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SEISMIC DESIGN BASED ON LOW CYCLE FATIGUE CRITERIA

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ABSTRACT

In this paper a consistent method including low cycle damage criteria in the seismic design is proposed. It is based on the estimation of the number of inelastic cycles and on the statistical characterization of the distribution of their amplitudes. A modified reduction factor is obtained that is also function of the earthquake damage potential by means of the index I_D and of the influence of the damage of the structural response by means of the low cycle fatigue parameter b. In this way damage spectra are obtained in order to utilise an equivalent elastic design using a reduced available ductility.

KEYWORDS

Seismic design, damage, low cycle fatigue, inelastic spectra, plastic cycles, reduction factor, ductility, seismic index, equivalent ductility.

INTRODUCTION

The first step of a correct seismic-resistant design of the structures is the reliable evaluation of the design forces in relation with the expected ground motions. Currently it is accepted that the structures could tolerate a certain level of damage and therefore the design is conducted using a Smoothed Inelastic Design Response Spectra obtained thought the use of reduction factor R_μ depending on the displacement ductility μ. This approach shows the advantage of allowing simple elastic analysis still and of being easily regulated (Eurocode, 1994), but presents the problem of the definition of reliable expressions of the reduction factor R_μ that represents a problem still open (Miranda and Bertero, 1994).

On the other hand, this method presents a conceptual very strong limitation, in fact the inelastic ductility spectra are significant of the only maximum demand of ductility and they not provide any information about the effective damage potential of the earthquake that is related also to the low cycle fatigue phenomena (Bertero and Uang, 1992). For this reason it is necessary to introduce the concept of damage functional (Cosenza *et al*, 1993), based on the simultaneous knowledge of the ductility μ and of the hysteretic energy E_H or on the definition of the cumulative ductility (Cosenza *et al*, 1993; Krawinkler and Zohrei, 1983; Park and Ang, 1985). This last concept, related with the low cycle fatigue functional, seems very promising and powerful (Cosenza and Manfredi, 1994) and will be adopted in the following.

Together the definition of the damage functionals, it is necessary to develop simplified design methods that allow to reach an improvement of the design without introducing excessive theoretical and operative difficulties.

The approach, which appears more convenient, is based on the definition of damage spectra in order to implement an elastic equivalent design still, generalizing the inelastic spectra methods, introducing the definition of "equivalent ductility." The "equivalent ductility" is a ductility value reduced respect to the monotonic one, in order to take into account the cyclic fatigue. This concept has been introduced independently by Cosenza and Manfredi (1991) with reference to the low cycle fatigue and by Fajfar (1993) with reference to the energy method.

This approach is advantageous, in fact the knowledge of the complex theory of the cyclic damage is not necessary for the designer, but it is sufficient the use of the well known structural ductility, modified on the basis of some fundamental data regarding the earthquake and the structure. In this paper a consistent method to evaluate the cumulative ductility from the inelastic ductility is proposed trough the introduction of an equivalent factor p. It is based on the assessment of the number of plastic cycles and on the characterization of their statistical distribution (Cosenza and Manfredi, 1992).

ELASTIC AND INELASTIC RESPONSE SPECTRA

The seismic records used in the statistical analysis represent the main Italian ground motions in the last years. The research has been extended to some destructive earthquakes happened in all the word that have been considered particularly significant: 82 Italian and 40 foreign records have been analyzed.

For each record the elastic exact spectrum has been obtained using the representation of Newmark-Hall (1982) that presents fixed values of the pseudo-acceleration Ae, of the pseudo-velocity Ve and of the pseudo-displacement De, in the field of the low, medium and long periods, respectively:

$$\begin{aligned} Ae &= C_a \cdot PGA & \quad 0.4 \cdot T_1 \le T \le T_1 \\ Ve &= C_v \cdot PVA & \quad T_1 \le T \le T_2 \\ De &= C_d \cdot PDA & \quad T_2 \le T \end{aligned}$$

where C_a, C_v and C_d are the amplification factors, while PGA, PVA and PDA are the peak values respectively of acceleration, velocity and displacement. For infinitely stiff structures ($T< 0.03$s.) the spectral acceleration is equal to PGA, while for very low periods ($0.04< T < 0.4T_1$) it varies with a linear law.

The construction of the ductility inelastic spectra could be developed through an exact way implementing a nonlinear step-by-step analysis, or through a very simple, although approximate, manner, introducing the concept of the reduction factor R_μ that represents the ratio between the elastic and the inelastic spectrum for fixed values of elastic period and inelastic ductility. In this last case, knowing the elastic spectrum and introducing a function that allows to determine R_μ from the allowable ductility μ, it is possible to obtain the inelastic spectra directly

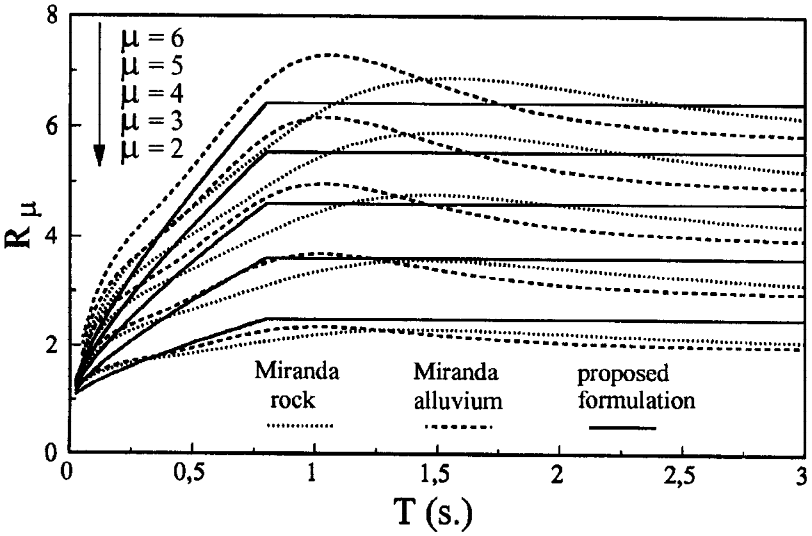

Fig. 1 Comparison among different formulations of R_μ.

without any nonlinear analysis. The definition of reliable expressions of R_μ is a still open question (Miranda and Bertero, 1994) and different formulations have been introduced in the scientific literature.

In this work, with the aim of defining a consistent procedure for the construction of the damage spectra, a simple relation for R_μ, suitable in the case of rock and low depth alluvium grounds, has been assessed, by means of a statistical analysis:

$$R_\mu = 1 + 1.5 \cdot (T/T_1)^{3/4} \cdot (\mu - 1)^{4/5} \quad per \quad T \leq T_1 \tag{1a}$$

$$R_\mu = 1 + 1.5 \cdot (\mu - 1)^{4/5} \quad per \quad T > T_1 \tag{1b}$$

This formulation provides an useful tool because of its independence from the period in the field of the medium and long periods and provides results similar in average with the results of more complex formulations (Miranda and Bertero, 1994), as the figure 1 shows for an earthquake characterized by a T_1 value equal to 0.7 s.

The inelastic spectra obtained using the (1) are generally acceptable for rock and low depth alluvium grounds and ductility μ lower than 8, as it could be observed in figure 2, where the "exact" spectra (Fig.2a) and the approximate ones (Fig.2b) are shown for the record of Llolleo (Cile, 1985)

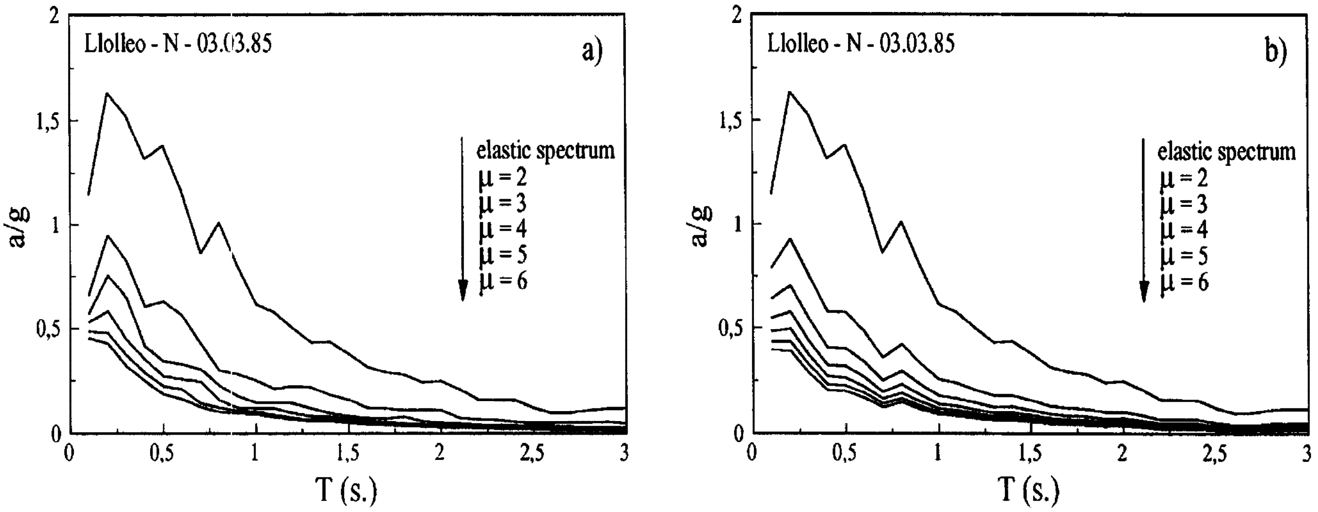

Fig.2 Inelastic spectra for a fixed ductility (records of Llolleo); a) exact, b) approximate.

THE EQUIVALENT DUCTILITY

The use of the ductility as cyclic collapse criterion is very common in the seismic design although it is acceptable only when the crisis is due to a limit of displacement. For this reason it seems useful to extend this concept to the cases in which the low cycle fatigue has a significant influence on the structural behaviour introducing the definition of equivalent ductility already proposed by (Cosenza and Manfredi, 1991; Fajfar, 1993). In fact, the experimental evidence shows that the accumulation of the plastic deformations carries on the structure to an early collapse respect to its property of resistance for monotonic actions. For this reason, the definition of effective criteria of collapse for cyclic actions is the second step in the development of a consistent seismic design. In particular, it is necessary to introduce appropriate damage functionals that, in their normalized form, assume the value 0 in absence of plastic damage and the value 1 at the collapse (Cosenza *et al*, 1993).

The most used damage functional are the functional of the ductility (kinematics or cyclic), the functional of the hysteretic energy, the functional of Park and Ang and the functional of the low cycle fatigue. The low cycle fatigue law is a very general criterium of cyclic collapse (Cosenza and Manfredi, 1993) and for these reasons this functional will be introduced in the formulation of the parameter of equivalent ductility.

The analytical expression of the functional D_F is:

$$D_F = A \sum_{i=1}^{n} (\Delta x_i)^b \quad with \quad b > 1 \tag{2}$$

where A and b are the two parameters necessary to define the model, n is the total number of the plastic cycles and Δx_i is the amplitude of the plastic displacement of the generic cycle. The case $D_F=0$ represents the absence of damage and the case $D_F=1$ the collapse under cyclic actions.

A different form of the eq.(2) could be introduced using the result of a test in which the element breaks down with an only cycle of amplitude $\Delta x_{1,u}$; eq.(2) becomes:

$$1 = A(\Delta x_{1,u})^b \quad \Rightarrow \quad A = \frac{1}{(\Delta x_{1,u})^b} \quad \Rightarrow \quad D_F = \sum_{i=1}^{n} \left(\frac{\Delta x_i}{\Delta x_{1,u}} \right)^b \tag{3}$$

The absence of collapse for cyclic actions is characterized by the relation:

$$\sum_{i=1}^{n} \left(\frac{\Delta x_i}{\Delta x_{1,u}} \right)^b = \left(\frac{\Delta x_{\max}}{\Delta x_{1,u}} \right)^b \left(1 + \sum_{i=1}^{n-1} x_i^b \right) < 1 \tag{4}$$

having defined the maximum plastic required amplitude $\Delta x_{\max}$, and the dimensionless ratio $x_i = \Delta x_i / \Delta x_{\max}$. The achievement of the collapse provides:

$$D_F = 1 \quad \Leftrightarrow \quad \frac{\Delta x_{\max}}{\Delta x_{1,u}} = \frac{1}{\left(1 + \sum_{i=1}^{n-1} x_i^b \right)^{1/b}} \tag{5}$$

The eq.(5) shows that, considering the effect of the low cycle fatigue, it is possible to use only the fraction $p<1$ of the maximum plastic monotonic available deformation $\Delta x_{1,u}$:

$$\Delta x_{\max} = p \Delta x_{1,u} \quad with \quad p = \frac{1}{\left(1 + \sum_{i=1}^{n-1} x_i^b \right)^{1/b}} \tag{6}$$

The value of p allows to define the equivalent ductility that the structure can supply under a cyclic displacement history:

$$\mu_{dan} = \frac{x_y + \Delta x_{\max}}{x_y} = 1 + \frac{\Delta x_{1,u}}{x_y} \cdot p = 1 + (\mu_{mon} - 1) \cdot p \tag{7}$$

and it depends on the parameter of structural damage b, on the number of plastic cycles n and, more in general, on the statistical distribution of the amplitude of the plastic displacements. Arranging these three informations, it is possible to formulate the factor p.

AN EXPLICIT FORMULATION OF THE EQUIVALENCE FACTOR

The choice of the statistical optimal distribution of the plastic displacements represents the first step in the formulation of the parameter of equivalent ductility p. In the following the statistical characterization of the distribution of plastic cyclic amplitudes of the elasto-plastic SDOF will be provided. More in detail, the random variable $x = \Delta x / \Delta x_{\max}$, Δx being the generic plastic deformation and $\Delta x_{\max}$ the maximum plastic amplitude (x belongs to the range [0,1]), will be analyzed.

In (Cosenza and Manfredi, 1991, 1992) a wide comparison between different probabilistic models is provided; the probabilistic models: truncated normal, lognormal, beta, gamma, exponential and pareto were extensively analyzed. The exponential probability distribution results sufficiently accurate for technical aims. Therefore the exponential model, that is defined by the following density function $f_X(x)$ ($f_X(x)$ is equal to the probability that the random variable assumes values in the range $x, x+dx$) and by the following distribution function $F_X(x)$ ($F_X(x)$ is equal to the probability that the random variable assumes values less or equal to x) seems the most suitable in the formulation of p:

$$f_X(x) = \nu e^{-\nu x} \quad ; \quad F_X(x) = 1 - e^{-\nu x} \quad (\nu, x \geq 0) \tag{8}$$

In fact the use of this probability model and of the damage functional of low cycle fatigue allow to obtain useful results, as it is shown in the following. It is well known that, if the random variables x_i are distributed with an exponential law, x_i^b is characterized by a Weibull probabilistic law. The random variable x is defined between 0 and 1 and, in the hypothesis of assuming the mean of the complete distribution $m=1/\nu$ as mean of the truncate distribution, the mean of the function of random variable $y=x^b$ truncate to 1 results equal to:

$$E[X^b] = \frac{\Gamma(b+1,\nu)}{1 - e^{-\nu} - \nu \cdot e^{-\nu}} \cdot \frac{1}{\nu^b} \tag{9}$$

where $\Gamma(b+1,\nu)$ is the truncated gamma function equal to:

$$\Gamma(b+1,\nu) = \int_0^\nu y^b e^{-y} dy \tag{10}$$

The limit study of the (9) underlines that $E[X^b]$ for $b \rightarrow 1$ tends to $E[X]=m$, while for $b \rightarrow 0$ tends to 0. Considering that n is known in determinist way and introducing for $E[X^b]$ the notation m_b, the (6) and (9) allow to obtain an explicit formulation for the factor of equivalence p:

$$p = \frac{1}{[1 + (n-1)m_b]^{1/b}} \tag{11}$$

The calculation of m_b is possible using an appropriate development in series for the truncated gamma function without excessive difficulties; nevertheless, in order to simplify the calculation of p, in Fig. 3 a graph is drawn that allows an easy evaluation of m_b for different values of b. The mean m varies in the range [0.1,0.5] that includes the values significant in the seismic field.

A simplified calculation of m_b can be performed by means of the following expression with a very low standard error:

$$m_b = e^{-0.06(b-1)} m^{1 + 0.87 \log b} \tag{12}$$

A further simplification can be obtained introducing the more simple formulation for m_b:

$$m_b = e^{-0.45(b-1)} m \tag{13}$$

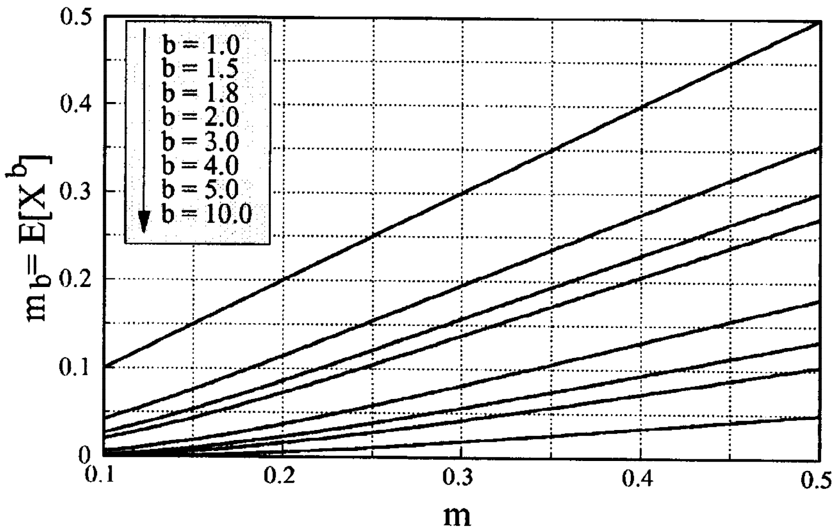

Fig.3 Graph for the calculation of m_b.

It is possible to observe that the (11) make correctly to depend the allowable ductility on the number of plastic cycles n, by the medium values of the distribution of these cycles m and also on the characteristics of structural damage by means of the b parameter. Furthermore it is worth to notice that in the cases characterized by $n \rightarrow 1$, $m \rightarrow 0$ or still $b \gg 1$ a value of $p=1$ is obtained, therefore the criteria of the plastic fatigue and of the ductility are coincident (Cosenza and Manfredi, 1993; Cosenza *et al*, 1993).

The knowledge of the number of reversal plastic cycles n performed by a structure owing to a ground motion results also necessary in the definition of the parameter of equivalent ductility p. The study of the behaviour of the SDOF shows that n results decreasing with the increase of the period and of the strength level. It is possible to notice, above all, that n varies largely varying the earthquake from values of 30 for the ground motion of Friuli to values of 200 for the ground motion of Cile with numerous intermediate cases. This remark underlines the narrow correlation between the value of n and the seismic characteristics of the earthquake.

The knowledge of the medium value of the plastic cycles m is also indispensable for the full definition of the parameter of equivalent ductility p, as it is shown by the (11). The analysis of the results shows that m increases slightly with the elastic period T and with the strength level; beside the influence of the seismic characteristics is present, but it is less important than in the case of the number of the cycles.

The definition of analytical formulations that allow to evaluate n and m must start from a selection of the principal parameters of functional dependence. In a first step it is necessary to introduce the elastics, the plastics and the seismic parameters of the system. The identification of the optimal parameters has been made on the base of an exponential type formulation. It has been assumed that the optimal parameters are obviously characterized by the higher partial correlation coefficients.

The elastic behaviour seems to be well described by the percentage of proportional damping ξ, that it is normalized respect to the reference value ξ_o=5%, and by the ratio T/T_1 (for $T<T_1$), where T is the elastic period of the system and T_1 is the initial period of the medium periods range in the spectral representation of Newmark and Hall. The plastic response is fully represented by $(R_\mu-1)$, where R_μ is the reduction factor.

The identification of the seismic parameters to introduce in the expression of n and m is more complex . For this purpose different parameters function of one or more seismic indexes have been used among which the I_D index:

$$I_D = \frac{I_E}{PGA^2} \cdot \frac{PGA}{PGV} = \frac{I_E}{PGA \cdot PGV} \qquad with \qquad I_E = \int_o^t a^2(t)dt \tag{14}$$

The I_D index has been also used in a different form by Fajfar and Vidic (1994) in the energy assessments and this remark can be verified implementing appropriate mathematical transformations. The statistical analysis has underlined that the parameter that is in absolute characterized by the highest correlation is just the I_D index that is function of three seismic parameters; among the single parameters, the effective duration t_d (Cosenza and Manfredi, 1994; Fajfar *et al.*, 1989; Jeong and Iwan, 1988) and the zero crossing number n_{zc} (Saragoni, 1990) show good results, as observed also by other researchers in relation to the assessment of the seismic damage potential.

Therefore it is finally possible to determine analytically n and m obtaining an explicit formulation of the equivalent ductility parameter p:

for $\xi \geq \xi_o = 5\%$ and $T \leq T_1$:

$$n = 1 + 1.05 \cdot \xi/\xi_o{}^{-1/3} \cdot (T/T_1)^{-2/3} \cdot (R-1)^{4/5} \cdot I_D^{4/5} \tag{15}$$

$$m = 0.17 \cdot \xi/\xi_o{}^{1/6} \cdot (T/T_1)^{1/6} \cdot (R-1)^{-1/5} \cdot I_D^{1/5} \tag{16}$$

Eqns. (15), (16) are valid also for $\xi \leq \xi_o$ and/or for $T \geq T_1$, with the position ξ/ξ_o=1 and/or T/T_1=1.

Introducing the relations (1) suggested for R_μ in the (15) and (16), it is possible to express n and m directly as a function of μ (ξ= 5% is implemented for simplicity):

$$n = 1 + 1.45 \cdot (\mu - 1)^{16/25} \cdot I_D^{4/5} \quad ; \quad m = 0.16 \cdot (\mu - 1)^{-4/25} \cdot I_D^{1/5} \tag{17}$$

The (17) are valid in all the field of periods, if a weak dependence on T/T_1 is neglected (exponent of the order of 1/15 for n and lower for m) in the field of low periods $T<T_1$.

The definition of the (17), that allows to assess n and m, represent the last step for defining the parameter of ductility equivalent p completely. In fact introducing in the (1) the values reduced by the effect of the cyclic damage, it is obtained, respectively for $T \leq T_1$ and $T > T_1$:

$$R_D = 1 + 1.5 \cdot (T/T_1)^{(3/4)} \cdot (\mu_{dan} - 1)^{4/5} \quad ; \quad R_D = 1 + 1.5 \cdot (\mu_{dan} - 1)^{4/5} \tag{18}$$

in which μ_{dan} could be evaluated using the p parameter through the (7) and the (11).

Therefore the construction of the damage spectra could be implemented in very simple way, as in the following (in the square brackets an example, referred to a SDOF with elastic period of 0.4 sec., is developed):

- the seismic characteristics in terms of I_D and the elastic spectrum of the expected ground motion are known in advance [i.e, with reference to the record of Llolleo-N, I_D=35.75 and T_1=0.45];
- the available monotonic ductility of the structure [i.e. μ_{mon}=4] and the characteristics of cyclic damageability introduced by means of the b parameter [i.e. b=1.5 that is considered a safe value (Cosenza and Manfredi, 1993)] are evaluated in advance;
- the expected number of cycles n and their average value m are evaluated by means of the (17) [i.e. n is equal to 42 and m to 0.284];
- using the graph of figure 3 (or eqns.(12)) and the eq.(11) it is possible to obtain the value of the p parameter [i.e. p equal to 0.217] and, then, to calculate by means of the (7) the value of the cyclic available ductility μ_{dan} [i.e μ_{dan} is equal to 1.65] ;

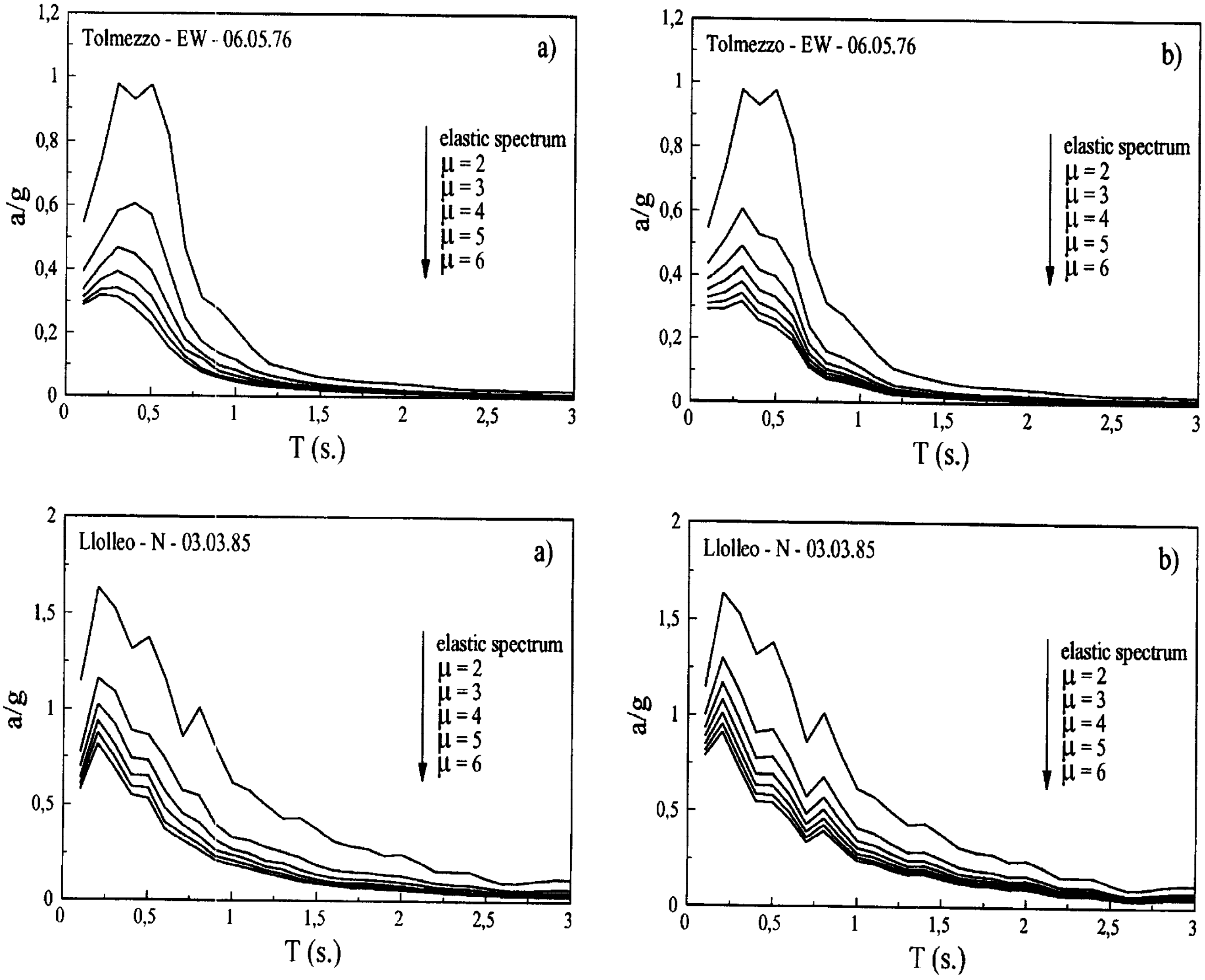

Fig.4 Inelastic spectra including low cycle fatigue criteria (b=1.5); a) exact, b) approssimate.

- using the eq.(18) it is possible to obtain the reduction factor R_D corresponding to the calculated μ_{dan} [i.e. R_D equal to 1.64 (exact value=1.73)];
- known R_D, the damage spectrum is obtained by means of a reduction of the elastic spectrum.

In figure 4 the low cycle fatigue spectra (b=1.5) for the record of Tolmezzo (I_D=7.25) and for the record of Llolleo (I_D=35.75) are shown. The comparison with the ductility spectra (Fig.2) underlines that the influence of the cyclic damage is very important for Llolleo: in fact a very low reduction of the elastic spectrum also for high monotonic ductility is evidenced . This result is obviously due to the high value of I_D that is indicative of a large number of plastic cycles. On the contrary, the ductility spectra show reduction factors similar for the two earthquakes and therefore they are not able to indicate the different required cyclic work.

CONCLUSIONS

The proposed design method including damage criteria allows to evaluate the design spectra as a function of the monotonic available ductility taking into account the influence of the low cycle fatigue on the structural response. The final formulation of R_D, supported by a consistent theoretical background and by extensive statistical analysis, is correctly dependent on the influence of the damage on the structure by means of the b coefficient, and on the cyclic work required by the ground motion by means of the I_D factor. The exact spectra are well fitted considering the complexity of the phenomena and the intrinsic simplicity of the proposed method: in conclusion, the proposed approach allows an easy extension of the equivalent elastic seismic design to the large field of the structures influenced by the low cycle fatigue phenomena.

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