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RESPONSE ANALYSIS OF SUSPENSION BRIDGES

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ABSTRACT

For suspension bridges which subjected to long period ground motions, the current response spectrum modal combination methods (such as SRSS, CQC, DSC and SMAM) often significantly underestimate or overestimate the maximum tower responses (displacements and bending moments) about 20-35%, compared with the results from time history method. Nevertheless, these methods continue to be used in seismic analyses of suspension bridges. In this paper, a new modal combination method is developed and applied to response spectrum analyses of two domestic suspension bridges — Shantou Bay Bridge and Tiger Gate Bridge. A comparison among the results abstracted from the new method, CQC method, SMAM method and time history method respectively is carried out. The results show that the new method can give good results.

KEYWORDS

response spectrum; modal combination; seismic response analysis; suspension bridges; imaginary excitation.

INTRODUCTION

Because the response spectra calculations have lost all information on sign and time when the maximum displacement etc. occurred, the use of response spectra techniques is complicated by the difficulty of combining the maximum modal responses for multiple degree-of-freedom structures. To find what contribution or sign each mode should have at the same time, several accepted combination methods have been developed and used in seismic analyses of structures. Among these methods, SRSS(Square Root of Sum of Squares) method is the most commonly used method. However, when some of the modes are closely spaced, the use of SRSS method may result in grossly underestimating or overestimating the maximum responses. In 1969, Rosenblueth *et al* suggested a Double Sum Combination(DSC) method assuming that the modal responses are statistically correlated. Afterwards, Humar(1984) and Gupta(1990) updated the original DSC method. Furthermore, a formulation known as the Complete Quadratic Combination(CQC) which is based on the theory of random vibrations, has been proposed by Kiureghian (1981). The method may be considered as an extension of the SRSS method. However, these methods mentioned above don't account for the perfect modal correlation when the frequencies are in the higher range. Hence, Lindley and Yow(1980), Hadjian(1981), Gupta(1984) proposed their own techniques to consider the correlation between modes below or greater than rigid frequency. In 1993, the original CQC rule was extended

(Kiureghian *et al.*) to account for the effects of high-frequency modes, narrow-band seismic input and cut-off frequency. Additionally, Tsai (1984) pointed out that the combination between the maximum responses of two modes converges to an algebraic sum when both modal frequencies are sufficiently low or high eventhough they may not be closely spaced with each other. Consequently, an Advanced Response Combination (ARC) technique was developed. The Sum of Modal Absolute Maximum (SMAM) response is also an acceptable method. But it's seldom used for ordinary structures because it's very conservative.

Among the methods above, the CQC modal combination rule for seismic analysis of ordinary structures has received wide recognition by researchers and practitioners. However, large errors have been found for these methods(including CQC method) in the seismic analyses of several domestic suspension bridges which dynamic characteristics are different from ordinary structures(Yuan *et al.*, 1995; Yutaka *et al.*, 1993). This reveals that the current response spectrum combination methods need to be improved for seismic analysis of suspension bridges.

THE NEW COMBINATION RULE

Consider a linear structure having N degrees of freedom and subjected to the base acceleration $\ddot{v}_g(t)$. Assume that the structure has classical modes with ω_j , ζ_j, denoting the *j*th modal frequency and damping ratio respectively. The well-know equation of motion is (Clough *et al.*, 1975)

$$[M]\{\ddot{v}(t)\}+[C]\{\dot{v}(t)\}+[K]\{v(t)\}=-[M]\{I\}\{\ddot{v}_g(t)\} \tag{1}$$

where [M], [C] and [K] are the N×N mass, damping and stiffness matrices respectively. $\{v(t)\}$ denotes the N vector of nodal displacements. {I} is the influence vector, containing the values of the nodal displacements when the base is excited by a unit static translation.

It's well known that, using a mode-superposition procedure, the nodal displacement in such a system can be expressed in terms of its modal responses as(Clough *et al.*, 1975)

$$\{v(t)\}=\sum_{j=1}^{N}\{\phi\}_j Y_j(t) \tag{2}$$

in which $Y_j(t)$, $\{\phi\}_j$ are the *j*th normalised modal response and mode shape vector respectively.

Substituting eq.(2) into eq.(1) and introducing the orthogonality property, we obtain

$$\ddot{Y}_j(t)+2\zeta_j\omega_j\dot{Y}_j(t)+\omega_j^2Y_j(t)=-\gamma_j\ddot{v}_g(t) \tag{3}$$

where $\gamma_j=\dfrac{\{\phi\}_j^T[\mathrm{M}]\{I\}}{\{\phi\}_j^T[\mathrm{M}]\{\phi\}_j}$, is the modal participation factor for mode j.

Assuming an imaginary sinusoidal excitation (Lin, 1992; Lin *et al.* 1993) $\ddot{v}_g(t)=\sqrt{S_{\ddot{v}_g}(\omega)}e^{i\omega t}$,which represents the sinusoidal component with frequency ω and strength $\sqrt{S_{\ddot{v}_g}(\omega)}$ of the actual ground motion. The substitution of $\ddot{v}_g(t)$ into eq.(3) gives the steady-state response as

$$Y_j(\omega,t)=-\gamma_jH_j(\omega)\sqrt{S_{\ddot{v}_g}(\omega)}e^{i\omega t}=-\gamma_j\left|H_j(\omega)\right|\sqrt{S_{\ddot{v}_g}(\omega)}e^{i[\omega t+\phi_j(\omega)]} \tag{4}$$

in which $S_{\ddot{v}_g}(\omega)$ is the power spectral density function of the ground acceleration $\ddot{v}_g(t)$. $i=\sqrt{-1}$, is the imaginary unit.

$H_j(\omega)=\left[\omega_j^2-\omega^2+2i\zeta_j\omega_j\omega\right]^{-1}$ is the frequency response function for mode j.

$$\phi_j(\omega) = tg^{-1}\frac{-2\zeta_j\omega_j\omega}{\omega_j^2-\omega^2}, \quad \text{especially, } \phi_j(\omega_j) = -\frac{\pi}{2}$$

Similarly, we can obtain the jth modal responses for each imaginary sinusoidal ground motion with discrete frequency $\cdots\omega_{-M}\cdots\omega_{C}\cdots\omega_{M}\cdots$ respectively. Consequently

$$Y_j(t) = \sum_{k=-\infty}^{+\infty} -\gamma_j H_j(\omega_k)\sqrt{S_{\ddot{v}_g}(\omega_k)}e^{i\omega_k t} \tag{5}$$

From eq.(5), we have

$$|Y_j(t)| = \sqrt{\sum_{k=-\infty}^{+\infty}\sum_{l=-\infty}^{+\infty}\gamma_j^2|H_j(\omega_k)||H_j(\omega_l)|\sqrt{S_{\ddot{v}_g}(\omega_k)}\sqrt{S_{\ddot{v}_g}(\omega_l)}\cos[(\omega_l-\omega_k)t+\phi_j(\omega_l)-\phi_j(\omega_k)]} \tag{6}$$

According to eq.(2), the total response of node r can be written as

$$v_r(t) = \sum_{j=1}^{N}\phi_{jr}Y_j(t) = \sum_{j=1}^{N}\sum_{k=-\infty}^{+\infty} -\gamma_j\phi_{jr}H_j(\omega_k)\sqrt{S_{\ddot{v}_g}(\omega_k)}e^{i\omega_k t} \tag{7}$$

and

$$|v_r(t)| = \sqrt{\sum_{j=1}^{N}\sum_{p=1}^{N}\sum_{k=-\infty}^{+\infty}\sum_{l=-\infty}^{+\infty}\gamma_j\gamma_p\phi_{jr}\phi_{pr}|H_j(\omega_k)||H_p(\omega_l)|\sqrt{S_{\ddot{v}_g}(\omega_k)}\sqrt{S_{\ddot{v}_g}(\omega_l)}\cos[(\omega_l-\omega_k)t+\phi_j(\omega_l)-\phi_p(\omega_k)]} \tag{8}$$

Introducing an assumption that each sinusoidal component is independent. Then, eqs.(6) and (8) can be given respectively by

$$|Y_j(t)| = \sqrt{\sum_{k=-\infty}^{+\infty}\gamma_j^2|H_j(\omega_k)|^2 S_{\ddot{v}_g}(\omega_k)} = \sigma_j \tag{9}$$

$$|v_r(t)| = \sqrt{\sum_{k=-\infty}^{+\infty}\sum_{j=1}^{N}\sum_{p=1}^{N}\gamma_j\gamma_p\phi_{jr}\phi_{pr}|H_j(\omega_k)||H_p(\omega_k)|S_{\ddot{v}_g}(\omega_k)} = \sigma_{v_r} \tag{10}$$

where σ_j and σ_{v_r} are the root mean squares of the jth modal response and the total response of node r under the actual ground acceleration $\ddot{v}_g(t)$.

Considering the assumption mentioned above, we can obtain the well-known CQC method easily. On the other hand, if we consider that each sinusoidal component is properly correlated, and that eqs.(6) and (8) approximately represent the root mean squares of the jth modal response and the total response of node r under actual ground motion $\ddot{v}_g(t)$, a new combination method can be easily derived as follows:

Because the damping ratios of most structures are very small in general, the structure is strongly selective to the sinusoidal components of the ground motion(Kuwamura *et al.*, 1994). Only those sinusoidal components which frequencies are identical to the natural frequencies of the structure have dominant contribution to corresponding modal response, the other contributions can be neglected. Hence, eq.(5) may be written approximately as

$$Y_j(t) \approx -\gamma_j|H_j(\omega_j)|\sqrt{S_{\ddot{v}_g}(\omega_j)}e^{i(\omega_j t-\frac{\pi}{2})} \approx -\sigma_j e^{i(\omega_j t-\frac{\pi}{2})} \tag{11}$$

The substitution of $D(\omega_j,\zeta_j) = p_j\sigma_j$ (Kiureghian, 1981) into eq.(11) results in

$$Y_j(t) \approx -\frac{D(\omega_j,\zeta_j)}{p_j}e^{i(\omega_j t-\frac{\pi}{2})} \tag{12}$$

in which $D(\omega_j,\zeta_j)$ is the ordinate of the mean displacement response spectrum (the 'design' spectrum) for mode j. p_j is the peak factor for mode j.

Substituting eq.(12) into eq.(7) yields

$$v_r(t) \approx \sum_{j=1}^{N} \frac{1}{p_j} \phi_{jr} D(\omega_j, \zeta_j) e^{i(\omega_j t - \frac{\pi}{2})} \tag{13}$$

Since eq.(13) can be used to calculate the mean square response of the structure subjected to actual ground acceleration $\ddot{v}_g(t)$ approximately, the mean of the absolute maximum response of the structure can be obtained from

$$E\left[\max\left|v_r^*(t)\right|\right] \approx \sqrt{\sum_{j=1}^{N}\sum_{k=1}^{N} \frac{p^2}{p_j p_k} R_{jr} R_{kr} \cos(\omega_k - \omega_l) t_m} \tag{14}$$

where $v_r^*(t)$ denotes the total response of node r to actual ground acceleration $\ddot{v}_g(t)$. p is the peak factor in relation to the total response of node r . $R_{jr} = \phi_{jr} D(\omega_j, \zeta_j)$. t_m represents the pseudo time which the maximum response arrives.

For most structures, the ratio $\frac{p}{p_j}$ would tend to be near unity. Thus, these ratios in the expression for the mean peak response can be discarded without much loss of accuracy (Kiureghian, 1981). With this simplification, eq.(14) reduces to

$$E\left[\max\left|v_r^*(t)\right|\right] \approx \sqrt{\sum_{j=1}^{N}\sum_{k=1}^{N} R_{jr} R_{kr} \cos(\omega_k - \omega_l) t_m} \tag{15}$$

DETERMINATION OF IMAGINARY TIME t_m FOR SUSPENSION BRIDGES

First of all, the characteristics of mode-superposition will be discussed. Using Duhamel's integral, the displacement, velocity and acceleration — $Y_j(t), \dot{Y}_j(t), \ddot{Y}_j(t)$ for mode j can be calculated from eq.(3). No loss of general, the displacement response of eq.(3) takes the following form

$$Y_j(t) = F_j(t)\sin(\omega_{jD} t - \varphi_j) \tag{16}$$

in which

$$F_j(t) = \frac{-\gamma_j}{\omega_{jD}} e^{-\zeta_j \omega_j t} \left\{ \left[\int_0^t \ddot{v}_g(\tau) e^{\zeta_j \omega_j \tau} \cos(\omega_{jD}\tau) d\tau \right]^2 + \left[\int_0^t \ddot{v}_g(\tau) e^{\zeta_j \omega_j \tau} \sin(\omega_{jD}\tau) d\tau \right]^2 \right\}^{\frac{1}{2}} \tag{17}$$

$$\varphi_j = tg^{-1} \frac{\int_0^t \ddot{v}_g(\tau) e^{\zeta_j \omega_j \tau} \sin(\omega_{jD}\tau) d\tau}{\int_0^t \ddot{v}_g(\tau) e^{\zeta_j \omega_j \tau} \cos(\omega_{jD}\tau) d\tau}$$, is the random phase angle.

$\omega_{jD} = \sqrt{1-\zeta_j^2}\,\omega_j$, is the damped circular frequency.

and the mean square of $Y_j(t)$ is

$$\sigma_{Y_j}^2 = E\left[F_j^2(t)\right] \tag{18}$$

From eqs.(16) and (18), it can be observed that the modal response at time t is mainly constituted by two parts, the vibration strength part $F_j(t) \geq 0$ which controls the amplitude and the periodic function part which controls the sign of the response. As a whole, the modal response is nearly a sinusoidal vibration with frequency ω_{jD}, and mean amplitude σ_{Y_j}.

Thus, the mode superposition has close relationship with the natural periods of the structure. The longer the periods are, the more probable the superposition near the peak modal responses is. The shorter the periods are, the more probable the peak responses of higher modes perfectly correlate with that of the long period modes. For long-span suspension bridges which fundamental period is longer than that of ordinary structures, the maximum responses of the structure are nearly the superposition of several peak modal responses which

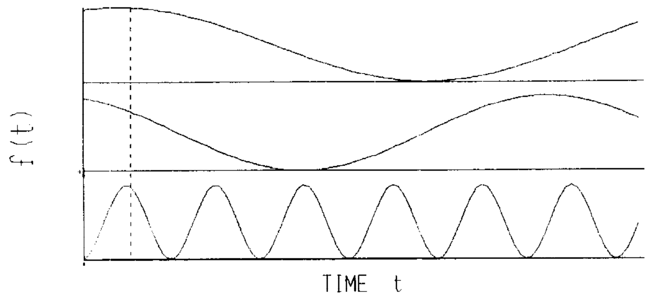

Fig. 1. Simulation of mode-superposition

have great contribution to the total responses (Table 1 and Table 2). Therefore, we can consider that the peak response takes place when the most important two modes (assuming mode k and mode l) have perfect correlation. Thus, we have

$$\cos(\omega_k - \omega_l)t_m = \frac{R_k R_l}{|R_k R_l|} \tag{19}$$

Hence

$$t_m = \frac{\pi}{\omega_k - \omega_l} \quad \text{or} \quad t_m = \frac{2\pi}{\omega_k - \omega_l} \tag{20}$$

RESPONSE SPECTRUM ANALYSES OF SUSPENSION BRIDGES

To assess the validity of the new response spectrum combination method, a comparison between the results of time history analysis and the results of response spectrum analysis with spectra generated from the same input time histories is often used. The ground acceleration used in this study is response spectrum-compatible ground motion of Shantou Bay Bridge, which peak acceleration is 0.2229g and duration 15 seconds (Fig. 2). The acceleration spectrum is computed from the same input ground motion (See Fig. 3).

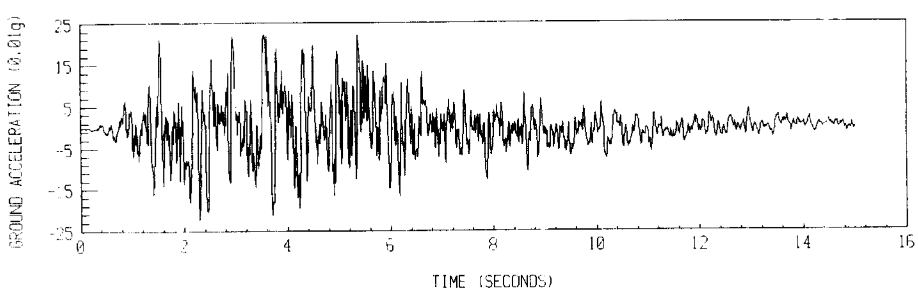

Fig. 2. Simulated earthquake ground motion of Shantou Bay Bridge

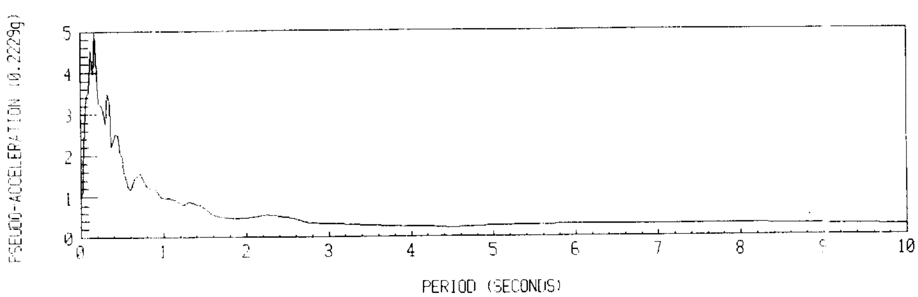

Fig. 3. Acceleration spectrum for generated earthquake of Shantou Bay Bridge

In order to compare, both the response spectrum method and the time history method are carried out on SAP-V. And a series of new response spectrum combination methods such as CQC, DSC, ARC, SMAM and the new method have been used to updated the original response spectrum segment of SAP-V program. On the other hand, the well-known mode-superposition method is used to do time history analysis.

Shantou Bay Bridge

This is a three-span suspension bridge. It has concrete towers, vertical hangers and a prestressed concrete box-deck of 452m main span. Its two side spans (each 154m) are supported by cable (Fig. 4).

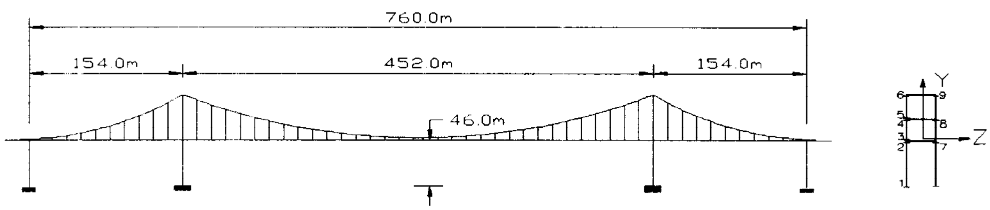

Fig. 4. Model of the Shantou Bay Suspension Bridge

The Table 1 and Fig. 5 give the results from response spectrum analysis and time history analysis, in which the responses based on CQC method, SMAM method, the new method and time history method are listed for the first 50 modes. From table 1, we can see that the new method gives a better result than the CQC and SMAM method. The CQC method significantly underestimates the tower bending moments by 24% while the SMAM

Table 1. Tower Bending Moment of Shantou Bay Bridge (the first 50 modes in-plane, KN-m)

Mode No.	Frequency	Mode-superposition			Response Spectrum		
	ω (rad/s)	contribution	maximum	time (s)	CQC	NEW	SMAM
1	1.137	-10940	-15790	9.02	15530	15530	15530
3	1.434	-27200	-31390	4.89	31520	31520	31520
13	7.889	-103900	-103900	5.49	104300	104300	104300
combination	values Σ	-146200		5.49	110800	152400	158800

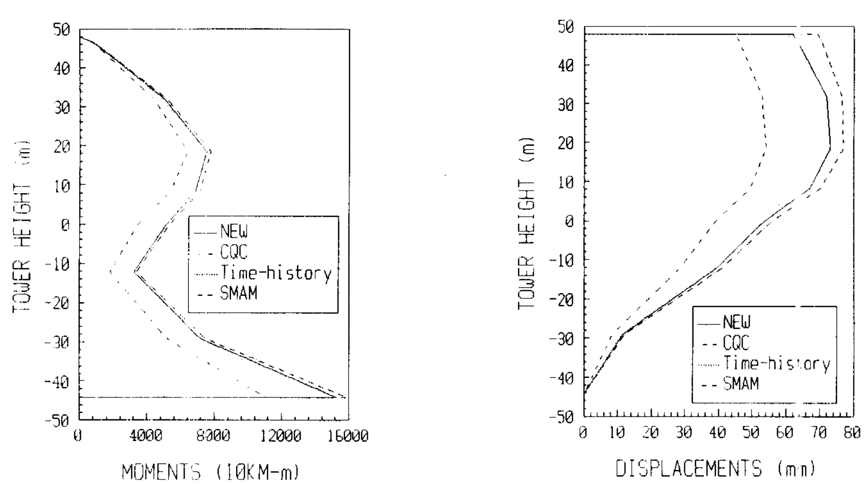

Fig. 5. Longitudinal seismic responses for the tower of Shantou Bay Bridge(the first 50 modes in-plane)

method yields an overestimation by 10%. Across the tower as a whole, it is parent that the four curves from the four methods discussed here have the similar trend. The new method values are very close to time history values elsewhere. However, the CQC and SMAM methods yield grossly discrepancy.

Tiger Gate Bridge

The Tiger Gate Bridge has a main span of 888m. The side spans are supported on piers(Fig. 6). The towers are of concrete, the deck a steel box and the hangers vertical.

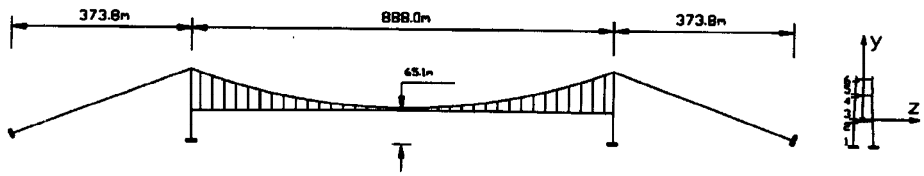

Fig. 6. Model of the Tiger Gate Suspension Bridge

The tower responses of Tiger Gate Bridge abstracted from time history and spectrum analyses for the first 150 modes are given in table 2 and Fig. 7. In the table and figure described above, three response spectrum methods, that is CQC, SMAM and the new method, are compared with the time history method.

Table 2. Tower Bending Moment of Tiger Gate Bridge (the first 150 modes in-plane, KN-m)

Mode No.	Frequency	Mode-superposition			Response Spectrum		
	ω (rad/s)	contribution	maximum	time (s)	CQC	NEW	SMAM
6	2.941	48710	94840	5.96	-91340	-91340	-91340
9	4.598	12120	19480	5.74	-19410	-19410	-19410
11	5.026	250200	252600	5.57	-253300	-253300	-253300
27	17.02	86320	87900	5.52	-88810	-88810	-88810
combination	values Σ	402100		5.53	276600	390500	478300

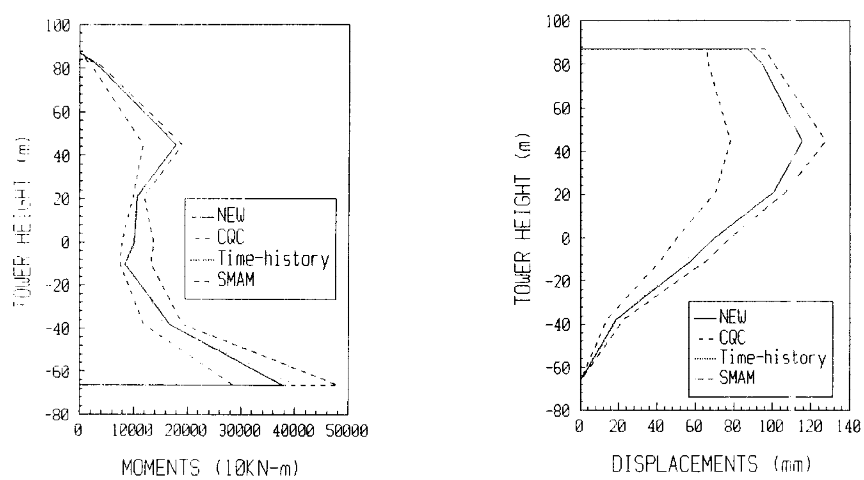

Fig. 7. Longitudinal seismic responses for the tower of Tiger Gate Bridge(the first 150 modes in-plane)

Table 2 tells us that the peak tower bending moment from the new method agrees well with that from time history analysis whereas the CQC value is smaller than time history value by 31% and the SMAM value is greater than time history value by 19%. From Fig. 7 which gives the longitudinal moments and displacements for the whole tower, we also can see that the present new method is the best one among the three response spectrum methods.

CONCLUSIONS

A new response spectrum combination method has been developed in this paper and applied to seismic analyses of two domestic suspension bridges — Shantou Bay Bridge and Tiger Gate Bridge. The comparison among the new method , CQC method, SMAM method and time history method shows that the results for present new method is in agreement with that abstracted from time history method and the discrepancy between them is less than 10%. It also shows that the CQC method underestimates the responses by 25~35% while the SMAM method overestimates the tower responses by 10~20%. From the discussion above, it can be concluded that the present new method can give better results which are satisfactory for suspension bridges. Additionally, to ensure the accuracy of the results, sufficient number of modes should be used for response spectrum analysis of suspension bridges.

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