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RANDOM VIBRATION ANALYSIS OF NON-CLASSICALLY DAMPED MULTI-SUPPORT STRUCTURAL SYSTEMS

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ABSTRACT

A random vibration methodology is formulated for the seismic response analysis of multi-support structural systems with non-classical damping. Using Foss's method to obtain the complex modal frequencies and shapes, it is shown that a dynamic response component of interest can be expressed as a linear combination of the response displacement, velocity and acceleration of equivalent modal oscillators. An expression is derived to evaluate the cross-correlation function between dynamic response components. Earthquake ground motions at the supports are modeled as zero mean jointly stationary random processes. A coherency function for shear waves propagating through a random medium is used. Wave-passage effects are taken into account by means of the phase spectrum for a train of plane waves and local amplification effects are modeled using modified Kanai-Tajimi spectral density functions. The methodology is applied to a case study to assess the influence of local soil conditions, and of incoherence and wave-passage effects on the dynamic response. The results are also used to test the appropriateness of assuming for the response analysis that support ground motions are perfectly correlated or statistically independent.

KEYWORDS

Random vibration; non-classical damping; ground motion spatial variation.

INTRODUCTION

A classical representation in seismic response analysis is that of a single -or multi- degree-of-freedom linear oscillator subjected to earthquake ground motion at its support. This model is used assuming that at all support points of a structure the ground motion excitation is the same, which may not be appropriate for structural systems such as bridges and lifelines extending over long distances. Empirical data obtained from dense seismographic arrays has shown that significant variability exists in ground motion parameters over distances of the same order of magnitude as those between the multiple supports of spatially extended structures. On the other hand, the operational integrity and serviceability of lifeline systems, after an earthquake, are essential for the safety of the built environment and for the efficient implementation of disaster mitigation strategies. Experience shows, however, that lifeline systems are often severely damaged during earthquakes.

Random vibration and random fields theory has been used for stochastic response analysis of multi-support systems (Zerva et al. 1986, 1988, Harichandran and Wang 1990, Zerva 1990, Berrah and Kausel 1990, Der Kiureghian and Neuenhofer 1991). More recently, Heredia-Zavoni and Vanmarcke (1994) developed a random-vibration methodology which reduces the seismic response analysis of linear multi-support multi-degree-of-freedom (MS-MDOF) structural systems to that of a series of one-degree modal oscillators in a way that fully accounts for the multi-support input and the ground motion space-time covariance structure. The main advantage of a random vibration methodology is that it provides, explicitly, statistical measures of the response applicable to a class of input ground motions.

In the case of structural systems with significantly different damping characteristics in different parts, such as the soil-structure system models for lifelines, the response analysis should account for the non-classical nature of damping. It has been shown that for MDOF systems subjected to a single support ground motion, the response can be expressed as a linear combination of the response displacement and velocity of equivalent modal oscillators (see e.g the review in A.K.Gupta, 1993). The formulation presented here is an extension to the case of multi-support structural systems where spatial variation of ground motion has to be taken into account. The general solution for the dynamic response of a MS-MDOF system is presented first. The paper then focuses on the dynamic response covariance and shows that it can be expressed in terms of the responses of equivalent modal oscillators. An example is given to assess the influence of local soil conditions, and wave-passage and incoherence effects on the response.

EQUATIONS OF MOTION

Consider a linear MS-MDOF system subjected to earthquake ground motions at each of the supports. Let n denote the number of response degrees of freedom and m the number of prescribed support ground motions. Let $\mathbf{u}=\{u_1,u_2,...,u_m\}^T$ be the (m x 1) vector of prescribed displacements at the supports and $\mathbf{x}$ the (n x 1) vector of dynamic response displacements at the response degrees of freedom. The equations of motion in terms of the dynamic response displacements can be written in matrix form as follows,

$$\boldsymbol{M}\ddot{\boldsymbol{x}}+\boldsymbol{C}\dot{\boldsymbol{x}}+\boldsymbol{K}\boldsymbol{x}=-(\boldsymbol{MR}+\boldsymbol{M}_c)\,\ddot{\boldsymbol{u}}-(\boldsymbol{CR}+\boldsymbol{C}_c)\,\dot{\boldsymbol{u}} \tag{1}$$

in which **M**, **C**, **K** are the (n x n) mass, damping and stiffness matrices associated with the response degrees of freedom, respectively; $\mathbf{M}_c$, $\mathbf{C}_c$, are the (n x m) mass and damping coupling matrices between the response degrees of freedom and the support displacements; and **R** is the so-called pseudo-static transfer matrix. Each column in **R** represents the displacements along the response degrees of freedom resulting from an unit static displacement of one support while keeping all other support displacements constrained.

The total response of the linear MS-MDOF system subjected to ground motion input, can be obtained by superposition of the responses to each separate support ground motion. Let $\mathbf{m}_j$ and $\mathbf{c}_j$ denote the j-th columns of the matrices $\mathbf{MR}+\mathbf{M}_c$ and $\mathbf{CR}+\mathbf{C}_c$, respectively. The equations of motion for the dynamic response displacements due to the j-th support ground motion u_j are then,

$$\boldsymbol{M}\ddot{\boldsymbol{x}}+\boldsymbol{C}\dot{\boldsymbol{x}}+\boldsymbol{K}\boldsymbol{x}=-\boldsymbol{m}_j\ddot{u}_j-\boldsymbol{c}_j\dot{u}_j \tag{2}$$

Using Foss's approach, equation (2) is transformed into a set of 2n first order linear differential equations whose free vibration solution yields a set of n complex eigenvalues, λ_i, along with their conjugates, and their corresponding eigenvectors, $\boldsymbol{\phi}_i$. The solution of the equation of motion in (2) can then be expressed as a linear combination of the eigenvectors $\boldsymbol{\phi}_i$,

$$\boldsymbol{x}_j=\sum_{i=1}^{n}\boldsymbol{\phi}_i z_{ij}(t)+\boldsymbol{\phi}_i^{*}z_{ij}^{*}(t) \tag{3}$$

where $\mathbf{x}_j$ is the dynamic response vector to the j-th support ground motion, the star denotes a complex conjugate and z_{ij} are complex generalized coordinates satisfying the equations,

$$\dot{z}_{ij}-\lambda_i z_{ij}=-F_{ij}\ddot{u}_j-G_{ij}\dot{u}_j \tag{4}$$

The forcing terms F_{ij} and G_{ij} in the right hand side of (4) are obtained in terms of the column vectors $\mathbf{m}_j$ and $\mathbf{c}_j$,

$$F_{ij}=\frac{1}{a_i}\boldsymbol{\phi}_i^T\boldsymbol{m}_j\ , \qquad G_{ij}=\frac{1}{a_i}\boldsymbol{\phi}_i^T\boldsymbol{c}_j \tag{5}$$

in which a_i is given by,

$$a_i=2\lambda_i\boldsymbol{\phi}_i^T\boldsymbol{M}\boldsymbol{\phi}_i+\boldsymbol{\phi}_i^T\boldsymbol{C}\boldsymbol{\phi}_i \tag{6}$$

The real and imaginary parts of the complex eigenvalues can be written as follows

$$\lambda_i=-\xi_i\omega_i+\iota\omega_{Di}\ , \qquad \omega_{Di}=\sqrt{1-\xi_i^2}\omega_i \tag{7}$$

where ξ_i, ω_i, ω_{Di} are equivalent modal oscillator natural frequencies, critical damping ratios and damped frequencies. Notice that for underdamped systems the real part of λ_i is negative. From (4) the following equations can be written for the real and imaginary parts of z_{ij}, $z_{ij}=p_{ij}+iq_{ij}$:

$$\begin{aligned}\dot{p}_{ij}+\xi_i\omega_i p_{ij}+\omega_{Di}q_{ij}&=-d_{ij}\ddot{u}_j-g_{ij}\dot{u}_j\\ \dot{q}_{ij}+\xi_i\omega_i q_{ij}-\omega_{Di}p_{ij}&=-f_{ij}\ddot{u}_j-h_{ij}\dot{u}_j\end{aligned} \tag{8}$$

where d_{ij}, g_{ij}, and f_{ij}, h_{ij}, are the real and imaginary parts of F_{ij} and G_{ij}, respectively.

Let x_{ikj} denote the i-th modal response along the k-th degree of freedom to the j-th support ground motion; x_{ikj} is the k-th component of the response vector $\mathbf{x}_{ij}$, where

$$\boldsymbol{x}_j=\sum_{i=1}^{n}\boldsymbol{x}_{ij} \tag{9}$$

From (3), it can be shown that,

$$X_{ikj}=2\,(p_{ij}Re[\phi_{ik}]-q_{ij}Im[\phi_{ik}]) \tag{10}$$

in which Re[ ] and Im[ ] denote the real and imaginary parts of the variable in the argument. After some algebraic manipulations of (10), the following equation, which is of the same form as the equation of motion for an equivalent modal oscillator, is derived for the response component x_{ikj},

$$\ddot{x}_{ikj}+2\xi_i\omega_i\dot{x}_{ikj}+\omega_i^2 x_{ikj}=-\alpha_{ikj}\dot{u}_j-\beta_{ikj}\ddot{u}_j-\gamma_{ikj}\dddot{u}_j \tag{11}$$

where α_{ikj} = -2Re[λ_i^* G_{ij} ϕ_{ik}], β_{ikj} = -2Re[λ_i^* F_{ij} ϕ_{ik} - G_{ij} ϕ_{ik}], and γ_{ikj} = 2Re[F_{ij} ϕ_{ik}]. The solution to (11) can be expressed as follows,

$$x_{ikj}=\alpha_{ikj}y_{ij}+\beta_{ikj}\dot{y}_{ij}+\gamma_{ikj}\ddot{y}_{ij} \tag{12}$$

where y_{ij} satisfies the equation,

$$\ddot{y}_{ij}+2\xi_i\omega_i\dot{y}_{ij}+\omega_i^2 y_{ij}=-\dot{u}_j \tag{13}$$

Equation (12) is now written in vector form to obtain the solution for the i-th modal response $\mathbf{x}_{ij}$,

$$\boldsymbol{x}_{ij}=\boldsymbol{\phi}^{d}_{ij}y_{ij}+\boldsymbol{\phi}^{v}_{ij}\dot{y}_{ij}+\boldsymbol{\phi}^{a}_{ij}\ddot{y}_{ij} \tag{14}$$

where the real mode shapes $\boldsymbol{\phi}_i^d$, $\boldsymbol{\phi}_i^v$, $\boldsymbol{\phi}_i^a$ are given by,

$$\begin{aligned}\boldsymbol{\phi}^{d}_{ij}&=-2Re[\lambda_i^{*}G_{ij}\boldsymbol{\phi}_i]\\ \boldsymbol{\phi}^{v}_{ij}&=-2Re[(\lambda_i^{*}F_{ij}-G_{ij})\boldsymbol{\phi}_i]\\ \boldsymbol{\phi}^{a}_{ij}&=\ 2Re[F_{ij}\boldsymbol{\phi}_i]\end{aligned} \tag{15}$$

Substituting (14) into (9) one obtains the solution for the dynamic response to the j-th support ground motion $\mathbf{x}_j$,

$$\boldsymbol{x}_j=\sum_{i=1}^{n}\ [\boldsymbol{\phi}^{d}_{ij}y_{ij}+\boldsymbol{\phi}^{v}_{ij}\dot{y}_{ij}+\boldsymbol{\phi}^{a}_{ij}\ddot{y}_{ij}] \tag{16}$$

Thus, the total response of the linear system to the multi-input earthquake ground motion is obtained by summing (16) over the responses to each support ground motion,

$$\boldsymbol{x}=\sum_{j=1}^{m}\sum_{i=1}^{n}\ \boldsymbol{\phi}^{d}_{ij}y_{ij}+\boldsymbol{\phi}^{v}_{ij}\dot{y}_{ij}+\boldsymbol{\phi}^{a}_{ij}\ddot{y}_{ij} \tag{17}$$

Consider now a dynamic response quantity of interest Z(t) of the linear MS-MDOF system. Z(t) may represent, for instance, the dynamic component of the bending moment, the shear force, the displacement or the rotation at some point of the structure. The dynamic response Z(t) can be expressed in general as a linear function of the response degrees of freedom by means of a response transfer vector $\mathbf{q}$ which depends on the geometry and stiffness properties of the structure,

$$\begin{aligned}Z(t)&=\boldsymbol{q}^{T}\boldsymbol{x}\\ &=\sum_{j=1}^{m}\sum_{i=1}^{n}\ a_{ij}\dot{y}_{ij}+b_{ij}\dot{y}_{ij}+c_{ij}\ddot{y}_{ij}\end{aligned} \tag{18}$$

where the influence coefficients a_{ij}, b_{ij}, c_{ij}, are given by,

$$\begin{aligned}a_{ij}&=\boldsymbol{q}^{T}\boldsymbol{\phi}^{d}_{ij}\\ b_{ij}&=\boldsymbol{q}^{T}\boldsymbol{\phi}^{v}_{ij}\\ c_{ij}&=\boldsymbol{q}^{T}\boldsymbol{\phi}^{a}_{ij}\end{aligned} \tag{19}$$

STOCHASTIC DYNAMIC RESPONSE

Assume now that the ground motions at the supports of the MS-MDOF system are zero mean jointly stationary (in the wide sense) random processes. Assume also that the duration of ground motion is long in comparison with the fundamental period of the system so that the transient response is damped out and stationarity is reached. Let Z(t) and V(t) denote two random dynamic response quantities of interest. Equation (18) can be used to find the cross-correlation function between these responses, $E[Z(t+\tau)V(t)]$. From the convolution integral for the responses of the equivalent modal oscillators y_{ij} in (13), and using the relationships for the cross-spectrum of a derivative process, the cross-correlation function between the dynamic responses can be written in terms of the cross spectral density function between ground accelerations at the k-th and l-th supports, $S_{kl}(\omega)$, as follows,

$$E[Z(t+\tau)V(t)] = \sum_{k=1}^{m}\sum_{l=1}^{m}\sum_{i=1}^{n}\sum_{j=1}^{n} a_{ki}a_{lj}^{1}\int_{-\infty}^{\infty}\frac{1}{\omega^2}H_i(\omega)H_j^*(\omega)S_{kl}(\omega)\,d\omega$$

$$+[b_{ki}a_{lj}^{1}-a_{ki}b_{lj}^{1}]\int_{-\infty}^{\infty}\frac{\iota}{\omega}H_i(\omega)H_j^*(\omega)S_{kl}(\omega)\,dw$$

$$+[b_{ki}b_{lj}^{1}-a_{ki}c_{lj}^{1}-c_{ki}a_{lj}^{1}]\int_{-\infty}^{\infty}H_i(\omega)H_j^*(\omega)S_{kl}(\omega)\,d\omega \qquad (20)$$

$$+[c_{ki}b_{lj}^{1}-b_{ki}c_{lj}^{1}]\int_{-\infty}^{\infty}\iota\omega H_i(\omega)H_j^*(\omega)S_{kl}(\omega)\,d\omega$$

$$+c_{ki}c_{lj}^{1}\int_{-\infty}^{\infty}\omega^2 H_i(\omega)H_j^*(\omega)S_{kl}(\omega)\,d\omega$$

where the superscript 1 in the influence coefficients refers to response V(t) and $H_i(\omega)$ is the frequency transfer function of an equivalent modal oscillator with parameters ω_i, ξ_i,

$$H_i(\omega) = [\omega_i^2-\omega^2+2\iota\xi_i\omega_i\omega]^{-1} \qquad (21)$$

LOCAL SPATIAL VARIATION OF EARTHQUAKE GROUND MOTION

The cross spectral density function between ground accelerations can be expressed as,

$$S_{ij}(\omega) = \sqrt{S_i(\omega)S_j(\omega)}\,\gamma_{ij}(\omega) \qquad (22)$$

where $S_i(\omega)$ is the ground acceleration density function and $\gamma_{ij}(\omega)$ is the coherency spectrum. Modified Kanai-Tajimi spectral density functions for ground accelerations have been used to model sites with different soil conditions (see Table 1). We have used the following functional form for the coherency spectrum,

$$\gamma_{ij}(\omega) = \exp[-(\omega\eta\frac{|\mathbf{r}_{ij}|}{V_s})^2]\exp(\iota\omega\frac{\mathbf{r}_{ij}\cdot\mathbf{V}}{V^2}) \qquad (23)$$

where η is an incoherence factor approximately in the range of 0 to 0.5, $\mathbf{r}_{ij}$ is the relative position vector between points P_i and P_j, V_s is a shear wave velocity representative of the medium, and **V** is the apparent wave velocity vector. The first exponential term in the right hand side of (23) is based on the coherence function for shear waves propagating through a random medium (Luco and Wong 1986) and takes into account the effect of ground motion coherency loss. The argument in the second exponential corresponds to the phase spectrum of a train of plane waves and accounts for ground motion spatial variation due to wave passage.

EXAMPLE

The two-span continuous beam shown in Fig. 1 is supported by springs and dashpots with stiffness and damping coefficients k_g and c_g, respectively. The beam is subjected to vertical ground accelerations at the supports. It is a steel beam with mass density ρ=1.064 kg sec^2/cm^2, elastic modulus E=2043050 kg/cm^2, moment of Inertia I=1.315x10^9 cm^4 and span length L=100 m. The damping matrix **C** is assembled assuming that it is of the Rayleigh type. To determine the corresponding factors for the combination of the mass and stiffness matrix, a real eigenvalue analysis was carried to determine the smallest and largest natural frequencies of the system. For this analysis a value of 5 was used for the ratio of support and beam stiffnesses k_g/k, $k=EI/L^3$, and 5% critical damping was considered for all modes. Once the damping matrix for the beam is obtained, the damping matrix for the entire beam-supports system can be assembled given values of the ratio of support to beam damping coefficients, c_g/c. Shear and axial deformations are neglected and the beam deforms due to bending only. The bending moment has been taken as the response quantity of interest and its variance at different locations along the axis of the beam has been evaluated. Whenever wave passage effects are considered, it is assumed that the direction of propagation is that from supports 1 to 3.

Based on reported values for η (Luco and Wong 1986) and for shear and apparent wave velocities (Lermo et al., 1993), values of V_s/η and V equal to 250 m/seg and 200 m/seg, respectively, have been considered for the analysis. To investigate the relative influence of wave passage and incoherence effects, the following five cases for the coherency model have been taken into account: (1) Fully coherent motions, $V_s/\eta=V=\infty$; (2) only incoherence effects, $V_s/\eta=250$ m/seg, $V=\infty$; (3) only wave passage effects, $V_s/\eta=\infty$, $V=200$ m/seg; (4) both wave passage and incohrence effects $V_s/\eta=250$ m/seg, $V=200$ m/seg; (5) statistically independent ground motions, $V_s/\eta=0$. The first and fifth cases are included for comparison purposes with possible assumptions in deterministic analysis such as uniform or independent motions at the supports.

Results are presented in terms of bending moment standard deviations normalized with respect to the maximum standard deviation for the case of fully coherent ground motions. Figures 2 and 3 show plots of the normalized response standard deviation along the axis of the beam for soft and firm soil conditions, respectively, and for a ratio $c_g/c=1$. For fully coherent motions and both soil conditions, the maximum response occurs at the middle support. This response is also the overall highest response among all five cases. When spatial variation is considered, the response is less than that for the fully coherent case at most locations along the axis of the beam, except for those close to the extreme supports. By comparing cases (2) and (3), it seems that wave passage effects have a greater influence on the responses along the left hand span (between supports 1 and 2); however the difference between these responses is not significant. The response to case (2) is relatively more significant for locations along the right hand span. Notice that when wave passage effects are not considered, the response of the continuous beam is symmetric with respect to the middle support. Moreover, case (2) produced moments larger than those corresponding to case (4) where both wave passage and incoherence effects are included. Furthermore, it can be seen that considering statistically independent ground motions at the supports underestimates the response to cases of spatial variation (2) and (4). For case (3), when only wave passage effects are considered, assuming statistically independent ground excitations will do fine for most locations along the beam, except for points on the left hand span close to the middle support where case (3) yielded the largest response among the spatial variation cases. On the other hand, assuming perfectly correlated support ground motions for the example illustrated here, gave conservative results. The maximum response at the middle support for case (2) -where the response is the largest among all cases of spatial variation- was about 87% and 85% that obtained assuming fully coherent motions for soft and firm soils, respectively.

Table 1. Spectral density function parameters

Soil Type	ω_f	ξ_f	ω_g	ξ_g
Firm	15.0	0.6	1.5	0.6
Soft	5.0	0.2	0.5	0.6

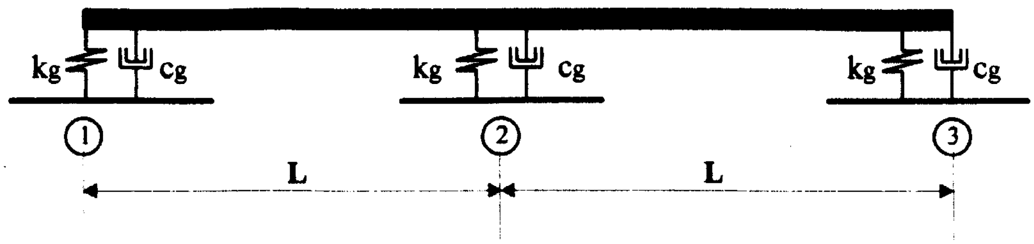

Fig. 1. Structural System

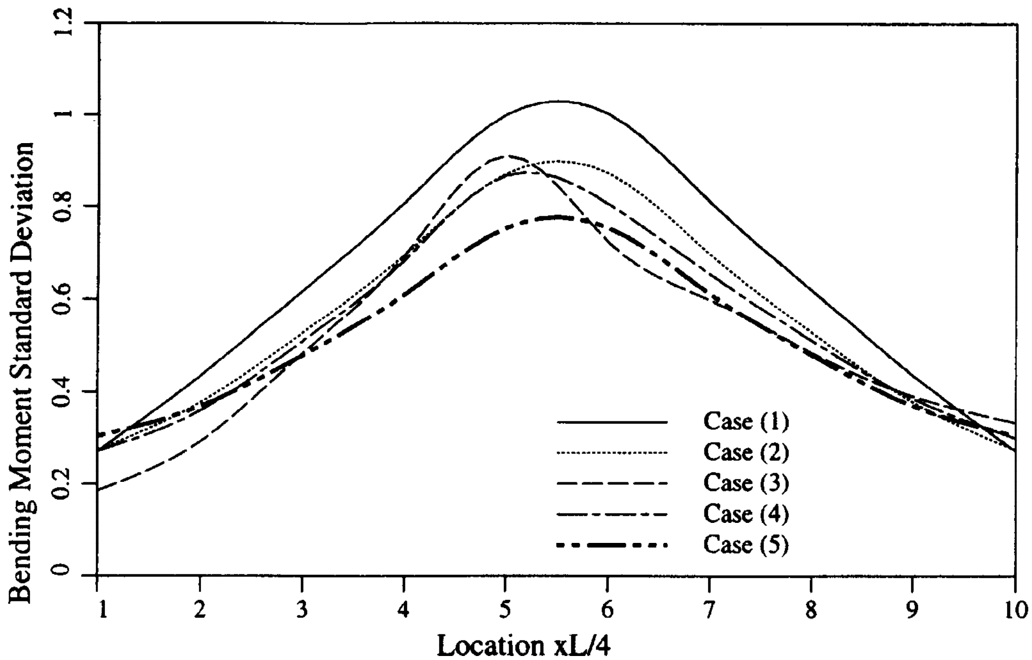

Fig. 2. Normalized bending moment standard deviations versus location; soft soil

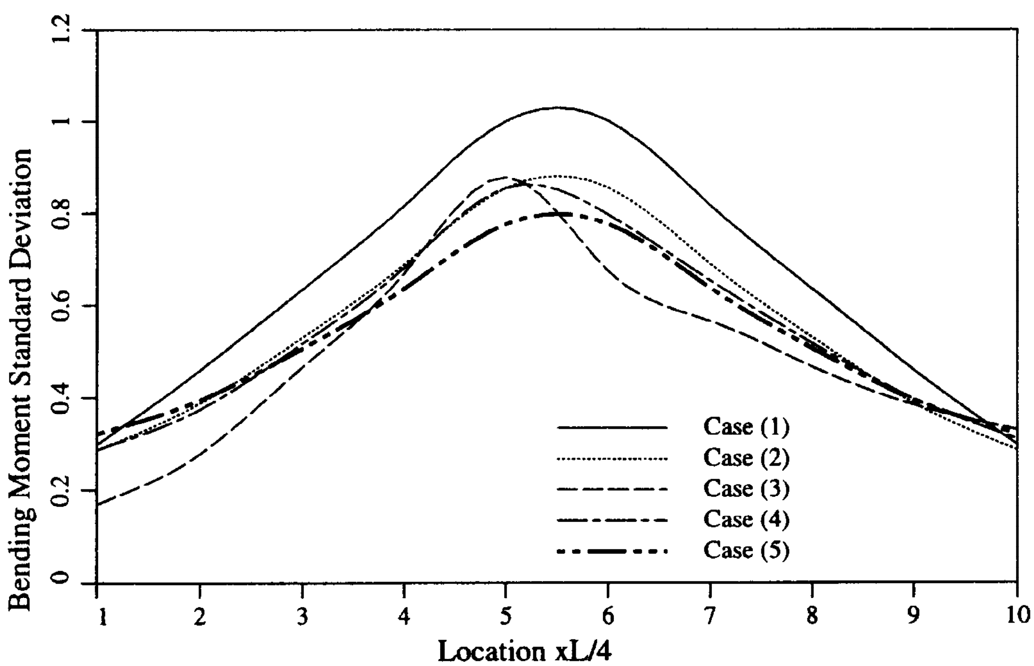

Fig. 3. Normalized bending moment standard deviations versus location; firm soil.

CONCLUSIONS

A random vibration method has been formulated for the response analysis of non-classically damped multi-support structural systems. It has been shown that the dynamic response is expressed as a linear combination of the response displacement, velocity and acceleration of equivalent modal oscillators. An expression has been derived for the dynamic response covariance in terms of equivalent modal transfer functions and the cross-spectral density function between ground accelerations at the supports. An example of a two-span continuous beam supported by springs and dashpots has been analyzed. Results showed that for all cases of ground motion spatial variation considered, the responses were less than for the fully coherent case. On the other hand, using statistically independent support ground motions for the analysis, underestimated the response. There was not a significant difference between the responses for the cases of spatial variation of ground motion. The reader should bear in mind that these conclusions are based on the results for a particular example and cannot be generalized a-priori.

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