into (29) yields the coherence between a point on the soil surface and a point at depth z_1 right below the surface point, and it may be shown to be expressed in a simple form as,

$$\Gamma_{12}(0, z_1, 0, \omega) = \left| \pi \sum_{n=0}^{\infty} d_{2n} J_{2n}\left(\frac{\omega z_1}{V_s}\right) \right|. \tag{37}$$

Yoshida and Mita (1988) studied the stochastic response of embedded foundations under an assumption that the motions at two points are random laterally but coherent in depth. The result of equation (37) suggests, however, that the assumption of perfect coherence in the vertical direction is not valid.

METHOD OF NUMERICAL CALCULATION

The amplitude distribution function is expressed in the form of infinite series as shown in equation (26). In the numerical calculation of the series, there arises a difficulty that the series tends to diverge for large value of n. The number of terms must be truncated at a certain value M, and it is reasonable to decide the number so as to minimize the squares of the difference defined as follows:

$$\varepsilon = \int_0^{2\pi f_{max}} \left| \Gamma(\lambda, \omega) - \tilde{\gamma}_{12}{}^2(\lambda, 0, 0, \omega) \right|^2 d\omega, \tag{38}$$

in which f_{max} is the maximum frequency under consideration: $\Gamma(\lambda, \omega)$ is the coherence given in advance: and

$$\tilde{\gamma}_{12}{}^2(\lambda, 0, 0, \omega) = \left| \pi \sum_{n=0}^{M} (-1)^n d_{2n} J_{2n}\left(\frac{\omega\lambda}{V_s}\right) - i\pi \sum_{n=0}^{M} (-1)^n d_{2n+1} J_{2n+1}\left(\frac{\omega\lambda}{V_s}\right) \right|^2, \tag{39}$$

which is given from equation (32) by letting $z_1 = z_2 = 0$. Fig. 2 shows the comparison between a given coherence and a computed result with $M = 3$ that makes equation (38) minimum. It may be seen from Fig . 2 that the coherence calculated based on the above described criterion gives the satisfactory result.

NUMERICAL RESULTS OF INTENSITY DISTRIBUTION FUNCTION

The intensity distribution functions, which is defined as the squares of the amplitude function shown in equation (26), were calculated for different values of the index of incoherence γ, which is considered to be between zero to 0.5 (Veletsos and Prasad, 1989), and for $\alpha = 80°$ and $90°$. The results which are normalized by the corresponding peak value are shown in Fig. 3(a) and (b). It is noticed from the results that with decrease of the index γ the intensity distribution tends to concentrate at a specific angle α. It is also noted that the distribution function has a very simple form. An empirical expression for the distribution function may be suggested as given by

$$g^2(\theta) = \frac{2}{\sqrt{\pi}a\left[\Phi((\pi - \alpha)/a) + \Phi(\alpha/a)\right]} e^{-(\alpha-\theta)^2/a^2}, \tag{40}$$

in which $a = 2.1\gamma$ and $\Phi(\,)$ represents the error function. The coefficient of the exponential term was decided so as to satisfy equation (14). It may be shown that as γ approaches zero $g^2(\theta)$ goes to $\delta(\alpha - \theta)$. Fig. 4 shows the comparison of results computed by equation (16) with results evaluated by use of the proposed empirical equation. The excellent agreement between two results may be noticed. In consequence, equation (40) may be suggested to use for estimation of the amplitude distribution function valid for $70° < \alpha < 90°$.

Once the function $g^2(\theta)$ is obtained, the coherence between two points may be evaluated by substituting from equations (13) and (28) into (27). Thus calculated results of coherence over two points on the soil surface are compared in Fig. 5 with the coherence $\Gamma(\lambda, \omega)$ given in equation (3). It is noticed from Fig. 5 that both results show excellent agreement.

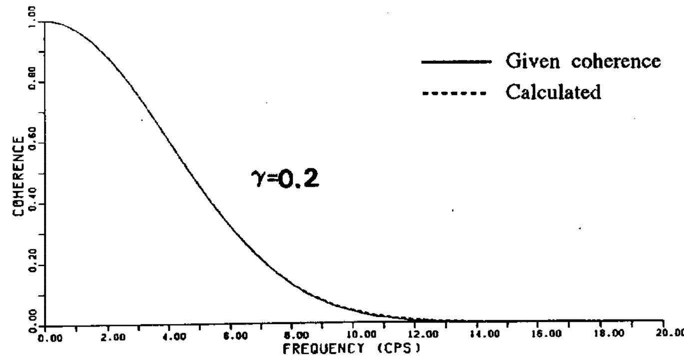
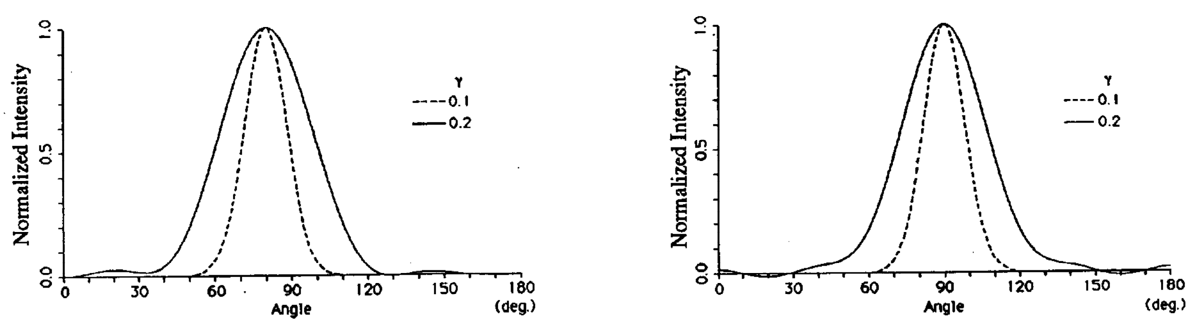


Fig. 2 Comparison of Intensity Distribution Function Computed by Eq. (16) with Empirical Result. ($M = 3$; $\alpha = 80°$ and $\lambda / V_s = 0.1\,\mathrm{sec}$)



(a) $\alpha = 80°$ (b) $\alpha = 90°$

Fig. 3 Normalized Intensity Distribution Functions with respect to θ.

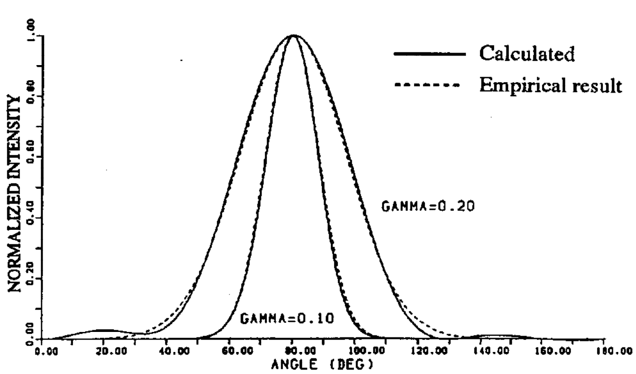


Fig. 4 Comparison of Intensity Distribution Functions Computed by Eq. (16) with Empirical Results. ($M = 3$; $\alpha = 80°$ and $\lambda / V_s = 0.1\,\mathrm{sec}$)

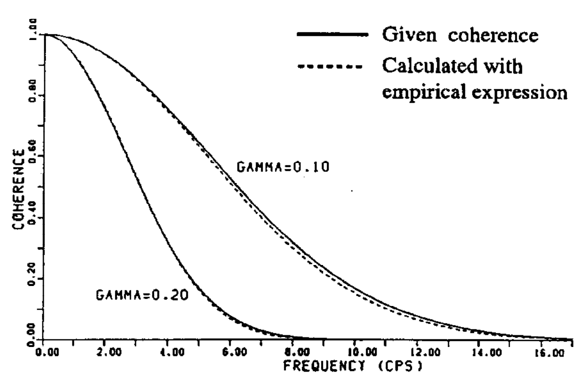


Fig. 5 Comparison of Given Coherence with One Calculated by use of Eq. (40). ($\alpha = 80°$ and $\lambda / V_s = 0.15\,\mathrm{sec}$)

CONCLUSIONS

The closed form solution for estimation of the amplitude distribution of incoming waves with respect to the angle of incidence that corresponds to the empirical model of spatial variation of the ground motions defined at the soil surface was presented for a simplified earthquake model which is composed of the superposition of plane stochastic SH-waves traveling with varying angles. In the numerical study, the amplitude distribution was calculated for different values of the index of incoherence. The computed results show that with decrease of an index of incoherency γ, the distribution of incident waves tends to concentrate around a specific angle. On the basis of the numerical results, an approximate but simple expression for estimation of the intensity distribution function of the incident waves was presented.

APPENDIX

This appendix gives first several terms of the coefficients d_n for obtaining the amplitude distribution function. The coefficient C_n was given by equation (22) and $C_0 = 1$.

$$d_0 = \frac{1}{\pi}C_0, \qquad d_1 = \frac{2}{\pi}C_1,$$

$$d_2 = -\frac{2}{\pi}(4C_2 + C_0), \qquad d_3 = -\frac{6}{\pi}(8C_3 + C_1),$$

$$d_4 = \frac{2}{\pi}(192C_4 + 16C_2 + C_0), \qquad d_5 = \frac{10}{\pi}(384C_5 + 24C_3 + C_1),$$

$$d_6 = -\frac{2}{\pi}(23040C_6 + 1152C_4 + 36C_2 + C_0), \quad d_7 = -\frac{14}{\pi}(46080C_7 + 1920C_5 + 48C_3 + C_1),$$

$$d_8 = \frac{2}{\pi}(5160960C_8 + 184320C_6 + 3840C_4 + 64C_2 + C_0).$$

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