



## OPTIMIZING THE ENERGY DISSIPATION IN COUPLED WALL STRUCTURAL SYSTEMS

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### ABSTRACT

In most building codes, there are very limited design guidelines regarding the proportioning of the shear walls and beam members in coupled wall assemblies. In general, members are proportioned to meet a desired level of strength and stiffness requirements, and at the same time provide sufficient ductility such that the structural elements dissipate the input energy of earthquake ground motion.

This paper will highlight some of the shortcomings of the current design approach. In particular, the seismic response of a coupled wall structural system that has been damaged during the Northridge Earthquake of 1994 will be studied. It will be demonstrated proportioning of the structural elements will have a significant effect on the level of energy dissipation in the system.

The objective of this paper is to establish a design criteria based on obtaining a desired level of damping ratio in the structural system, and optimizing the hysteretic energy loss. First an analytical procedure will be formulated and the response of the coupled wall assembly will be studied through closed form solutions. Then, an assessment will be made of the magnitude of the hysteretic energy loss and the equivalent viscous damping ratio. Finally, a design guideline will be formulated for proportioning of the shear wall and the coupling beam members with the goal of optimizing the hysteretic energy loss in the system.

### KEYWORDS

Optimization, energy dissipation, coupled walls, reinforced concrete.

### INTRODUCTION

In the conceptual design guidelines for reinforced concrete coupled wall structural systems, the primary emphasis has been on the strength, stiffness, and ductility of coupling beams and structural walls. Generally, the coupling beams are proportioned to achieve a desired level of stiffness or strength, with the implicit understanding that by providing sufficient ductility, the structural elements will experience yield excursions and dissipate the input energy of dynamic vibrations. In particular, one of the most typical approaches in the design and proportioning of the coupling beams is to concentrate on the stiffness aspect of design (Aktan and Bertero, 81). Generally, the idea is to proportion the coupling beams such that the entire coupled wall assembly is nearly (about 80% or higher) as stiff as a hypothetical solid wall that spans the entire width of the coupled wall assembly and has no openings. The problem encountered with this approach is that after proportioning the connecting beams, a structural analysis will be conducted to estimate the distribution of the loads to the structural members. With the beam element selected to be super stiff, a large shear and bending moment will be attracted to the connecting members. Consequently, a



## MECHANISM OF LOAD RESISTANCE

In a coupled wall assembly, there are two mechanisms of load resistance: (1) the cantilever action of the shear wall members which provides shear and flexural resistance to the applied loads, and (2) the relative displacement of the coupling beams which provides an overturning moment resistance (Fig. 3). In general, the global equilibrium equation that governs the response of the coupled wall structural systems may be stated as follows:

$$M_{ot} = (M_1 + M_2) + T \cdot L \quad (1)$$

where  $M_{ot}$  is the total applied overturning moment;  $M_1$  and  $M_2$  are the internal moment resistances of walls 1 and 2;  $T$  is the axial coupling force at the base of the walls; and  $L$  is the center-to-center distance between the coupling walls. In equation (1), the axial coupling force  $T$  is the result of the accumulation of the internal shear forces of the connecting beams. Hence, the component  $T \cdot L$  is considered to be the contribution of the beam element to the overturning moment resistance of the structural wall system.

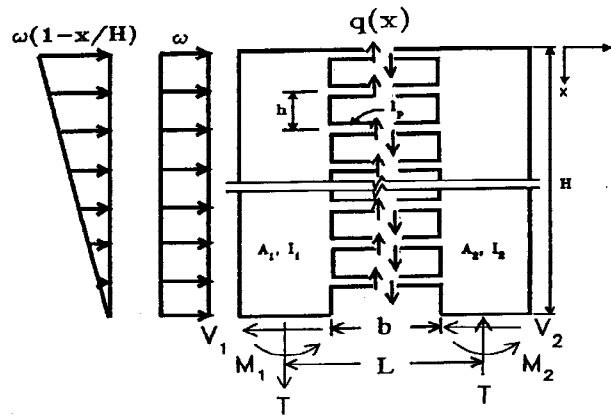


Fig. 3 - Internal resistance mechanisms of coupled wall assemblies

## ELASTIC ANALYSIS

Generally the elastic response of a structural system is determined through finite element analytical procedures. But in order to establish a closed form solution to the elastic response of coupled wall assemblies, the method of laminar analysis will be employed here. The primary advantage of the latter method is that it reduces a highly statically indeterminate system into a solution of a single differential equation. In the laminar analysis, the coupling beams and the shear wall members are treated as continuous mediums, and the interaction of these two media's is governed by equations of equilibrium and compatibility. The outcome of the laminar analysis (Fig. 3) are closed form solutions for lateral displacement pattern,  $y(x)$ , of the shear wall members, and the internal shear distribution,  $q(x)$ , of the coupling beams. A detailed discussion of this method is covered in other references (Coull, *et al*, 67a, 67b, 67c), and will not be studied further here. Only the results of the analysis will be employed here to study the elastic response of the coupled wall assemblies.

In order to study the elastic distribution of the load between the beams and the shear walls, the coupling axial force  $T$  will be evaluated through integration of shear force distribution:

$$T = \int_0^x q(x) dx \quad (2)$$

With the knowledge of axial coupling force  $T$ , the contribution of the beam and the shear wall members to the overall strength of the coupled wall assembly may be determined:

$$F_b = T \cdot L = L \int_0^H q(x) dx \quad (3)$$

$$F_{sw} = M_{ot} - L \int_0^H q(x) dx \quad (4)$$

Considering the fact that the elastic response of the beams and the shear walls may be defined in terms of their stiffnesses, a normalized stiffness ratio, SR, of the two members may be defined as follows:

$$SR = \frac{F_b}{F_{sw}} = \frac{K_b \cdot \Delta_b}{K_{sw} \cdot \Delta_{sw}} = \frac{L \int_0^H q(x) dx}{M_{ot} - L \int_0^H q(x) dx} \quad (5)$$

where  $\Delta_b$ ,  $\Delta_{sw}$ ,  $K_b$ , and  $K_{sw}$  are the relative displacement and the lateral stiffness of the coupling beams and shear walls, respectively.

It is important to note that with the knowledge of the stiffness ratio, SR, the equations of global equilibrium (1) may be stated in terms of the displacement and stiffness of the walls, i.e.:

$$F_{total} = K_b \cdot \Delta_b + K_{sw} \cdot \Delta_{sw} = (SR \cdot K_{sw}) \cdot \Delta_{sw} + K_{sw} \cdot \Delta_{sw} \quad (6)$$

Using the laminar analysis of coupled wall assemblies and considering uniform and inverted triangular load patterns, the stiffness ratio of the coupling beams to shear walls has been determined and plotted in Fig. 4.

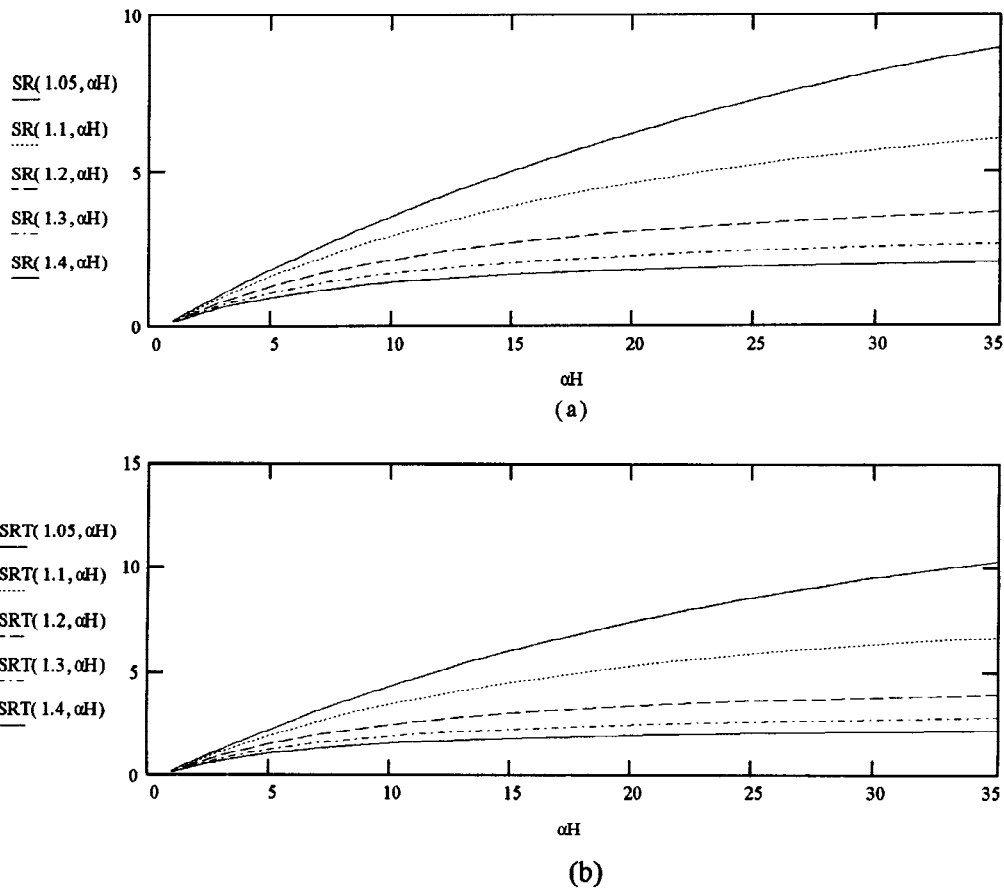


Fig. 4- The stiffness ratio of coupling beams to shear walls for coupled wall assemblies subjected to (a) uniform and (b) inverted triangular load patterns.

It is clear from these illustrations that the ratio SR depends on the geometric properties of members. In particular, two parameters influence the response of the coupled wall systems to lateral loads. The first one is the compliance coefficient,  $\mu$ , which is defined as follows:

$$\mu = 1 + \left( \frac{1}{A_1} + \frac{1}{A_2} \right) \cdot \frac{I}{L^2} \quad (7)$$

where,  $A_1$  and  $A_2$  are the cross sectional areas of walls,  $I$  is the sum of the moment of inertias of the two walls, and  $L$  is the distance between the centroidal axes of the two walls.

The second important parameter is the dimensionless quantity  $\alpha H$ , which measures the relative stiffness of the individual beam members with respect to that of the shear walls:

$$\alpha H = \left[ \frac{12I_p}{hb^3} \cdot \frac{L^2}{I} \cdot \mu \right]^{1/2} \cdot H \quad (8)$$

### ENERGY DISSIPATION IN DUAL STRUCTURAL SYSTEMS

Considering the fact that the primary intention of this study is to determine the response of the structure to cyclic loads, the focus of this analytical investigations is on the magnitude of the hysteretic energy loss incurred by the structural elements. For structural systems that experience cyclic yield excursions, the energy dissipated in the system may be defined in terms of the equivalent viscous damping ratio  $\xi$  (Clough and Penzien, 75):

$$\xi = \frac{E_d}{4\pi \cdot E_s} \quad (9)$$

According to this relationship, only two parameters are significant in the measurement of the equivalent viscous damping: the strain energy  $E_s$ , which is a measure of the elastic behavior of the structure, and the energy loss  $E_d$  which is a measure of the plastic behavior of the structure. More importantly, equation (9) reveals that it is not the absolute values of these quantities that is significant, but rather the relative values of the energy loss as compared to the strain energy that plays as important role. Consequently, the analytical investigations is primarily focused on the relative strength, stiffness and ductility of structural components (i.e., the shear walls and the coupling beams).

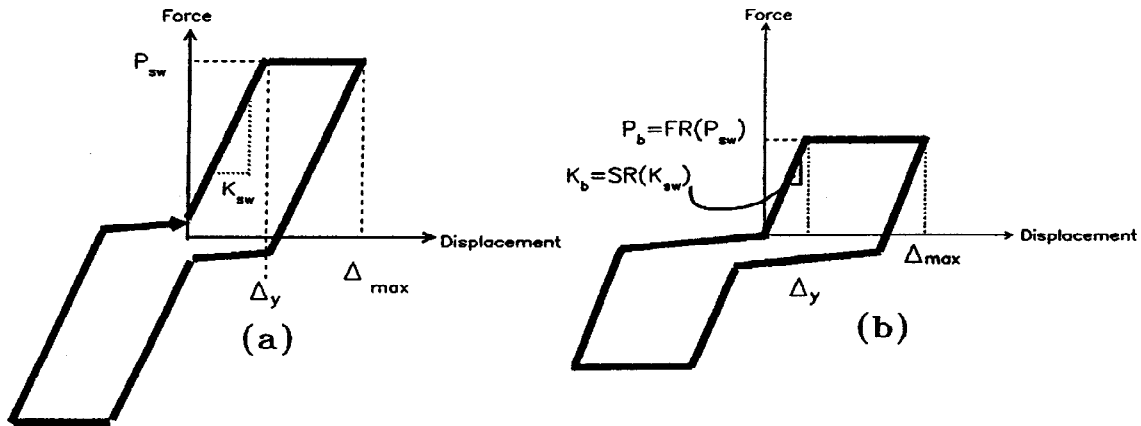


Fig. 5 - Idealized force-deformation relationships for (a) shear walls, and (b) coupling beams.

With the structural system comprising of two primary component, an idealized forced deformation relationship may be set up for each component (Fig. 5). For concrete members, the hysteretic force-deformation relationship might exhibit a pinch in the second and fourth quadrant. Hence, for an initial estimation of the energy dissipation, the idealized force-deformation relationship is drawn such that the energy loss in these regions is completely ignored. More importantly, in order to compare the response of the beams to the walls, the force deformation relationship of the beams will be defined in terms of that of the walls. For the elastic response of the structure, the overall stiffness of the coupling beams will be defined as follows:

$$K_b \cdot \left( \frac{\Delta_b}{\Delta_{sw}} \right) = SR \cdot K_{sw} \quad (10)$$

For the inelastic response of the structure, the yield strength of all the coupling beam will be expressed in terms of that of the shear wall:

$$P_b = FR \cdot P_{sw} \quad (11)$$

where  $P_b$ , and  $P_{sw}$  are the overall yield strength of the beams and shear wall members, respectively.

With these force deformation relationships, the hysteretic energy loss in the system may be determined at two different limit states: the damageability limit state and the ultimate limit state. At the damageability limit state, it is expected that the shear wall members remain elastic while the coupling beams yield and dissipate the input energy of dynamic vibration. For the ultimate limit state, it is anticipated that the shear walls have yielded as well, and are contributing to the energy dissipation in the structural systems.

For the damageability limit state, the energy loss will comprise of the energy dissipated in the coupling beams, but the elastic strain energy  $E_s$  consists of two components:

$$E_d = 2 P_b [ (\Delta_y)_{sw} - (\Delta_y)_b ] \quad (12)$$

$$E_s = (E_s)_{sw} + (E_s)_b = \frac{P_{sw}^2}{2K_{sw}} + \frac{P_b^2}{2K_b} \quad (13)$$

With the elastic strain energy and the energy loss defined, the equivalent viscous damping ratio  $\xi_{\text{damageability}}$  may be determined in terms of the stiffness and strength ratios (SR, and FR) of the coupling beams to the shear walls:

$$\xi_{\text{damageability}} = \frac{FR(SR - FR)}{\pi(SR + FR^2)} \quad (14)$$

A graph of this equation (Fig. 6a) reveals the magnitude of the damping ratio as a function of the stiffness and strength ratios, SR and FR. This Fig. clearly illustrates the trend in the values of FR and SR for optimum energy dissipation in the system.

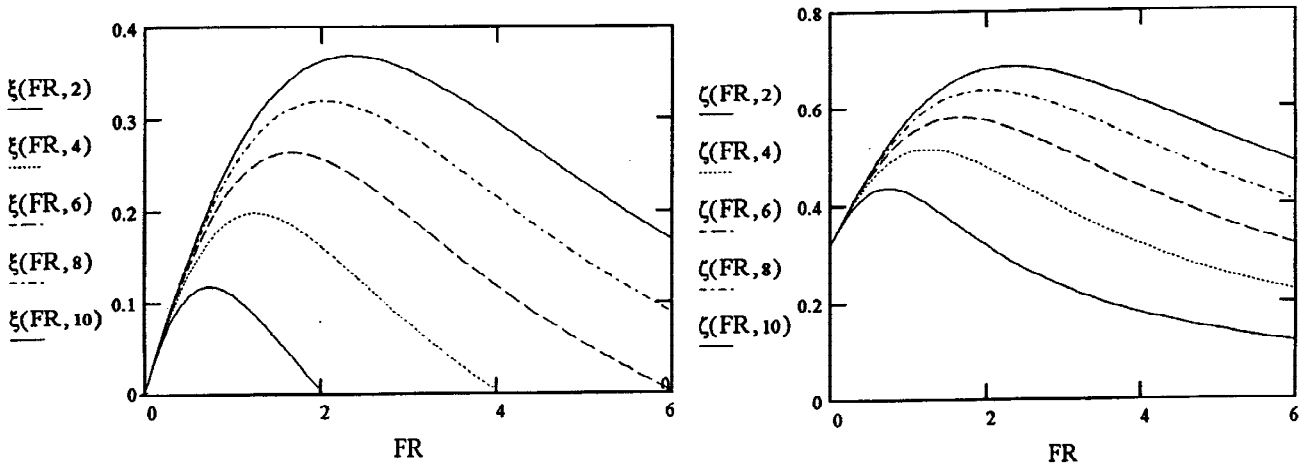


Fig. 6-(a) Equivalent viscous damping ratio at damageability, and (b) dependence of additional damping at ultimate limit state on the values of FR and SR.

For the ultimate limit state, a higher damping ratio  $\xi_{\text{ultimate}}$  is expected as a consequence of the yield excursions of the shear walls. The additional energy dissipated in the coupling beams and the shear walls may be determined from the force-deformation relationships in Fig. 5:

$$E_d = 2P_{sw}(\Delta_{\max} - \Delta_{y \cdot sw}) + 2FR \cdot P_{sw}(\Delta_{\max} - \Delta_{y \cdot sw}) \quad (15)$$

$$E_d = 2P_{sw} \cdot (\Delta_{y \cdot sw})(1 + FR)(\mu_{sw} - 1) \quad (16)$$

where  $\mu_{sw} = \Delta_{\max} / \Delta_{y \cdot sw}$  is the displacement ductility of shear walls. Considering the fact that the same magnitude of strain energy is stored in the structure as before (13), the additional damping in the structure would be:

$$\xi_{\text{add}} = \frac{(1 + FR)}{\pi(1 + \frac{FR^2}{SR})} (\mu_{sw} - 1) = \zeta(FR, SR)(\mu_{sw} - 1) \quad (17)$$

where  $\zeta(FR, SR) = (1 + FR) / \pi(1 + FR^2 / SR)$ . According to this relationship, the additional damping  $\xi_{\text{add}}$  at the ultimate limit state is a function of the strength and stiffness ratios (FR and SR) as well as the displacement

ductility of the shear wall elements  $\mu_{sw}$ . Fig. 6b is a graph of the function  $\zeta(FR,SR)$  which illustrates the interaction of the two parameters SR and FR on the additional damping ratio  $\xi_{add}$ . In general, Fig. 6 can be used to determine the upper and lower bounds on the magnitude of the damping ratio  $\xi$  for any existing structure:

$$\xi_{damageability} < \xi < \xi_{ultimate} = \xi_{damageability} + \xi_{add} \quad (18)$$

Similarly, they can be employed in a design process where the structural members are proportioned for the appropriate level of stiffness, strength and ductility demands.

## COMPARISON OF THE SEISMIC RESPONSE OF TWO COUPLED WALL ASSEMBLIES

In this section, the seismic response of two coupled wall assemblies in the 16-story apartment complex (Fig. 1-2) will be compared. In order to establish an upper and lower bounds on the maximum lateral resistance of the two coupled wall assemblies, two limits were considered for the axial coupling load, T. An upper limit of 1957 kips, corresponding to the maximum shear resistance of all the beams, and a lower limit of 968 kips, corresponding to a net axial load of zero on the tension wall of the coupled wall assembly. The lower limit was set after the assessment of the damage on the walls. Based on the observation that the walls showed no evidence of cracking, and the coupling beams on the south wall from 12th through 16th floors and on the 2nd floor showed no sign of cracking, it was presumed that not all beams have reached their maximum shear capacity. Hence, a lower bound for axial coupling load, T, based on the shear capacity of the beams that have failed was considered. Table 1 provides the bending moment resistance ( $M_1 + M_2$ ) and flexural resistance ( $EI_1 + EI_2$ ) of the tension and compression walls of the coupled wall assemblies on the north and south sides of the building. It is evident that because of the higher stiffnesses of the south wall, the coupled wall assembly on the south side of the building attracts more of the applied overturning moment. As a result, most of the beams on this side of the building have experienced shear failure.

But there is another factor that has contributed to the failure of coupling beams on the south walls. The force ratio, FR, of the south wall is half as much as that of the north wall. Basically, the yield capacity of the coupling beams on the south wall (as compared to the yield strength of the walls) is much lower than those in the north wall. One might consider that a lower value of FR provides a measure of safety against failure of the walls (i.e., with the beams dissipating the input energy of vibration). But in reality, in most coupled wall assemblies, the beams yield prior to shear walls. This trend in the behavior of the members is due to the fact that the coupling beams usually have a higher stiffness (i.e.,  $SR * K_{wall}$ ) than the shear walls, but their yield strength (i.e.,  $FR * P_{wall}$ ) is comparable to that of the walls.

Table 1 - Strength, stiffness and damping properties of coupled wall assemblies

| Coupled wall system |        | Strength         |           |      | Stiffness |       |            |                 | Damping               |
|---------------------|--------|------------------|-----------|------|-----------|-------|------------|-----------------|-----------------------|
| Walls               | T(kip) | T.L <sup>#</sup> | $M_1+M_2$ | FR   | $EI_w$    | $\mu$ | $\alpha H$ | SR <sup>*</sup> | $\xi_{damageability}$ |
| North Wall          | 1952   | 100,052          | 38,839    | 2.58 | 160       | 1.06  | 53         | 8.9(UN)         | 0.27 to 0.40          |
|                     | 968    | 49,489           | 44,221    | 1.12 | 112       | 1.06  | 53         | 11.3(TR)        |                       |
| South Wall          | 1952   | 89,777           | 65,517    | 1.37 | 282       | 1.07  | 33         | 8.2(TR)         | 0.17 to 0.30          |
|                     | 968    | 44,407           | 71751     | 0.62 | 246       | 1.07  | 33         | 7.3(UN)         |                       |

<sup>#</sup> TL and  $M_1+M_2$  are in K.ft,  $EI_w$  is in  $10^6$  K.ft<sup>2</sup>.

<sup>\*</sup> Stiffness ratio is determined for either an inverted triangular (TR) or uniform (UN) load pattern.

In Table 1, the stiffness ratio, SR, of the coupled wall assemblies on the north and south side of the building has been tabulated, as well. In both cases, the individual beam members have much higher flexural stiffness compared to the wall members (i.e.,  $\alpha H$  is very high). Consequently, during elastic response of the structure, the beams contribution to the overall bending moment resistance is one order of magnitude higher than those of the walls (i.e.,  $7 < SR < 11$ ). Based on the stiffness and force ratios (SR, and FR), the energy dissipation capacity of the two walls on the north and south sides of the building have been provided in Table 1. The coupled wall assembly on the north side, certainly has a higher potential for energy dissipation..

## RECOMMENDATIONS FOR PROPORTIONING AND DESIGN OF MEMBERS

There is no question that the seismic response of coupling beams on the south walls showed evidence of shear failure, and this is certainly an undesirable mode of failure. However, no clear guidelines have ever been provided in the technical literature for proportioning structural members for optimum seismic response. In this section, an attempt has been made to make recommendations for proportioning and design of structural members in a coupled wall assembly.

Generally, the coupled wall systems are expected to resist a portion of the lateral loads applied to the structure. It has been recommended (Aktan and Bertero, 81) that if the coupling axial force on the tension wall does not result in a net tensile force, a more balanced shear and overturning moment resistance can be expected from the tension and the compression walls. Hence, for an initial estimation of axial coupling force, T, the net tensile force on the walls may be assumed to be zero. After estimating the total overturning moment ( $M_{ot}$ ) and the axial coupling force (T), the strength ratio (FR) should be estimated:

$$FR = \frac{T \cdot L}{M_{ot} - T \cdot L} \quad (19)$$

Based on the value of the force ratio, FR, and a desired level of energy dissipation (i.e., damping ratio,  $\xi$ ), the stiffness ratio SR may be selected from Fig. 6. Knowing that the elastic response of the structure must be governed by the condition of equilibrium and compatibility, the value of the stiffness ratio, SR, may be used in Fig. 4 to select the parameter  $\alpha H$ . It is important to point out that the curves of Fig. 4 asymptotically reach a particular value of SR for any given compliance coefficient  $\mu$ . It is certainly not beneficial to select a very high value of  $\alpha H$ , just to make marginal improvement on the value of SR. It is apparent that proportioning a beam member for high values of  $\alpha H$  would result in a very stiff coupling beam with poor ductility levels.

Once the parameter  $\alpha H$  has been selected, the flexural stiffness of the coupling beam member ( $12EI_p/b^3$ ) can be determined from the flexural properties of the shear wall member (equation 8), and the beam member may be proportioned accordingly. Finally, the coupling beams should be designed for flexural and shear reinforcement such that the maximum axial coupling force developed at the base of the wall is T.

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