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SHEAR FAILURE OF REINFORCED CONCRETE BEAMS SUBJECTED TO DIAGONAL TENSION

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ABSTRACT

In most of all previous studies on shear behavior of reinforced concrete beams, the beams were tested on condition that shear force was generated by transverse compressive loading. The shear transfer mechanism greatly depends on the diagonal compressive field in case of such loading condition, so concrete would be significant in the shear resistance. The past proposed equations for shear strength of reinforced concrete members are derived from the test results in the above mentioned loading condition. But there are members that are subjected to shear with the diagonal tension like as footing beams in a high rise building, and the members should be designed considering the shear transfer mechanism depended on the diagonal tension field. The experimental study was carried out to investigate the shear behavior of such members,. In this paper, the influence of the loading conditions in RC beams on the shear resistance and strength is presented.

KEYWORDS

reinforced concrete; shear resistance; shear cracking; transverse reinforcement;
diagonal tension; diagonal compression; arch mechanism; truss mechanism; dowel action;

INTRODUCTION

Most of all previous experimental and analytical studies on shear behavior of reinforced concrete beams were carried out on stress condition of that subjected to diagonal compression field was dominant. When a RC beam is subjected to shear generated by pushing forces at loading points (called type-CC loading), compression stress results in each diagonal concrete strut between each couple of the loading points as the arch action. The understanding of shear resisting mechanism of RC beams, which consists of the arch action by the diagonal concrete strut and the truss action by transverse reinforcements, was resulted from the performance of the beams subjected to shear with type-CC loading. However, some structural members in actual buildings are forced on condition that diagonal tension is dominant, for example a

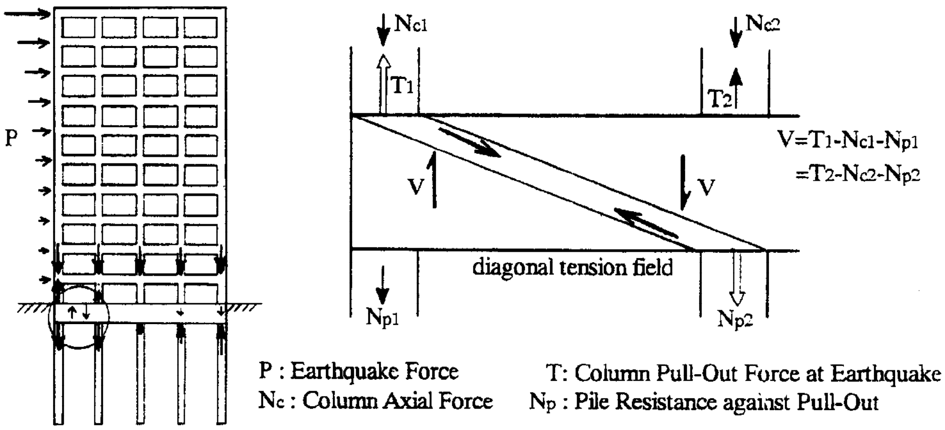


Fig. 1 Stress Condition of Footing Beam

footing beam loaded by pull-out forces at an exterior column and at a pile below the next interior column. The shear resisting mechanism of such members must be remarkably different from that of the preceding members, because the diagonal compression does not appear directly between the bottom of column and the top of pile. In spite of it, such members have been designed according to the theory based on the diagonal compression mechanism, consequently these members might turn to be critical. The rational design method for the members under diagonal tension is necessary to be established.

EXPERIMENTAL WORK

The experimental study was carried out to investigate the influence of loading conditions on the shear strength of RC beams. The footing beam in high-rise building was chosen as an example of the experiment. the beam was subjected to shear with diagonal tension because the additional tension of the exterior column by rotation of the building exceeded the pull-out capacity of exterior pile and the axial force of the column and the exceeded force must be resisted by the next interior pile (see Fig. 1).

The Specimens

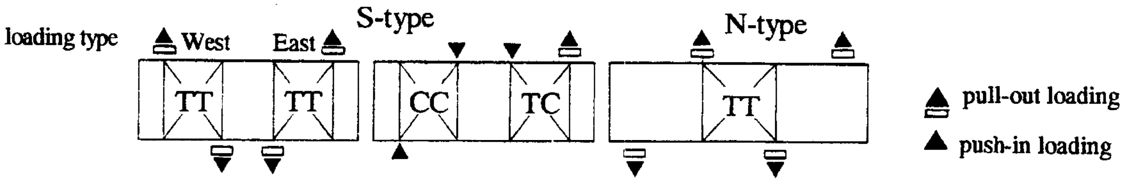
Eighteen specimens were tested, which were about one tenth of the actual size and had difference in loading condition, shear span ratio (M/Vd), concrete compressive strength (Fc), ratio of column or pile depth to shear span (b/a), anchorage length of loading steel bars, the volume (p_w) and strength (σ_{wy}) of transverse reinforcement (see Table 1 and Fig. 2). Specimens had enough beam longitudinal bar volume in their top and bottom that beam yielding would never occur before shear failure.

Loading Condition. Specimens were forced at the positions assumed to be columns and piles, in compression directly or tension through each loading reinforcement. Both the simple beam loading type (S-type loading) and Ohno's loading type (N-type loading proposed by K. Ohno who had supervised the author's group, see Ref.) were adopted in the experiment. Four loading types were considered in combination of simple beam type or Ohno's type and compression or tension. The standard loading condition in the experiment was the combination of simple beam type and tension at both column and pile positions (S-type TT). The others were simple beam type with tension at column and compression at pile

Table 1 Specimens and Properties of Materials

specimen	portion	loading	anchorage model	hook	σ_B	σ_{wy}	p_t	notation variant
BST-1	East West	S-TT S-TT	B A	no no	310	3270	0.91	prototype
BST-2	East West	S-TC S-CC	B A	no no	327	3270	0.91	loading type
BST-3	---	N-TT	B	no	336	3270	0.91	loading type
BST-9	---	N-TT	A	no	337	3270	0.91	loading type
BST-4	E/W	S-TT	B/A	no	322	3270	0.68	M/Vd= 0.5
BST-10	E/W	S-TT	B/A	no	329	3920	0.91	M/Vd=0.75
BST-11	E/W	S-TT	B/A	no	327	3920	0.91	M/Vd=1.5
BST-12	E/W	S-TT	B/A	no	327	3920	0.91	M/Vd=2.5
BST-13	E/W	S-TT	B/A	no	245	3270	0.91	Fc=200(designed)
BST-5	E/W	S-TT	B/A	no	436	3270	0.91	Fc=400(designed)
BST-14	East West	S-TC S-TT	B A	no no	548	3270	1.68	Fc=600(designed)
BST-6	E/W	S-TT	B/A	no	343	3270	0.91	p_w=0%
BST-7	E/W	S-TT	B/A	no	343	3270	0.91	p_w=1.2%
BST-15	East West	S-TT S-TT	A A	no no	225	3270 8960	1.63	p_w=1.8% p_w=0.76%
BST-16	East West	S-TT S-TT	A A	no no	305	3270 8960	1.63	p_w=2.4% p_w=0.89%
BST-17	East West	S-TT S-TT	B A	with with	283	3920	0.91	Anchorage type/ hook
BST-18	East West	S-TT S-TT	C C	no with	334	3920	0.91	Anchorage type/ hook
BST-8	East West	S-TT S-TT	B A	no no	349	3270	0.91	b/a=0.2

note: protoype(BST-1) M/Vd=1.0, Fc=300kgf/cm ^{2} , p_w=0.6%, b/a=0.1
M/Vd : shear span ratio p_t: beam reinforcement ratio,%
concrete compressive strength F_c : (designed) σ_B : (at the test) ,kgf/cm ^{2}
p_w : shear reinforcement ratio,% σ_{wy} : yield strength of shear reinforcement ,kgf/cm ^{2}



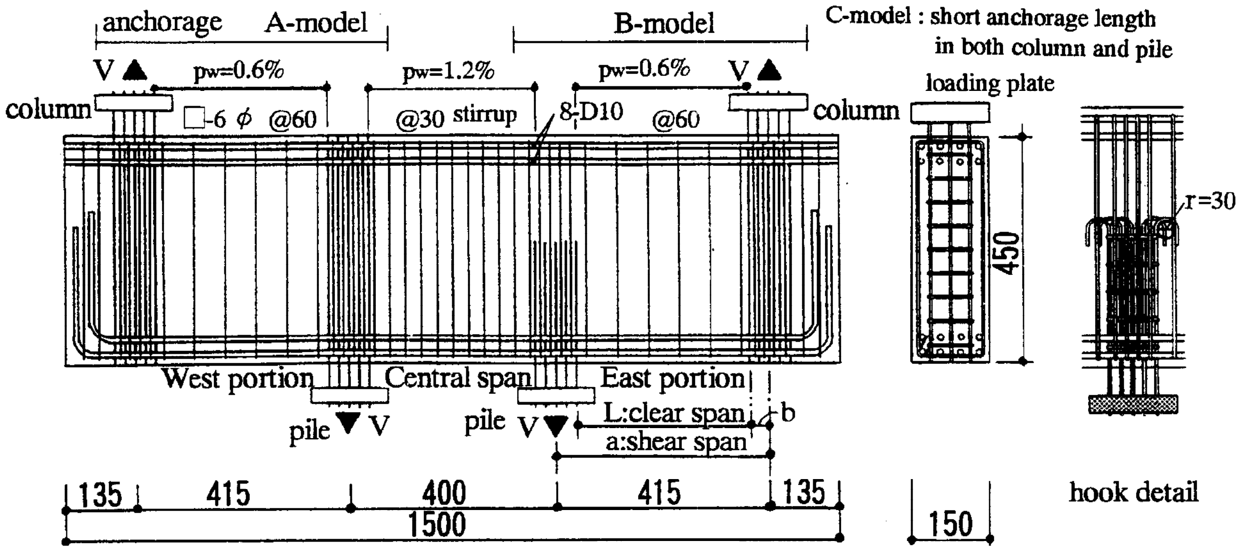
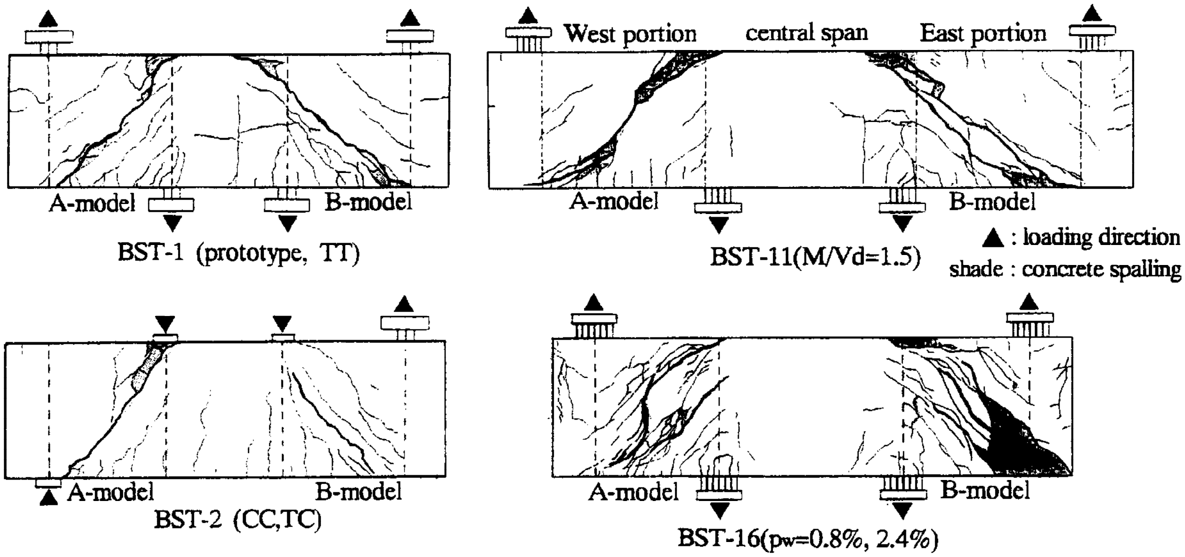


Fig. 2 Specimen Details



BST-1 (prototype, TT)

BST-11(M/Vd=1.5)

BST-2 (CC,TC)

BST-16(p_w=0.8%, 2.4%)

Fig. 3 Crack Pattern after Test

(S-type TC), simple beam type with compression at both points (S-type CC) and Ohno's type with tension at both positions (N-type TT), as shown in Table 1. Repeated loading was given to the specimen in only one direction, which was controlled by the shear distortion between the column and pile positions at 2/1000, 4/1000 and 8/1000 for the first 3 cycles, and at ultimate stage at the 4th cycle. In S-type loading, when one side of specimen (i.e. west or east side) was not failed, such side was reached to ultimate strength by "the extra loading cycle" after strengthen the failed side with steel plates.

Anchorage of Loading Bars. Anchorage length of column bars at both ends in beam was 45cm (75d_b; d_b is bar diameter) in all specimens except for C-model of 24cm (40d_b). That of pile bars had also two variants of 45cm (75d_b ; A-model) and 24cm(40d_b ; B-model, C-model), and 180-degree hooked bars end were used in two specimens 'BST-17, 18'.

EXPERIMENTAL RESULTS AND DISCUSSION

Crack Patterns and Failure

The aspect of typical specimens after the test included the extra loading cycle are shown in Fig. 3. In the S-type TT loading, initial shear cracking was observed in the lower corner close to pile position regardless of anchorage type. After the diagonal cracking penetrated the entire beam section, concrete crashing due to the dowel action of beam bars occurred when the specimen reached the ultimate. The principal diagonal crack did not grow into the central span in the specimen of A-type anchorage, and no shear cracking except initial crack was observed in 'BST-2W' (W : West portion) of S-type CC loading and concrete crashing occurred near the loading position at the ultimate.

Mechanical Behavior

Relationship between Shear Stress and Shear Distortion. Shear stress vs. shear distortion relationship of typical specimens are shown in Fig. 4 as the envelope curves. The behavior of all specimen was almost

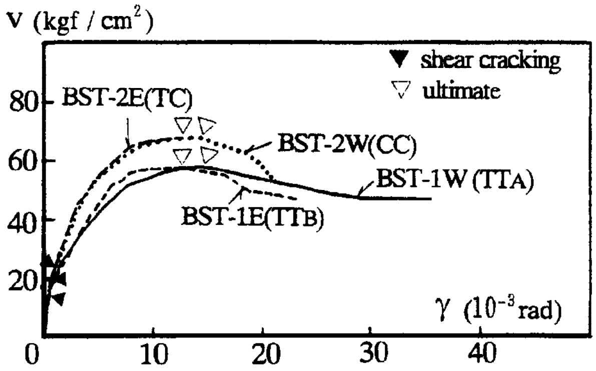


Fig. 4 Shear Stress vs. Distortion Relationship

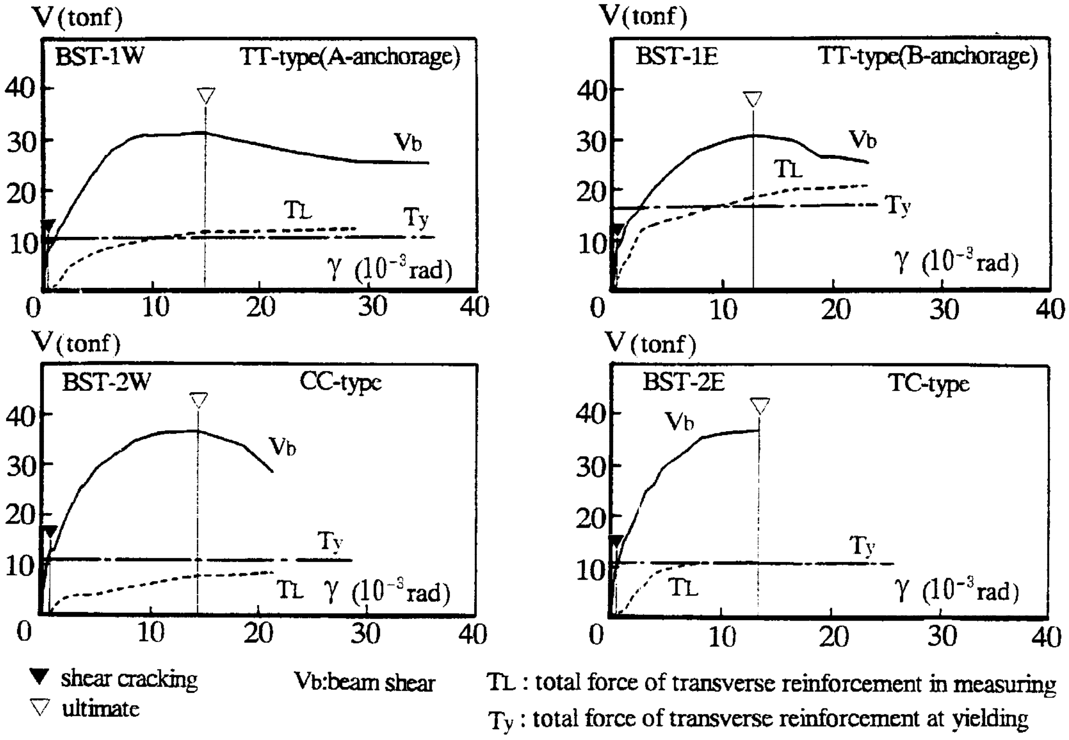


Fig. 5 Contribution of Transverse Reinforcement

similar before shear cracking, but the stiffness of the CC and TC specimens (BST-2E, 2W) became larger than TT specimen (BST-1) and also reached larger strength. However, the degradation of the former two specimens after reaching the ultimate strength are more remarkable than the latter.

Shear Force Resisted by Transverse Reinforcement. The total tensile force acting in transverse reinforcement located within the area where the principal shear cracking penetrated is shown in Fig. 5, as the relationship to the shear distortion. In the figure, V_b, T_L and T_y represent beam shear, the total force of transverse reinforcement measured in the test and that calculated with yield strength, respectively. The ratios of T_L to V_b in the TT specimen (BST-1W, 1E) are larger than that of the CC specimen (BST-2W), so the contribution of transverse reinforcement to the shear resistance is significant in the type-TT loading. The ultimate strength of the TC specimen (BST-2E) is larger than that of the TT specimen (BST-1W), but

TL reached to the yield strength at the ultimate as the TT specimen, while TL of the type-CC specimen did not reach. This results in that the replacement of compression by tension gives rise to the large contribution of transverse reinforcement to the shear resistance of RC beams. The reinforcement contribution was larger in B-model anchorage specimen than that in A-model anchorage.

Strength

Experimental results are shown in Table 2, where strength are expressed in shear stress. The shear strengths obtained from the Ohno and Arakawa's equation (see Table) are also indicated correspondingly.

Strength at Initial Shear Cracking. The ratio of experimental value to calculated value by Eq.(1) is 0.93 for the specimen 'BST-2W' which was subjected to CC-type loading, 0.79 for TC specimens 'BST-2E, BST-14E' and 0.67 for average of all TT specimens. That is, when the loading by compression is replaced by tension, the cracking strength is decreased obviously. Discussion about the influence of other parameters on shear cracking strength in S-type TT loading specimens are followings: 1) The strength increases in proportion to concrete compressive strength, but the rate of increase is smaller than that of type-CC. 2) The smaller the shear span ratio is, the larger the strength is in the same manner as the CC specimen. 3) Transverse reinforcement volume does not influence the strength, but the strength increases apparently as beam longitudinal bar volume increases.

The factor α to modify Eq.(1) is defined for the difference of loading type shown as Fig. 6(a). A computing examination between the experimental and calculated values, Eq. (A) was derived. The comparison with the experiment is shown in Table 2. The calculated values shows in approximate agreement, except for the specimen which was small in shear span ratio or large in beam bar volume.

$$ {}_{cal}v_{cr} = \alpha \times \frac{0.085 \cdot k_c \cdot (F_c + 500)}{M / Vd + 1.7} \tag{A} $$

The Ultimate Shear Strength. The ratio of the experimental result to that calculated by Eq.(3) is 0.82 for specimen 'BST-2W', CC loading. The obvious strength reduction is expected because the arch action could not arise due to the absence of diagonal concrete compression in the specimens subjected to TT loading. The truss mechanism by transverse reinforcement and the dowel action by beam bars are considered for the main resistance mechanism in that case. However the experimental result did not indicate so large reduction in strength, it is considered that the arch action might be produced or the resistance of above two might be greater than in case of CC loading. Discussion about the influence of other parameters on the ultimate shear strength in S-type TT loading specimen are followings: 1) The ultimate strength increases linearly as concrete compressive strength. 2) The strength increases when shear span ratio decreases in the range of M/Vd less than 1.0, because the compressive stress regions which extended like as a cone shape from loading bars of both column and pile overlapped to each other and diagonal compression field was produced. 3) The increase of transverse reinforcement volume and strength gives the increase of the ultimate strength proportionally in the range of $p_w \cdot \sigma_{wy}$ less than $80\,kgf / cm^2$, but it is different that the component resisted by concrete decreases with the increase of $p_w \cdot \sigma_{wy}$ from the characteristic of Eqs.(2) and (3) where the component is constant. 4) The difference of

Table 2 Experimental Results

(unit: kgf/cm²)

variant	specimen	portion	loading	shear cracking strength exp.	cal.(1)	exp./cal.(1)	/cal.(A)	ultimate strength exp.	cal.(2)	cal.(3)	exp./cal.(2)	/cal.(3)	/cal.(B)	
proto type	BST-1	E	TT	(17.0)	25.5	(0.67)	(1.03)	56.6	52.4	79.8	1.08	0.71	1.15	S
		W	TT	(17.0)		(0.67)	(1.03)	57.5			1.10	0.72	1.06	S
loading type	BST-2	E	TC	{20.2}	26.0	{0.78}	{0.94}	67.5	53.8	82.3	1.25	0.82	1.20	S
		W	CC	24.3		0.93	0.93	67.5			1.25	0.82	1.20	S
	BST-3	–	TT	15.9	26.3	0.60	0.93	36.9	54.5	83.7	0.68	0.44	0.63	Jb
		EX	–	–		–	–	50.2			0.92	0.60	0.86	S
	BST-9	–	TT	15.8	26.4	0.60	0.92	52.1	54.6	83.8	0.95	0.62	0.89	S
shear span ratio	BST-4	E	TT	24.8	31.8	0.78	1.20	65.9	86.7	133.2	0.76	0.49	0.93	S
		W	TT	–		–	–	(65.9)			–	–	–	–
	BST-10	E	TT	18.9	28.8	0.66	1.01	64.0	67.1	102.7	0.95	0.62	0.94	S
		W	TT	25.7		0.89	1.37	70.7			1.05	0.69	1.10	S
	BST-11	E	TT	13.7	21.6	0.63	0.98	51.2	45.2	67.0	1.13	0.76	0.87	S
		W	TT	12.6		0.58	0.90	41.6			0.92	0.62	0.99	S
	BST-12	E	TT	14.1	17.3	0.82	1.25	{53.0}	34.6	48.0	{1.53}	{1.10}	{0.87}	–
		W	TT	12.9		0.75	1.15	45.6			1.32	0.95	1.03	S
concrete strength	BST-13	E	TT	16.9	23.5	0.72	1.11	47.1	47.0	70.0	1.00	0.67	1.02	S
		W	TT	14.9		0.63	0.98	51.9			1.10	0.74	1.11	S
	BST-5	E	TT	18.4	29.5	0.62	0.96	61.3	62.7	98.7	0.98	0.62	1.10	S
		W	TT	18.4		0.62	0.96	(65.3)			–	–	–	–
	BST-14	E	TC	26.3	33.0	0.80	0.96	89.6	80.6	131.2	1.11	0.68	0.97	S
		W	TT	22.3		0.68	1.04	90.0			1.12	0.69	0.97	S
shear reinforcement	BST-6	E	TT	15.4	26.5	0.58	0.89	38.5	43.1	78.5	0.89	0.49	0.93	S
		W	TT	16.5		0.62	0.96	38.5			0.89	0.49	0.95	S
	BST-7	E	TT	18.4	26.3	0.70	1.08	67.0	59.3	85.9	1.13	0.78	1.16	Ja
		W	TT	16.3		0.62	0.95	(71.6)			(1.21)	(0.83)	1.04	–
	BST-15	E	TT	20.9	22.8	0.92	1.41	64.3	58.6	80.1	1.10	0.80	0.89	S
		W	TT	20.9		0.92	1.41	79.7	60.5	81.1	1.32	0.98	1.02	S
	BST-16	E	TT	23.5	25.3	0.93	1.43	93.4	69.3	95.5	1.35	0.98	1.09	S
		W	TT	20.6		0.81	1.25	75.2	69.8	95.8	1.08	0.78	0.86	S
anchorage	BST-17	E	TT	16.7	24.7	0.68	1.04	56.1	51.3	76.3	1.09	0.74	1.03	S
		W	TT	15.8		0.64	0.98	59.3			1.16	0.78	1.08	S
	BST-18	E	TT	17.3	26.3	0.66	1.01	59.3	55.5	84.0	1.07	0.71	1.04	B
		W	TT	20.2		0.79	1.18	64.4			1.16	0.77	1.13	S
b	BST-8	E	TT	14.7	26.7	0.55	0.85	(53.6)	55.6	85.6	(0.96)	(0.63)	(1.30)	–
a		W	TT	15.1		0.57	0.87	[62.6]			[1.13]	[0.73]	[1.16]	S

shear cracking	ultimate strength	failure mode
( ) : uncertain due to loading manner	( ) : stop loading due to the jack capacity	S : shear failure
{ } : decline of the flexural crack	{ } : beam yielding	Ja : pile bars pull-out
--- : mismesuring	[ ] : shear failure with concrete compressive fracture at the opposite side of pile	Jb : column bar pull-out
		B : beam yielding

■the equations for estimation

(1) shear cracking strength

$$ {}_{cal}V_{cr} = \frac{0.085 \cdot k_c (500 + \sigma_B)}{M/Vd + 1.7} $$

(2) shear strength for N-type loading

$$ {}_{cal}V_u = \frac{0.115 \cdot k_u \cdot k_p (\sigma_B + 180)}{M/Vd + 0.12} + 2.7\sqrt{p_w \cdot \sigma_{wy}} $$

(3) shear strength for S-type loading

$$ {}_{cal}V_u = \frac{0.23 \cdot k_u \cdot k_p (\sigma_B + 180)}{M/Vd + 0.23} + 1.4\sqrt{p_w \cdot \sigma_{wy}} $$

d : effective depth (cm) j : moment arm (=7d/8)

σ_B : concrete compressive strength (kgf/cm^2)

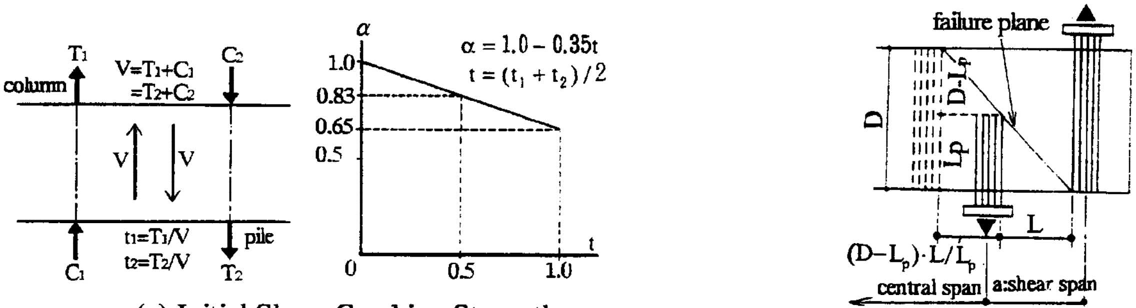
M/Vd : shear span ratio

k_c, k_u : coefficient for scale effect (= unity in this experiment)

p_t : beam bar ratio $k_p = 0.82(100 \cdot p_t)^{0.23}$

p_w : shear reinforcement ratio

σ_{wy} : yield strength of shear reinforcement (kgf/cm^2)



(a) Initial Shear Cracking Strength

(b) Failure Model for Estimation

Fig. 6 Application for the Past Proposed Equations

anchorage type gives no influence on the results. That is, the contribution of truss mechanism is increased because the transverse reinforcement in the central span became effective for shear resistance in the specimens of B and C-model anchorage (short length).

From general consideration of results mentioned above, the equation predicting the shear ultimate strength for S-type TT loading is derived as follow.

$$_{cal}V_{bu} = T_c + T_w \qquad \text{(B)}$$

where

$$T_c = 0.14 \cdot k_u \cdot k_p \cdot k_s \cdot (1-k_w) \cdot \sigma_B \cdot b \cdot j \qquad T_w = p_w \cdot \sigma_{wy} \cdot b \cdot j \cdot \cot\phi$$

$$k_s = 1.26 \cdot (1.89 - M/Vd) \quad [M/Vd \leq 1.5]$$

$$k_w = 0.0013 \cdot (p_w \cdot \sigma_{wy})^{1.427} \quad , \quad \cot\phi = L/j \leq 1.0 \qquad \text{(others ref. Table 2)}$$

The strength in case of the short anchorage like as B or C-model is evaluated by the equation through the procedure shown in Fig. 6(b). The calculated values by Eq.(B) shows good agreement with the experiment. It is said that shear transfer mechanism in RC beams is depended on the loading type, and if shear forces are generated by tension, the truss mechanism becomes significant in shear resistance because concrete diagonal compressive strut would not be formed apparently comparing with in case of CC loading.

CONCLUSIONS

The reinforced concrete beams subjected to shear on condition that diagonal tension is dominant behaved different in shear cracking strength, ultimate shear strength and stress transfer mechanism from those in diagonal compression field. In particular, there was remarkable reduction in shear cracking strength of almost 30% from the past proposed equation derived from the compressive loading test results. Therefore, such members have to be designed on the basis of the proposed evaluation of shear strength.

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