

# ANALYSIS OF REINFORCED CONCRETE INTERIOR BEAM-TO-COLUMN-JOINT SUBASSEMBLAGE WITH POOR BOND SUBJECTED TO LATERAL LOAD

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## ABSTRACT

This paper proposes a model to derive the failure criteria of the 2-D reinforced concrete beam-to-column joint subassembly, considering the bond deterioration of the reinforcement through the joint core. This criteria is used to derive a analytical equation to predict a maximum joint shear in beam-column subassembly. A beam-column-joint subassembly is modelled by strut and ties model. For the purpose of obtaining strength due to concrete failure, the longitudinal reinforcement in beams and columns are assumed to have infinite large strength. Concrete is assumed to be plastic material with reduced compressive strength. The maximum joint shear is derived based on the theory of plasticity and the lower bound theorem. The ultimate joint shear was predicted as a function of bond deterioration. As the bond in joint deteriorate progresses, ultimate joint shear decreases. The major predictions of this model are ; i) Ultimate joint shear is not constant and affected by bond capacity, ii) The ultimate joint shear decrease due to thrust force in beam, as in case of unbonded prestressed beam.

## KEYWORD

beam-column joint; reinforced concrete, joint shear failure; bond strength; theory of plasticity; lower bound theorem; bond deterioration,

## INTRODUCTION

In earthquake resistant design of reinforced concrete moment resisting frames, brittle diagonal failure of beam-column-joint should be prevented. ACI Committee 352 proposed design procedure (1976) based on the concept of limiting joint shear stress, where design joint shear should be smaller than ultimate joint shear capacity to prevent joint shear failure before beams yields. Although, the values of joint shears to prevent joint shear failure has not been obtained theoretically and the most design codes adapt empirical equations. The objective of this paper is to demonstrate different approach to understand of phenomena of joint shear failure not by concept of joint shear strength.

Paulay *et al.* (1978) showed a model depicting the shear transfer mechanism in beam-column-joint in which the joint shear is transferred by combination of two mechanism; (a) diagonal strut mechanism and (b) truss mechanism. The model is suitable to take into consideration the equilibrium of the resultant forces from beams and column within joint. However, the model neglect a concrete compressive stress continuity between beam end and joint panel for simplification. So the model may overlook the failure of over reinforced beam at beam ends.

The depth of concrete compressive stress block at beam ends is generally different from that predicted by flexural theory, provided bond deteriorate through joint core. By examining the test specimens reported in past, we found that the beam-column-joint subassembly specimens failing in joint shear are heavily reinforced and longitudinal bars of large diameter is used. The reason is to put a quite large amount of reinforcement in the small beam section. Hence the bond stress of those specimens are usually much higher than actual structure. It is also well known that the compressive stress in compressive reinforcement of beams at the column face decreases as the bond in joint core deteriorate. It means that the compressive

reinforcements do not act as compressive reinforcement and the increase of concrete stress block depth need to be larger. It may leads to compressive failure at beam end. Therefore, the compressive failure of beam need to be examined provided the bond of beam longitudinal bar is poor.

This study places a special emphasize on the compressive failure of beam end. This study derives the failure criteria of the concrete failure of beam, considering the bond deterioration of the reinforcement through the joint core. This criteria is used to derive a analytical equation to predict a maximum joint shear in beam-column subassemblage. The calculated joint shears are compared to joint shear strengths obtained in tests in literatures reported as joint shear failure.

## MODELLING OF TWO-DIMENSIONAL BEAM-COLUMN-JOINT SUBASSEMBLAGE

A consistent modelling tool is necessary for a beam-column-joint subassemblage in order to consider the equilibrium in the members as well as boundary between them. Thus, the struts and ties model (Marti, 1985) are employed. To make the problem simple, beam-column-joint subassemblage is modelled as a two-dimensional member with uniform thickness  $t$ . For the purpose of obtaining strength due to concrete failure, the longitudinal reinforcement in beams and columns are assumed to have infinite large strength. Concrete is assumed to be plastic material with reduced compressive strength. The maximum joint shear is derived based on the theory of plasticity and the lower bound theorem.

### *Statically Permissible Stress Field*

Lower bound theorem in the plastic theory is applied to crucial beam-column-subassemblage subjected to equivalent statically seismic lateral force to predict the maximum joint shear. All of the longitudinal reinforcing bars in beams are assumed to be laid straight through the joint core. Figure 1 shows some possible statically permissible stress field. The models in Fig. 1 assumes symmetrical beam-column-joint as well as the followings;

- a) The distances between column center to the contraflexure points in beams are same for the left beam and the right beam. The distances between beam center to the contra flexure points in columns are same as well.
- b) The longitudinal reinforcement in beams and columns go through the joint.
- c) Thickness is uniform for beams, columns, and joint.
- d) Story shear is transferred by only column, thus no thrust force in beams.
- e) The shears in the beams are identical.
- f) The equilibrium in the joint shear panel is maintained by redistribution of stress after concrete cracking and bond unloading. The typical example for redistribution is shown in Fig. 1(a) and (b), where the shears from beams and columns are in equilibrium in beam-column-joint, where the compression, shear and tensile forces transferred from beams and columns are equilibrated by two mechanism described as follows;

(A) a pair of sub-strut mechanism (IJBH and EDFG) with tensile force in column longitudinal bars

(B) main diagonal strut mechanism (ABCD)

For example, increase in axial load cause the main diagonal strut mechanism smaller and increase the contributions of the sub-struts mechanism as shown in Fig. 1(b).

Obviously, the assumption f) is a different approach from many researcher. Under assumptions above, the maximum story shear transferred by the statically permissible stress field is derived as follows.

Joint shear  $V_j$  of crucial beam-column-joint without prestressing in beam is usually defined as follow.

$$V_j = T_s + C_s + C_c - V_c \quad (1)$$

where,  $T_s$ : tensile force in top reinforcing bars at a beam end,  $C_s$ : compressive force in the top reinforcing bars at the opposite beam end,  $C_c$ : compressive force in concrete at the opposite beam end and  $V_c$ : column shear.

The compressive force in concrete  $C_c$  is the function of the  $x$ , which denotes height of the strut in beam at beam ends and written as Eq. (2)

$$C_c = bx\sigma_c \quad (2)$$

By applying the lower bound theorem of plastic theory to the beam modelled by single strut as shown Fig. 1, it is obvious that the maximum shear in beam and the maximum possible contribution of concrete compression at beam ends is obtained when Eq. (3) is satisfied.

$$x = \frac{D_B}{2} \quad (3)$$

where,  $\sigma_0$ : concrete compressive stress in concrete strut,  $b$ : beam width and column width (thickness of the member),  $D_B$ : total beam height.

Concrete crushing occurs if Eq. (3) is satisfied. So compressive force  $C_c$  in concrete at the concrete failure is derived as Eq. (4) by substituting Eq. (3) into Eq. (2).

$$C_c = \frac{1}{2} b D_B \sigma_c \quad (4)$$

The compressive force  $C_c$  cannot exceed the value in Eq. (4) and if loading is continued beyond the point of Eq. (4), concrete crushing occurs. Hence, the three unknown variable remaining in Eq. (1) are  $C_s$ ,  $T_s$  and  $V_j$ . We use here three equations to determine these values.

The first equation is equation for equilibrium in beam force. We had assumed that the axial force in beam is zero and beam-column joint is symmetrical. So, the resultant of concrete compression at beam ends  $C'_c$  (positive in compression), resultant of compressive force in tensile reinforcement at the beam end  $C_s$  and the resultant of tensile force in tensile reinforcement  $T_s$  (positive in tension) should be in equilibrium. Eq. (4) shown the equilibrium of the section forces at the end of beam in horizontal direction.

$$C_c + C_s = T_s \quad (5)$$

The second equation is an assumption of relation between  $C_s$  and  $T_s$ . It is assumed to be simply modelled as Eq. (6).

$$C_s = -\alpha T_s \quad (6)$$

Hereafter, a variable  $\alpha$  is introduced so that the effect of bond deterioration is included in this model. Equation (8) is rewritten from Eq. (7) as a ratio of  $C_s$  and  $T_s$ .

$$\alpha = -\frac{C_s}{T_s} \quad (7)$$

Figure 2 depicts the relation between the value  $\alpha$  and the stress conditions in longitudinal reinforcement. If the bond is perfectly lost through the beam-column joint, then bond stress along longitudinal reinforcing bars becomes uniformly tensile and thus the value of  $\alpha$  is estimated as 1.0 as shown in Fig.2 (c). This is extreme case realized only by test or finite element analysis with debonded reinforcement. When the value of  $\alpha$  is exactly zero, it means the stress in the longitudinal reinforcement in compression fibre is zero, at the end of the beam as shown in Fig. 2(b). On the contrary, In case bond is good enough, the value of  $\alpha$  may be less than zero as shown in Fig. 2(a). As the value  $\alpha$  corresponds to the bond deterioration,  $\alpha$  is hereafter called "bond deterioration factor."

The third equation is derived from the equilibrium of tensile force in reinforcement and column shear. Taking into it consideration that the equilibrium of the free-body of column as shown Fig. 3, the column shear  $V_c$  is calculated from the tensile force in longitudinal reinforcement  $T_s$ .

$$V_c = \frac{l_c}{L_c} \frac{L_B}{L_B - D_c} (2T_s) = 2\lambda T_s \quad (8)$$

where,  $l_c$ : the distance from tensile reinforcement to compressive resultant in beam,  $L_c$ : the distance between the contra flexure points of columns,  $L_B$ : the distance between the contra flexure points of beams,  $D_c$ : the total depth of column.

By substituting Eq. (4) and (6) into Eq. (5), we obtain the Eq. (9).

$$\frac{1}{2} b D_B \sigma_c - \alpha T_s = T_s \quad (9)$$

Therefore, the tensile force in longitudinal reinforcement  $T_s$  at the beam end, at which the beam-column-joint subassemblage causes concrete crushing, is obtained by Eq. (10) (positive in tension).

$$T_s = \frac{1}{1 + \alpha} \frac{b D_B \sigma_c}{2} \quad (10)$$

By substituting Eqs. (8) and (10) into the definition of joint shear Eq. (1), the ultimate joint shear  $V_j$  at concrete crushing is derived as Eq. (11).

$$V_j = T_s + C_s + C_c - V_c = 2T_s - V_c = (1 - \lambda) \frac{1}{1 + \alpha} bD_b \sigma_c \quad (11)$$

where,  $\lambda : (l_c / L_c)(L_b / (L_b - D_c))$ .

The result of Eq. (11) indicates physical meanings as follows:

- The ultimate joint shear changes as a function of bond deterioration factor. As the bond in joint deteriorate progresses, the value  $\alpha$  increases from -1 to 1 and the ultimate joint shear decreases. For example, after of the reversals of plastic deformation the bond stress decreases and the value  $\alpha$  increase because compressive stress in compressive reinforcement changed to tensile. Thus the ultimate joint shear decrease. The least joint shear is obtained as  $(1 - \lambda)bD_b \sigma_c / 2$  by substituting 1.0 to  $\alpha$ . This may be verified by a test using debonded reinforcement or a finite element analysis assuming bond link without strength.
- The ultimate joint shear is affected by the length of column and beams slightly.
- The thrust force in beam affects the ultimate joint shear. For example, in case of compression, the ultimate joint shear becomes smaller than that of no thrust force in beams. It is easily derived considering the effect of thrust force in Eq. (5). Therefore, provided beam-column-joint subassembly having unprestressed reinforcement and thrust for using debonded prestressing tendon, the reduction of ultimate joint shear is predicted.

In traditional paradigm of joint shear design of reinforced concrete beam column joint, the ultimate joint shear is assumed to be constant and it is not the function of column shear, bond deterioration of beam reinforcement nor beam prestressing. But the result of this model clearly contradicts to the idea.

### *Modelling of Failure Criteria for Concrete, Steel and Bond deterioration factor*

The following assumptions for failure criteria for materials are used to apply plastic theory.

**Concrete:** The square failure criteria are used as shown in Fig. 4, where tensile strength is zero, and the increase of compressive strength due to biaxial compression is neglected. The effective concrete compressive strength is reduced from concrete cylinder compressive strength using the Eq. (12) which had been proposed by CEB [7]. The Eq. (13) gives good prediction for wide range of concrete strength including high strength concrete. Figure 5 shows the relation of strength reduction factor and concrete compressive strength.

$$\sigma_c = \nu \sigma_B = 3.67 \sigma_B^{2/3} \quad \text{unit in kgf/cm}^2 \quad (12)$$

where,  $\sigma_c$ : effective concrete compressive strength,  $\nu$ : strength reduction factor,  $\sigma_B$ : concrete cylinder compressive strength.

When the beam-column-joint specimen has a transverse beams which secure confinement of joint core concrete by longitudinal reinforcement in transverse beams, the increase of ultimate joint shear due to confinement is take into account by multiplying a factor  $k$  to a effective concrete strength. The value 1.6 for the factor  $\kappa$  is empirical value.

$$k = 1 + \kappa \frac{p_w f_{wy} + p_g f_{sy}}{\sigma_B} \quad (13)$$

where,  $f_y$ : tensile yield strength of longitudinal reinforcement,  $f_{sy}$ : tensile yield strength of joint core transverse reinforcement,  $p_g$ : the ratio of total sectional area of transverse beam longitudinal reinforcing bar going through the joint core to sectional area of transverse beam and  $p_w$ : the ratio of total sectional area of joint core transverse reinforcement to sectional area of transverse beam.

**Reinforcing Bars:** The longitudinal bars in beams and columns are assumed to have infinite high strength in tension and compression.

**Bond Deterioration Factor  $\alpha$ :** The value  $\alpha$  has already defined in Eq. (7). There is no existing model to predict the value. The value is evaluated only by experimental measurement. So, in this study, a model represent by Eq. (14) is arbitrary chosen to predict the value  $\alpha$ . It is defined as a function of Bond Index  $\mu$ , a non-dimensional index for induced bond stress, adopted in commentary of AIJ's recently revised guideline for earthquake resistant design of reinforced concrete building based on a capacity design concept (AIJ, 1991). Figure 6 shows the relation of bond factor  $\alpha$  and bond index  $\mu$ .

$$\alpha = \begin{cases} 0.1\mu - 1 & (0 < \mu \leq 20) \\ 1 & (20 < \mu) \end{cases} \quad (14)$$

The bond index  $\mu$  is defined as shown in Eq. (15). In the AIJ's guideline (AIJ, 1991), the bar diameter is recommended to be designed so that  $\mu$  is less than 10 to prevent bond deterioration which leads to significant

pinching in hysteresis loop under cyclic loading after beams yield.

$$\mu = \frac{d_B \sigma_y}{D_c \sqrt{\sigma_B}} \quad (\text{unit in kgf/cm}^2) \quad (15)$$

where,  $d_B$ : nominal diameter of longitudinal beam reinforcement through joint,  $\sigma_y$ : tensile yield strength of longitudinal beam reinforcement ( in this study, the realistic value ( $= T_s / \Sigma a_t$ ) is used for unrealistic infinite high strength assumption for beam reinforcing bar ),  $D_c$ : depth of column,  $\sigma_B$ : compressive strength of concrete cylinder. It should be noticed that the bond factor  $\alpha$  is calculated from Eqs. (14) and (15), where the tensile force in beam reinforcement  $T_s$  is unknown. Thus iterative try-and-error basis procedure should be taken for actual calculation of Eq. (11).

## COMPARISON OF PREDICTED ULTIMATE JOINT SHEAR AND TEST RESULTS

In this section, the predicted ultimate joint shear calculated using Eq. (11) are compared with test results, and several equations proposed before.

### *Specimens and methods*

In total 113 beam-column-joint subassemblage specimens were examined from 28 references. Among them 46 specimens were reported to be failed in joint shear before beam or column had yielded. The specimens excluded were a) specimens in which steel fiber reinforced concrete is used, and b) specimen which are with more than two layers longitudinal reinforcing bars in beams. The observed maximum joint shear was calculated from the maximum story shear reported with assumptions that the  $j$ ; distance of two resultant forces  $C$  and  $T$  at a beam end is equal to the  $(7/8)d$ ;  $d$ : the beam effective depth. This definition leads to slightly small value compared with the definition of Eq. (1). Because the distance  $j$  of over reinforced concrete beam is smaller than the assumed value of  $(7/8)d$ . The averaged joint shear stress was derived from the ultimate joint shear divided by the product of column depth  $D_c$  and effective joint width  $b_{eq}$  defined in Eq. (16). When the column width is not identical to beam width, joint effective width  $b_{eq}$  is used. As shown in Fig. 6, the joint effective width is average of column width and beam width.

$$b_{eq} = \frac{1}{2}(b_B + b_C) \quad (16)$$

where,  $b_B$ : beam width, and  $b_C$ : column width.

### *Correlation of joint strength obtained by analysis and test*

The reliability of ultimate joint shear predicted by the Eq. (17) is compared with empirical equation as follows;

$$\tau_j = 5.7\sqrt{\sigma_B} \quad \text{unit in kgf/cm}^2 \quad (17)$$

Figure 9(a) shows the plots of ultimate joint shear predicted and calculated by Eq. (2.11). Figure 8(b) shows the predictions obtained from Eq. (18). The correlation of the model to experiments were 0.95 in average, and 0.10 in standard deviation. On the other hand, for the Eq. (12) the average is 0.99 and the standard deviation is 0.12 respectively.

## CONCLUSION

Interior beam-column-joints usually have longitudinal reinforcing bars going through the joint core. Most of the tests on beam-column-joint subassemblages aiming to fail in joint shear usually have a large amount of longitudinal beam rebars and sometimes large diameter reinforcement are used so as to prevent congested arrangement of rebars in joint. So the bond stress condition is severer. If the bond stress exceeds the bond strength, bond deterioration will occur, which cause the compressive reinforcement in tensile at the critical section. Taking into consideration of these situation, it is possible to occur a compressive failure of beam concrete even if the beam have longitudinal reinforcement in compressive fibre.

This paper focuses on this type of beam column joint failure criteria. Plastic theory and the strut and tie model were applied to derive the maximum joint shear which is attained at the compressive failure of beam, taking into account the bond deterioration. This kind of failure have not been considered before.

The major predictions of this model are

- i) Ultimate joint shear is not constant and affected by bond capacity.

ii) Ultimate joint shear is affected by column length

iii) The ultimate joint shear decrease due to thrust force in beam, as in case of unbond prestressed beam.

These predictions contradict to existing paradigm of joint shear design that ultimate joint shear is constant and is not effected by column shear, bond deterioration nor beam prestressing.

This model was used to predict the ultimate joint shear of 46 tests of beam-column-joint subassemblage reported to fail in joint shear before beams yield. Although, the prediction is based on the criteria of the balanced failure of the beam, and not based on the equilibrium of shear in joint shear. The predicted joint shear strength show a good correlation with the test results.

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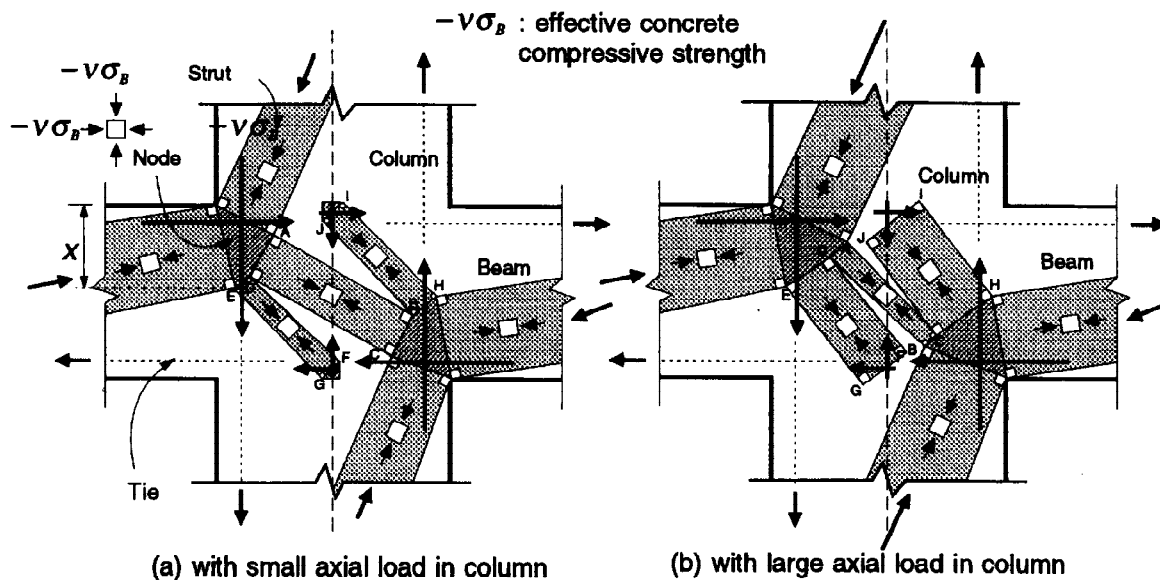


Fig. 1: Permissible stress field for interior R/C beam-column joint

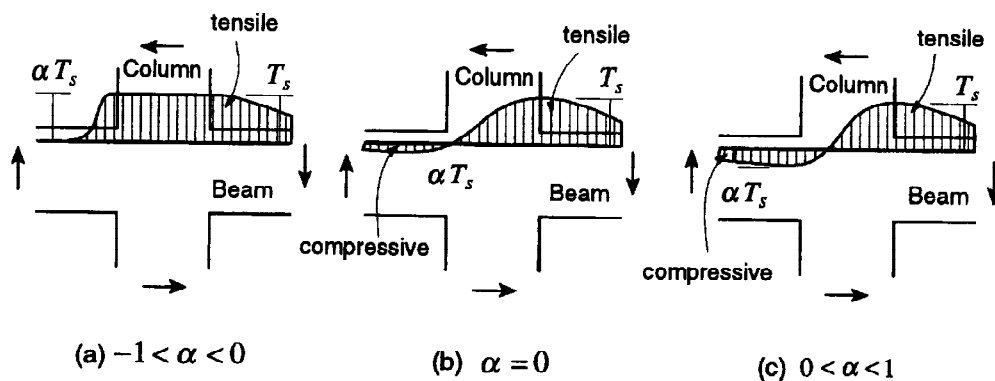


Fig. 2 : Definition of bond factor  $\alpha$

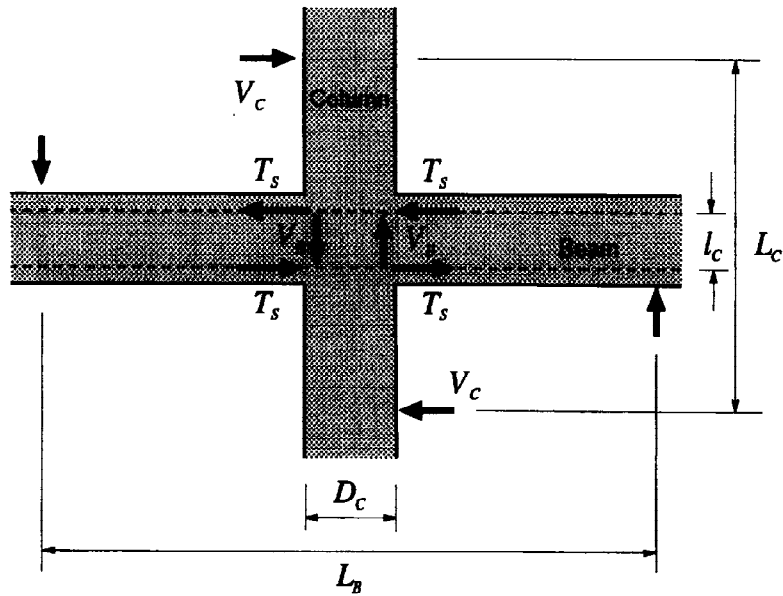


Fig. 3 : Equilibrium of Column shear and force in beam bars

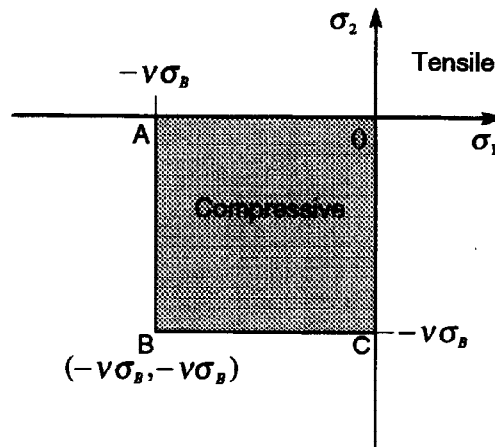


Fig. 4 : Failure Locus for Concrete

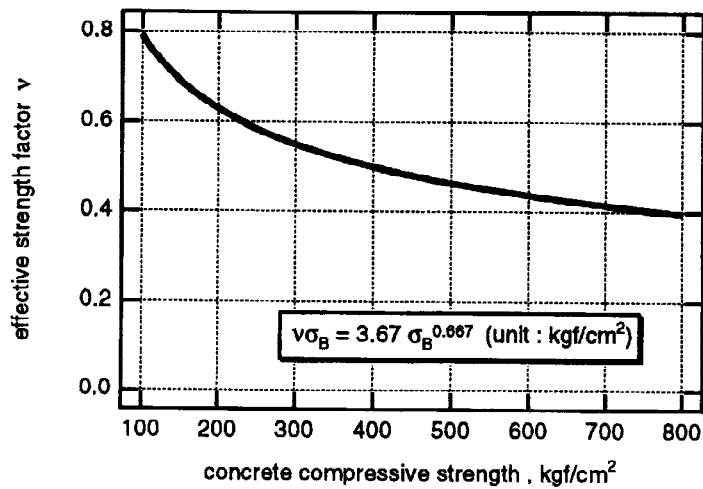


Fig. 5 : Effective strength reduction of Concrete

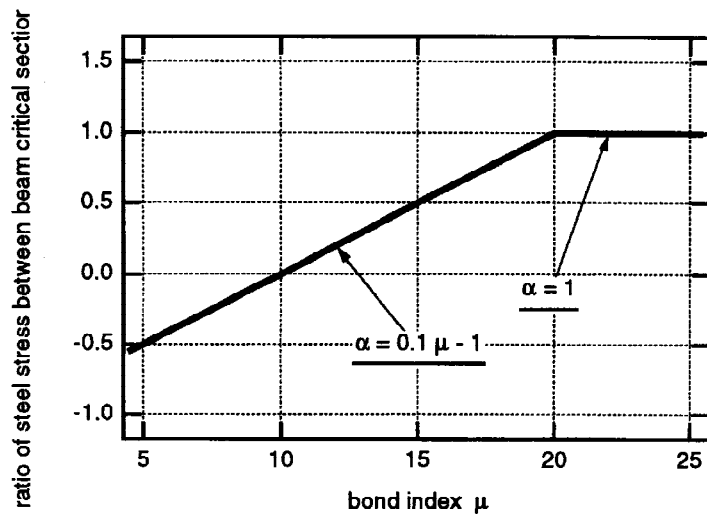


Fig. 6 : Modeling of relation between Bond Index  $\mu$  and Bond Factor  $\alpha$

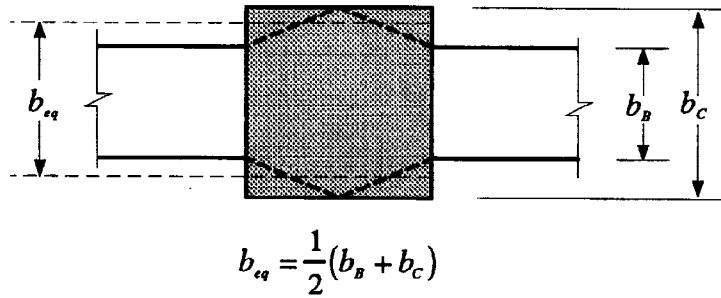
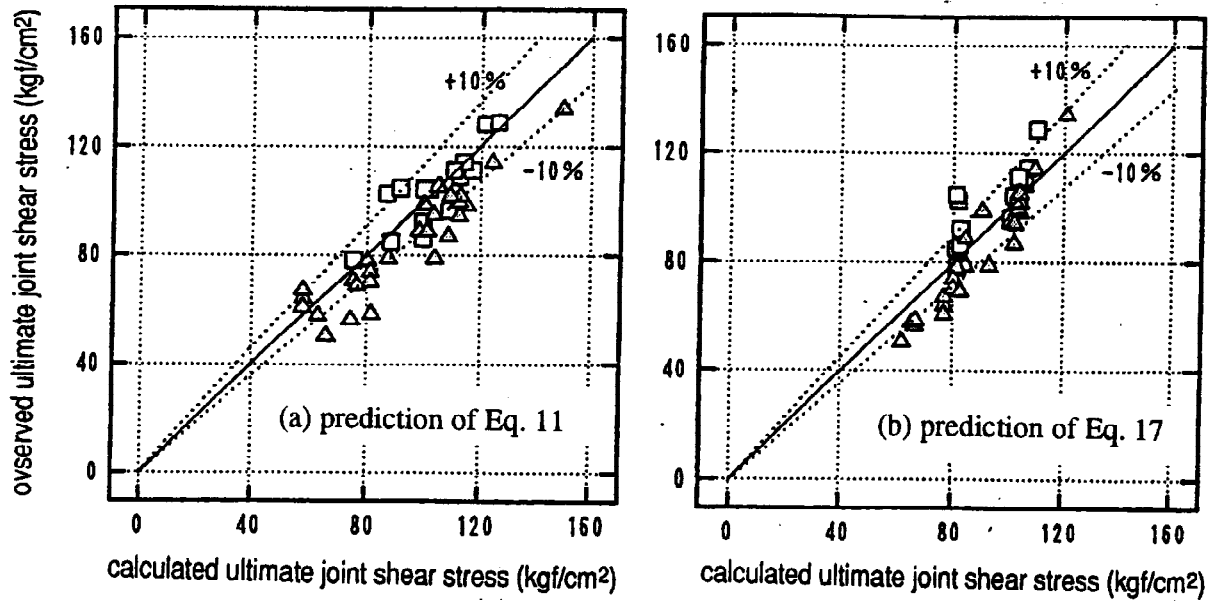


Fig. 7 : Definition of the effective beam-column-joint width



- transverse confining reinforcement exceeds 1.0%
- △ others

Fig. 8 : Comparison of calculated and observed joint shear capacity