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EVALUATION OF CODE FORMULAS FOR FUNDAMENTAL PERIOD OF BUILDINGS

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ABSTRACT

The formulas specified in US building codes to estimate the fundamental vibration period are based largely on motions of buildings recorded during the 1971 San Fernando earthquake. These formulas should be re-evaluated in light of the new data that has become available from recent earthquakes. For this purpose, a comprehensive database has been developed on periods of buildings identified (or measured) from their motions recorded during the 1971 and subsequent earthquakes. From this database, measured periods of three categories of buildings were extracted and compared with those obtained from the code formulas. This comparison showed that the code formulas for concrete and steel moment resisting frame buildings generally lead to periods that are shorter than measured periods, and periods estimated from these formulas almost always lead to higher values of seismic coefficients compared to the values based on measured periods. The comparison for shear wall buildings showed that the formula based on a coefficient independent of the shear wall area leads to periods much longer than the measured periods which in turn results in unconservative design forces. The formula based on shear wall area, on the other hand, results in periods much shorter than measured periods and hence overly conservative design forces.

KEYWORDS

Building codes; building period; dynamic analysis; earthquake analysis; earthquake design; fundamental period; period database; seismic codes; system identification; vibration period.

INTRODUCTION

The fundamental vibration period is needed to calculate the design base shear and lateral forces according to building codes. Because this building property can not be computed for a structure that is yet to be designed, building codes provide empirical formulas that depend on the building material (steel, concrete, etc.), building type (frame, shear wall, etc.), and overall dimensions. The empirical formula specified in US building codes -- UBC-94 (*Uniform Building Code*, 1994), ATC3-06 (*Tentative Provisions*, 1978), SEAOC-90 (*Recommended Lateral Force Requirements*, 1990), and NEHRP-91 (*NEHRP*, 1991) -- is of the form:

$$T = C_t (h_n)^{3/4} \qquad (1)$$

where h_n is the height of the building in feet above the base and C_t is a numerical coefficient related to the lateral-force-resisting system. The values of C_t specified in these codes are summarized in Table 1.

Table 1. Code-specified values of numerical coefficient C_t.

Code	Values of C_t			
	Steel Moment Resisting Frames	Concrete Moment Resisting Frames	Eccentrically Braced Steel Frames	Others
UBC-94, SEAOC-90, NEHRP-91	0.035	0.030	0.030	0.02
ATC3-06	0.035	0.025	--	--

For concrete or masonry shear wall buildings, UBC-94 and SEAOC-90 provide an alternative value of the coefficient:

$$C_t = 0.1/\sqrt{A_c} \tag{2}$$

where

$$A_c = \sum A_e\left[0.2 + (D_e/h_n)^2\right]; \qquad D_e/h_n \le 0.9 \tag{3}$$

in which A_e = the effective horizontal cross-sectional area, in square feet, of a shear wall in the first story of the structure, and D_e = the length, in feet, of a shear wall element in the first story in the direction under consideration. ATC3-06 specifies that the fundamental period of all other buildings, including building with shear walls and eccentric braced frames, be calculated by the formula:

$$T = \frac{0.05\,h_n}{\sqrt{L}} \tag{4}$$

where L = overall length, in feet, of the building at the base in the direction under consideration.

For preliminary design of a building, it is desirable to use a conservative estimate of the base shear. As described later, such a conservative estimate would be obtained if estimated period of the building is smaller than its true period. Therefore, the code formulas for fundamental period (Eqs. 1, 3, and 4) were intentionally calibrated to underestimate the period (*Tentative Provisions*, 1978).

The codes also recommend that whenever feasible the fundamental period should be calculated using the structural properties and deformation characteristics of the resisting elements in a properly substantiated analysis. This requirement may be satisfied by using the following formula based on Rayleigh's method:

$$T = 2\pi\sqrt{\left(\sum_{i=1}^{n} w_i \delta_i^2\right) \div \left(g\sum_{i=1}^{n} f_i \delta_i\right)} \tag{5}$$

where w_i is the seismic dead load at level *i*, *g* is the acceleration due to gravity, and f_i (i = 1,2, ...N) are a set of selected lateral forces with reasonable height-wise distribution and δ_i are the resulting deflections.

The codes specify that the period calculated from rational analysis (Eq. 5) should not be longer than that estimated from empirical formulas (Eqs. 1 and 3) by a certain factor. The factors specified in various US codes are: 1.2 in ATC3-06, 1.3 for high seismic region (Zone 4) and 1.4 for other regions (Zones 3, 2, and 1) in UBC-94; and a range of values with 1.2 for regions of high seismicity to 1.7 for regions of very low seismicity in NEHRP-91. SEAOC-90 specifies that the base shear calculated using the period from the rational analysis shall not be less than 80 percent of the value obtained by using the period from the empirical formulas, which corresponds to a value of 1.4 (Cole et al., 1992).

As mentioned previously, the codes recognize that the true period of a building may be longer than its period estimated from code specified empirical formulas (Eqs. 1 and 3). Therefore, the codes permit the period computed from rational analysis (Eq. 5) to be longer than that from empirical equations (Eqs. 1 and 3). However, in order to safeguard against unreasonable assumptions in the rational analysis, which may lead to uncoservatively long period, codes impose the aforementioned restrictions on the period computed from Eq. (5). These limiting values are indicative of by how much the codes expect the true period of a building to be longer than its period estimated from the empirical formulas.

The US seismic codes, for example UBC-94 and SEAOC-90, specify the base shear as:

$$V = \frac{ZIC}{R_w} W \tag{6}$$

where Z is the seismic zone factor, I is the importance factor, C is the elastic seismic coefficient, W is the total seismic dead load, and R_w is the reduction factor to account for energy dissipation capacity of the building. The elastic seismic coefficient C is a function of the fundamental period T and is given as:

$$C = \frac{1.25S}{T^{2/3}} \leq 2.75 \tag{7}$$

in which S is the site coefficient depending on the soil characteristics. The formulas specified in NEHRP-91 and ATC3-06 are similar in nature to that given by Eqs. (6) and (7) but differ slightly in details.

The empirical formula for fundamental period (Eqs. 1 and 3) in US building codes is based largely on motions of buildings recorded during the 1971 San Fernando earthquake (*Tentative Provisions*, 1978). These formulas should be re-evaluated in light of the wealth of data that has become available from recent earthquakes. This research investigation is aimed towards filling this need. For this purpose, a comprehensive database has been developed on periods of buildings identified (measured) from their strong motions recorded during the 1971 and subsequent earthquakes. This database is then used to evaluate the code formulas for estimating the fundamental period of buildings with steel and concrete moment resisting frames, and concrete shear walls.

DATABASE FOR BUILDING PERIODS

The vibration periods and modal damping ratios of about twenty buildings have been identified from their recorded motions during the 1994 Northridge earthquake using parametric system identification techniques (Beck, 1978; Li and Mau, 1990; Safak, 1988). These results are combined with similar data available from the 1971 San Fernando earthquake and other recent California earthquakes to form a comprehensive set of data. These data are compiled on an electronic database that can be easily updated after every major earthquake.

Contents of this database are arranged into the following five broad categories: (1) General information, (2) structure characteristics, (3) excitation characteristics, (4) recorded motions, and (5) vibration properties. For each of these general categories, several individual parameters are defined. The general information includes building location, identification number, occupancy, name, address, and reference that reported the data. The structural characteristics include height, plan dimensions, number of stories above and below the ground, material, and longitudinal and transverse resisting systems. The excitation is characterized by the earthquake name and date. The recorded motion data include peak values of roof and base accelerations, roof displacement relative to the base, and drift ratio in each of the two building directions. Vibration properties considered include period and damping ratio for up to two longitudinal, transverse, and torsional modes. A separate set of data (or record) is established for each building. In addition, a separate record is created for data obtained for the same building but for different earthquakes or by different investigators for the same earthquake.

From this comprehensive database, the information on fundamental vibration period has been extracted for three categories of buildings: (1) Concrete moment resisting frame buildings, (2) steel moment resisting frame buildings, and (3) concrete shear wall buildings. The extracted information included 37 data points for 27 concrete moment resisting frame buildings, 53 data points for 42 steel moment resisting frame buildings, and 27 data points for 16 concrete shear wall buildings. The number of data points exceed the number of buildings because some buildings yielded data for more than one earthquake or because data were reported by more than one investigator for the same earthquake. This information is then used to evaluate the code formulas for fundamental period. Although, the database contains information on buildings with other types of material and structural system, this information is not presented in this paper for brevity.

EVALUATION OF CODE PERIOD FORMULAS

In order to evaluate the code period formulas, we compare for each of the aforementioned categories of buildings:

- The measured building periods identified from their strong motion records with those obtained from the empirical code formulas (Eqs. 1 and 3), and
- The seismic coefficients calculated according to UBC-94 (Eq. 7) using the measured and code periods.

The measured and code periods are compared in Figures 1, 3, and 5 where they are plotted against the building height. The measured periods in two orthogonal directions are shown by solid circles connected by a vertical line, whereas code periods are shown by the curve denoted as T. Note that the code formula (Eq. 1) would lead to same period in the two directions as long as the lateral resisting systems in these directions are identical. Also included are curves for $1.2T$ and $1.4T$ representing restrictions on the period from rational analysis (Eq. 5) imposed by various US codes for high seismic regions like California. The former corresponds to the limit specified in ATC3-06 and NEHRP-91, whereas the latter is obtained from SEAOC-90 provisions; the limiting value of $1.3T$ specified in UBC-94 falls between these two extremes.

The seismic coefficients calculated according to UBC-94 (Eq. 7) using measured and code periods are compared in Figures 2, 4, and 6. These values are calculated for rock sites (S = 1 in Eq. 7). The seismic coefficients calculated using measured periods in the two orthogonal directions are shown by solid circles connected by a vertical line whereas those calculated using code periods are shown by a solid line. In order to calculate the seismic coefficient as a function of the building height, Eqs. (1) and (7) were utilized to get:

$$C = \frac{1.25S}{\left(C_t h_n^{3/4}\right)^{2/3}} \leq 2.75 \tag{8}$$

Concrete Moment Resisting Frame Buildings

The following trends can be gleaned from the results presented in Figures 1 and 2 for 27 concrete moment resisting frame (MRF) buildings:

- As intended by the code writers, the code formula leads to periods that are generally shorter than measured periods (Figure 1).
- The code formula leads to periods that form the lower bound of measured periods for building up to 160 ft tall.
- The lower bound of measured periods for buildings taller than 160 ft is about twenty percent higher, that is, the lower bound for such buildings is $1.2T$ rather than T. However, this conclusion is based on limited data as there are few concrete MRF buildings taller than 225 ft in the database.
- In general, measured periods of concrete MRF buildings fall between the curves for $1.2T$ and $1.4T$ indicating that the code imposed limits on the period from the rational analysis (Eq. 5) are reasonable.

- The code formula for estimating the building periods almost always leads to higher values of seismic coefficient compared to the values based on the measured period (Figure 2).

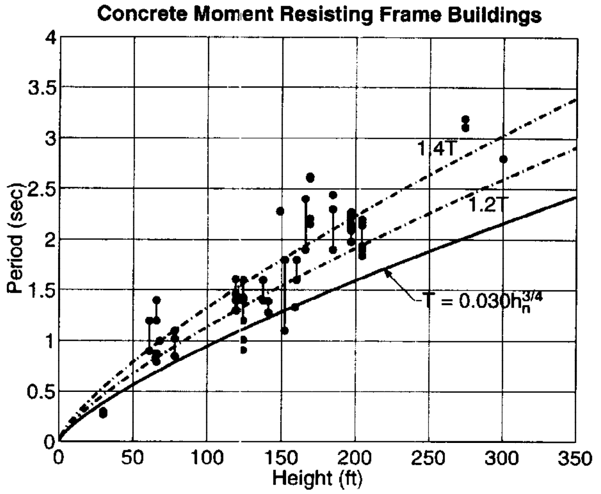

Figure 1. Measured and code periods for concrete moment resisting frame buildings.

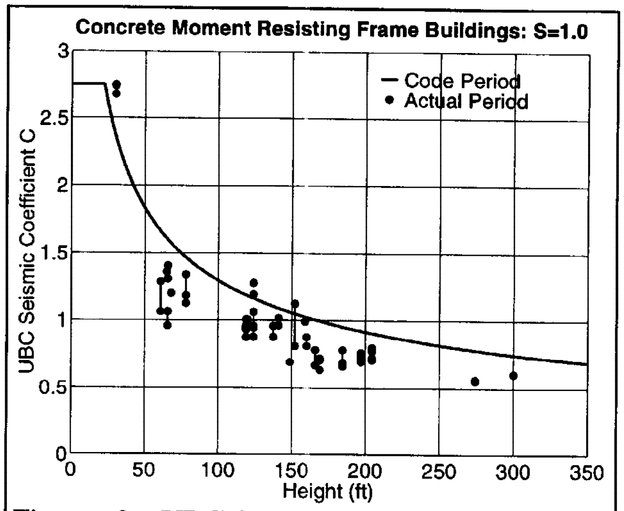

Figure 2. UBC-94 seismic coefficients for concrete moment resisting frame buildings.

Steel Moment Resisting Frame Buildings

The results presented in Figures 3 and 4 for 42 steel MRF buildings lead to the following conclusions:

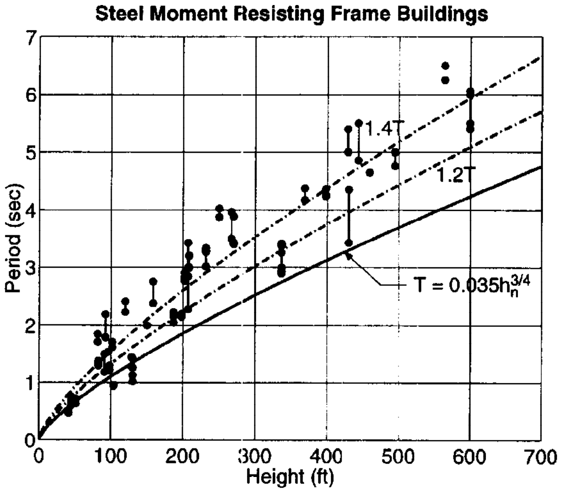

Figure 3. Measured and code periods for steel moment resisting frame buildings.

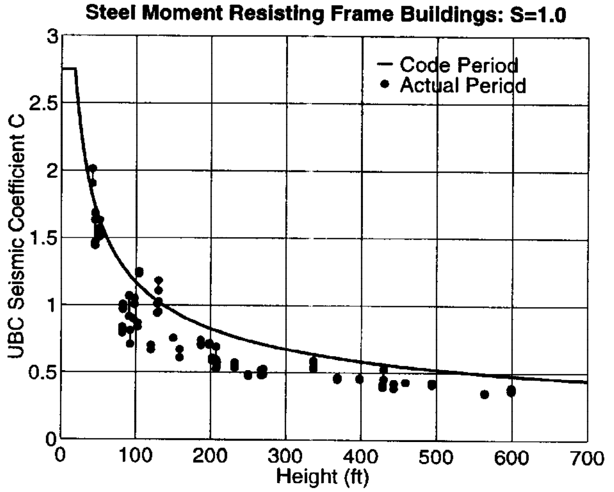

Figure 4. UBC-94 seismic coefficients for steel moment frame buildings.

- As noted previously for concrete MRF buildings, the code formula leads to periods that are generally shorter than measured periods (Figure 3). However, the margin between the code and measured periods is much larger for steel MRF buildings.
- The code formula leads to periods close to the lower bound of measured periods for buildings up to about 120 ft tall.
- With a few exceptions, the lower bound of measured periods for steel MRF buildings taller than 120 ft is twenty to thirty percent higher. Unlike concrete MRF buildings, many more data points support this conclusion for steel MRF buildings.
- Many measured period values exceed 1.2*T* and even 1.4*T* indicating that these limits are unrealistically restrictive for steel MRF buildings.

- As noted for concrete MRF buildings, the code formula for estimating periods of steel MRF buildings leads to seismic coefficients that are almost always higher than those from measured periods (Figure 4).

Concrete Shear Wall Buildings

The trends observed for concrete shear wall buildings (Figures 5 and 6) are quite different compared to those noted previously for concrete and steel MRF buildings. The trends gleaned from results presented for 16 concrete shear wall buildings are:

- The scatter in the plotted data is significantly larger (Figure 5) compared to both concrete and steel MRF buildings (Figures 1 and 3). This indicates that building height alone may not be a good parameter for estimating the periods of shear wall buildings and other parameters should be investigated.
- Unlike MRF buildings, the code formula based on $C_t = 0.02$ may lead to periods of shear wall buildings that are longer than measured periods (Figure 5).
- Obviously, the limiting values of $1.2T$ and $1.4T$ imposed by the codes on the period from rational analysis are unreasonable for shear wall buildings.
- The code formula based on $C_t = 0.02$ also leads to seismic coefficients that are smaller and hence unconservative compared to those from measured periods (Figure 6).

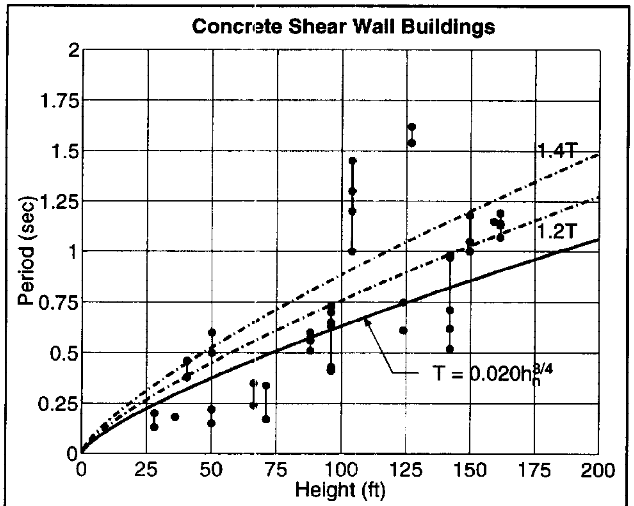

Figure 5. Measured and code periods for concrete shear wall buildings: $C_t = 0.02$.

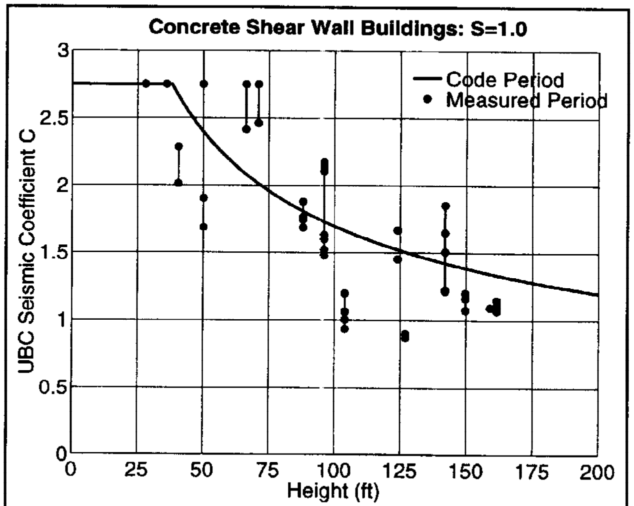

Figure 6. UBC-94 seismic coefficients for concrete shear wall buildings.

As mentioned previously, US seismic codes permit use of an alternative value of C_t (Eq. 2), that involves area of shear walls, to estimate the periods of concrete shear wall buildings. In order to evaluate this formula, measured and code periods are compared in Figure 7 for 10 selected buildings for which information on area of shear walls is available. The measured periods are shown by circles whereas code periods are shown by squares; the period values in the two orthogonal directions are connected by a vertical line. Since, areas of shear walls in the two orthogonal directions can be different, the code formula may lead to different periods in these directions. Also compared in Figure 8 are seismic coefficients calculated according to UBC-94 using the measured and code periods. The following trends emerge from these comparisons:

- Code Eq. (2) predicts periods that are generally much shorter than measured periods (Figure 7).
- The periods based on Eq. (2) also lead to seismic coefficients that may be significantly larger than calculated from measured periods (Figure 8). To reduce this resulting conservatism, Eq. (2) should be modified.

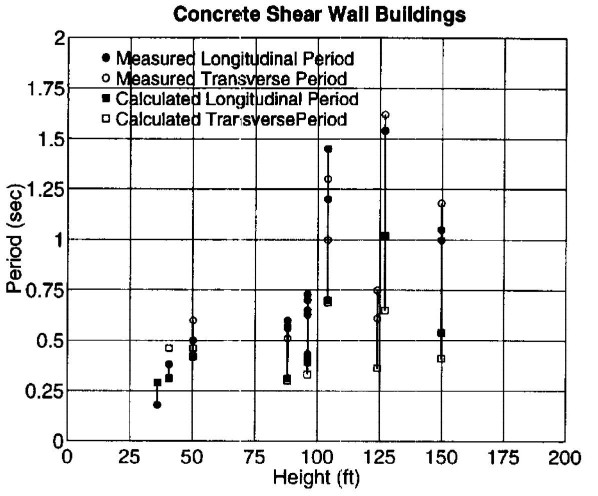

Figure 7. Measured and code periods for concrete shear wall buildings: $C_t = 0.1/\sqrt{A_c}$.

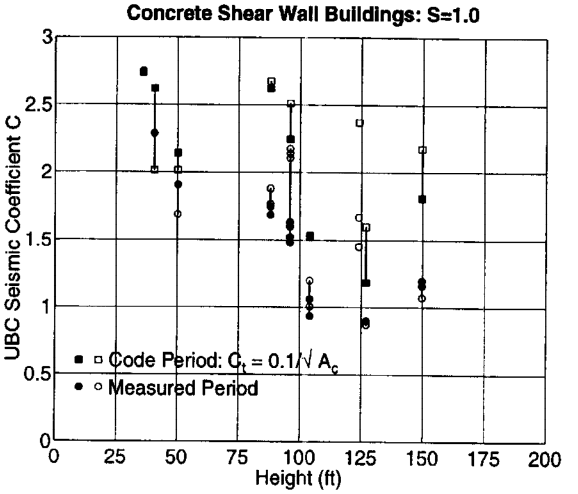

Figure 8. UBC-94 seismic coefficients for concrete shear wall buildings.

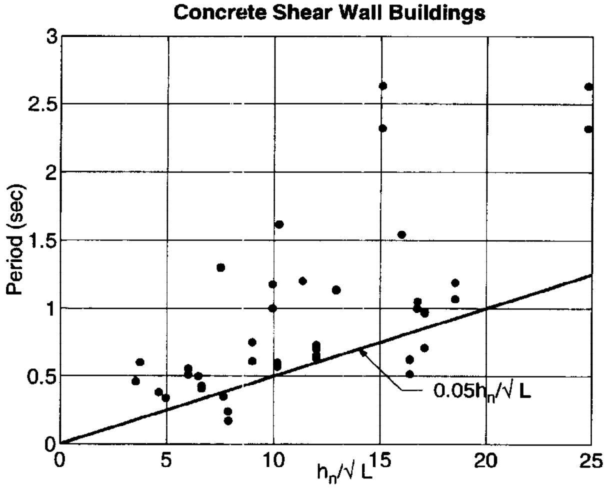

Figure 9. Measured and code periods for concrete shear wall buildings: Code period from Eq. (4).

ATC3-06 specifies that the period of a shear wall building be calculated from Eq. (4) that involves not only the building height but also the building base dimension. In Figure 9, the measured periods of 14 selected buildings are compared with the values calculated from Eq. (4). Comparison of the two sets of data indicates that:

- There is significant scatter in the plotted data indicating that $h_n/\sqrt{L}$ may not be a good parameter to estimate periods of shear wall buildings.
- Eq. (4) underestimates significantly the periods of buildings and leads to unnecessarily large seismic coefficients.

CONCLUSIONS

This investigation on evaluation of code formulas for fundamental period of buildings has led to the following conclusions:

- Code formulas for concrete and steel MRF buildings lead to periods that are generally shorter than periods measured from strong motion records of buildings. The difference between the code and measured periods, however, is much larger for taller buildings.
- The code formulas for estimating the periods of steel and concrete MRF buildings almost always lead to higher values of seismic coefficients compared to the values based on measured periods.
- The code imposed limits on the period from rational analysis are reasonable for concrete MRF buildings, but they are unrealistically restrictive for steel MRF buildings.
- For shear wall buildings, code formula based on $C_t = 0.02$ leads to periods much longer than measured periods which in turn results in unconservative design forces. The code formula based on $C_t = 0.1/\sqrt{A_c}$, on the other hand, results in much shorter periods and hence overly conservative design forces.

- The height, or height along with the base dimension, of a shear wall building alone may not be good parameter(s) to estimate its fundamental period.
- Based on this data and observations, the period formulas in US seismic codes should be revised.

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