

## IDENTIFICATION OF DAMPING OF STRUCTURES IN INELASTIC RANGE

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#### ABSTRACT

Based on the data of dynamic-static comparison tests of three pairs of small steel frames, damping in dynamic response time-history analysis of structures is investigated. The sum of squares of displacement response SSD, the maximum displacement response MD, and the hysteretic energy response EH are selected as equivalent objects in damping identification. Four developed Rayleigh-type damping representations are considered. It is shown that the damping features of structures are independent of stress or deformation states as soon as the actual hysteretic restoring forces of structures are used in dynamic analysis.

### **KEYWORDS**

Damping models; damping ratio; damping identification; dynamic time-history analysis; dynamic-static comparison test; inelastic dynamic response.

### INTRODUCTION

It is assumed in earthquake response analysis that the energy absorbed by a structure is dissipated in part by inelastic deformation of the structure, and in part by damping, due to mathematical simplicity. Actually, damping is just a mathematic model for equivalent response of the structure, and damping information can only be obtained by system identification from the data of input and output of the structure. Therefore, damping is related to both the equivalent object and analytical method used in damping identification, as well as to phsical properties of the structure.

Traditional identification methods of structural damping such as the logarithmic decrement method, the half-power band-width method, and the Fourier transform techniques of response transfer functions, are based on linear-elastic vibration theory. They are quite effective in identifying damping of structures in elastic range, but when they are applied to structures in inelastic range, the result that the damping increases as the amplitude of structural response increases, is always got (Newmark and Hall, 1969; Hart and Vasudevan, 1975). Obviously, such a damping ratio is not suitable for inelastic time-history response analysis in which the actual hysteretic restoring force is used. Traditional damping identification methods take only the

maximum displacement response as equivalent object, that is, the damping identified by these methods is maximum displacement equivalent damping, it remains a problem whether the displacement time-history and the hysteretic energy analysed with such a damping are creditable. Furthermore, recent results (Léger and Dussult, 1992) show that the effect of different damping models on structural responses in inelastic time-history analysis is not negligible. But how to select damping models and corresponding damping ratio values in inelastic time-history analysis remains a problem, due to the lack of convincing data of structure tests.

Based on the data of dynamic-static comparison tests, damping in earthquake response time-history analysis, especially in inelastic time-history analysis, is investigated in this paper.

#### TEST CONFIGURATION

### **Fundamentals**

Through dynamic test only inertia force, that is , the sum of damping and restoring force can be measured. It is impossible to distinguish between the two. In this study, dynamic tests and corresponding static tests were performed to solve the problem. The structures that were tested were designed in pairs, one used as dynamic testing structure, the other as static testing structure. Corresponding to each inelastic dynamic test, a static test which experienced the same displacement history with the dynamic test was performed. The force-displacement hysteretic curve measured in the static test, and the area surrounded by the curve are considered the restoring force and hysteretic energy (EH) of the dynamic test respectively; the area surrounded by the inertia force-displacement curve measured in the dynamic test is the total input energy EI. Virtually, this definition indicates that damping is the energy difference between dissipated in dynamic loading and static loading under the same displacement history. Tests were performed in National Disaster-Prevetion Lab. of Civil Engineering, located in Tongji University, Shanghai, China.

### Structures

Three pairs of small SDOF structures with four steel legs supporting a concrete block were designed. All the structures shared the same concrete block. The legs were clamped with bolts at both ends to approximate fixed connection. The section dimension of the legs is the same (20mm × 10mm), but the length of the legs is different for different pairs of structures. The three pairs are named T1,T2 and T3 for 275mm, 435mm and 600mm length of legs respectively. The natural frequencies and damping ratios of the three pairs of structures as shown in Table 1 were measured by free vibration tests, where the elastic limit displacement X<sub>c</sub> represents the turning point from straight line to curve in the load-deflection relationship curves measured by static tests.

Table 1. Properties of tested structures

Structure	X <sub>e</sub> (mm)	Natural Frequency(Hz)	Damping Ratio
Т1	9. 0	5. 38	0. 010
Т2	20. 0	2.75	0.011
Т3	30.0	1.75	0. 011

### Dynamic tests

Modified Imperial Valley Earthquake (May 18, 1940) El-Centro S00E record, Bear Valley Earthquake (Spet. 4, 1972) Melendy Ranch N29W record, and several sine acceleration waves with different frequencies were used as inputs. The magnitude of the input acceleration wave was increased step by step in the tests so that different levels of structural responses were measured. Ductility  $\mu = X_{max}/X_{\bullet}$  was diffined, where  $X_{max}$  is the maximum displacement response of structures. It can be considered that tests with  $\mu \leq 1$  are elastic ones and those with  $\mu > 1$  are inelastic ones. All together 20 elastic tests with  $\mu$  ranging 0. 1~0. 93 and 8 inelastic tests with  $\mu$  ranging 1. 44~2. 95 were performed.

### Static tests

Corresponding to each inelastic dynamic test, a static test which experienced the same displacement history with the dynamic test was performed one by one. Load-deflection curve was measured in the static test, and the curve was used in the following damping identification process as restoring force of the corresponding dynamic test.

#### **IDENTIFICATION PROCEDURE**

## Objective Functions

Three different indexes of structural response were selected separately as equivalent objects in the damping identification. They are the maximum displacement (MD), the sum of squares of displacements in the whole vibrating history (SSD), and the total hysteretic energy (EH). Errors were defined as following to discribe the difference between analysed structural response and corresponding objective response:

$$ERR_{MD} = \frac{X_{0max} - X_{max}}{X_{0max}}$$
 (1)

$$ERR_{SSD} = \frac{\sum X_0^2 - \sum X^2}{\sum X_0^2}$$
 (2)

$$ERR_{EH} = \frac{Eh_0 - Eh}{Eh_0}$$
 (3)

Where  $X_{0max}$ ,  $\Sigma X_0^2$  and Eh<sub>0</sub> are actually measured MD,SSD and EH respectively;  $X_{max}$ ,  $\Sigma X^2$ , and Eh are analysed MD,SSD and EH respectively. Each time one of the three errors was chosen as objective function, and identification completed when the error reduced to less than 5% for ERR<sub>MD</sub>, 10% for ERR<sub>SSD</sub> and ERR<sub>EH</sub>.

#### Identification Procedure

Dynamic time-history analysis theory is used combined with the trial and error method. With a given damping model, first a primary damping ratio  $\zeta_0 = 0$ . 01 is selected, and dynamic analysis proceeds. One of the three structural response indexes, e.g. SSD is chosen as equivalent object. If ERR<sub>SSD</sub> < 0, it indicates that the analysed SSD is greater than the tested one, then with a higher damping ratio dynamic analysis is performed again; Otherwise, with a lower damping ratio the same work is done. The damping ratio adjustment process does not stop until  $|ERR_{SSD}|$  is reduced to within 10%. In dynamic analysis, make the system follow the hysteretic restoring force loops measured in the static test corresponding to each dynamic test

# Damping models

Rayleigh-type damping representation is used extensively in the dynamic analysis of structure. In inelastic analysis for SDOF system, Rayleigh-damping can be the following four different forms (Huang, 1995):

$$C_1(a,b_0) = 2\xi\omega_0 m \tag{4}$$

$$C_2(b) = 2\xi \omega_0 m \left(\frac{\omega_1}{\omega_0}\right)^2 \tag{5}$$

$$C_2(a,b) = \xi \omega_0 m \left(1 + \left(\frac{\omega_t}{\omega_0}\right)^2\right) \tag{6}$$

$$C(a_t,b_t) = 2\xi\omega_0 m \frac{\omega_t}{\omega_0} \tag{7}$$

Where  $\zeta$ =the damping ratio of the system;  $\omega_0$ =the initial natural frequency;  $\omega$ =the tangent frequency. Obviously,  $C_1(a,b_0)=2\zeta\omega_0$ m is the constant damping influence coefficient, usually used in elastic dynamic analysis, while the other three are time-dependent.

### **IDENTIFICATION RESULTS**

## Influence of Equivalent Object

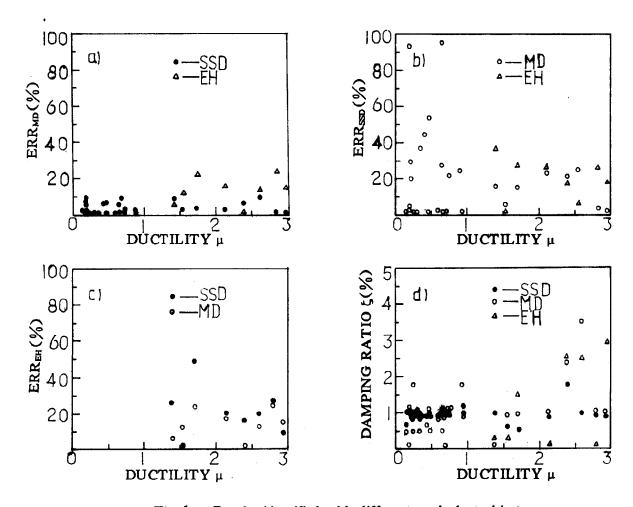


Fig. 1. Results identified with different equivalent objects

Fig. 1 shows the effect of different equivalent objects on the identified results, where the horizontal ordinate  $\mu$  indicates the measured displacement ductility of the tests. There are 20 elastic dynamic tests (those  $\mu \leq 1$ ) and 8 inelastic dynamic tests (those  $\mu > 1$ ) all together. Only for inelastic tests, identifications taking EH as equivalent object can be performed. It can bee seen from Fig. 1 that SSD is a better equivalent object than MD and EH in damping identification. Since SSD reflects the whole feture of displacement-history response of the structure while MD just a single peak value, the deviation of the damping ratios identified with the equivalent object SSD is much smaller than that identified with the usual equivalent object MD (Fig. 1(d)). Furthermore, the displacement and absolute acceleration history analysed with the damping ratio corresponding to SSD can meet the tested results very well(Fig. 2) and therefore meet the requirement of MD(Fig. 1(a)). Conversely in most case, the results analysed with the damping ratio corresponding to MD can not meet the reqirement of SSD(Fig. 1(b)), and the displacement history has obvious discrepancy with the tested result. The deviation of the damping ratios identified with the equivalent object EH is the highest compared with the that corresponding to SSD and MD(Fig. 1(d)). EH is not suitable for equivalent object in damping identification.

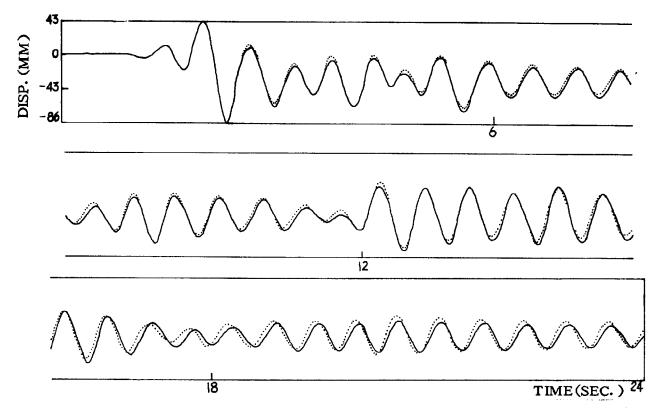


Fig. 2. Comparison of displacement response time-history between the tested and the analysed (c=2m $\omega_0\xi$ ,  $\xi$ =0.010,  $\mu$ =2.95)

## Influence of Damping Models

Analysed results show that the effect of damping models on the nonlinear response of structures is not negligible when  $\mu \ge 2$  and the strong ground-motion duration is relatively long. From Table 2 it can be considered that  $C_1(a,b_0)$  is the most suitable damping model of the four for inelastic dynamic analysis. Model  $C_1(a,b_0)$  leaded to the smallest C.O.V. of identified damping ratios, and more important, it indicates that both in elastic and in inelastic dynamic analysis the same damping model can be chosen.

Table 2. Mean value  $\mu$  and coefficients of variation C. O. V. of damping ratio for different damping models

Damping Model	$C_1(a,b_0)$	C <sub>2</sub> (b)	C <sub>2</sub> (a,b)	$C_3(a_t,b_t)$
μ .	0. 010	0. 015	0. 011	0. 012
C.O.V.	0. 39	0. 59	0. 55	0. 50

# Influence of Stress or Deformation State

Fig. 1(d) shows that the magnitude of damping ratio is independent of the stress or deformation state of structures, and there is no tendency that the value of damping ratio increases as the deformation of the system increases. From 20 elastic tests, the average value of damping ratio is 0.010 with C.O.V. = 0.14; From 8 inelastic tests, as shown in Table 2, the average value of damping ratio is also 0.010, but with a greater C.O.V. = 0.39.

### **CONCLUSIONS**

In this study, appropriate equivalent object for damping ratio identification, suitable damping model and corresponding damping value in dynamic time-history analysis of structures have been investigated. Dynamic tests and corresponding static tests of three pairs of SDOF small steel frames have been performed. Three equivalent objects and four damping models have been considered in the identification.

The sum of squares of displacement response SSD is a better equivalent object than the conventional maximum displacement response MD. The deviation of the damping ratios identified with equivalent object SSD is much smaller than that identified with equivalent object MD. The displacement histories analysed with the damping ratio corresponding to SSD meet the tested results very well and therefore meet the requirement of MD, while the results analysed with the damping ratio corresponding to MD can not meet the requirement of SSD.

The damping features of structures are independent of stress or deformation states as soon as the actual hysteretic restoring forces are used in dynamic analysis. Whether in elastic or in inelastic dynamic analysis, the constant damping influence coefficient model  $C=2m\omega_0\zeta$  is suitable, and the magnitude of damping ratio may be the same.

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