

RELATIVE DISPLACEMENT RESPONSE SPECTRUM AND ITS APPLICATION

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ABSTRACT

Relative displacement response spectra is proposed to clarify the relative displacement developed between two structural segments with different natural period and damping ratio. Relative displacement is important in bridges where two structural segments are connected at a hinge. When excessive relative displacement is developed at the hinge, the deck supported by the other deck falls from the supports. To simplify the problem, two structural segments are idealized by two single-degree-of-freedom oscillators, and relative displacements are computed for oscillators with various natural period and damping ratio. Because of the analogy to the displacement response spectra, it was proposed to call *relative displacement response spectra*. Effect of earthquake magnitude, epicentral distance and site condition was clarified based on 63 strong motion records, and a design value for the relative displacement response spectra was proposed based on a series of analysis. A numerical example is also presented.

Key Word: Seismic design, Bridge, Building, Structure, Earthquake response Spectrum, Response spectral analysis, Dynamic response analysis, Ground motion, Relative displacement, Seismic damage

INTRODUCTION

Relative displacement developed between two adjacent structural segments is important when a structural segment is supported by the other at a hinge. The segment falls if the hinge seat length is not sufficient for the relative displacement developed between the two segments. This evidence actually happened when bridges felt from their hinge seats in the past earthquakes. Because the relative displacement depends on various parameters such as natural period and damping ratio of two segments and ground motion characteristics, it is useful to clarify its characteristics based on a simplified analytical model. For this purpose, the structures are idealized by two single-degree-of-freedom oscillators, and the relative displacement between the two oscillators was computed. The relative displacements between the two oscillators with various natural period and damping ratio is proposed to call *relative displacement response spectra* from its analogy to the displacement response spectra.

DEFINITION OF RELATIVE DISPLACEMENT RESPONSE SPECTRA

To study the relative displacement between two structural segments, it is assumed to idealize the structures as shown in Fig. 1. Structures are idealized by two single-degree-of-freedom oscillators subjected to the same ground motion at their bases. Natural period and damping ratio are denoted as T_1 and h_1 for No. 1 oscillator and T_2 and h_2 for No. 2 oscillator. Although friction and restrainers may cause some interactions between the two structures, it is assumed here that they are negligibly small for simplicity of analysis. Effect of spatial variation of ground motions to each structure (Kawashima, 1994 a) is also disregarded here because it requires analysis on ground strain from different point of view.

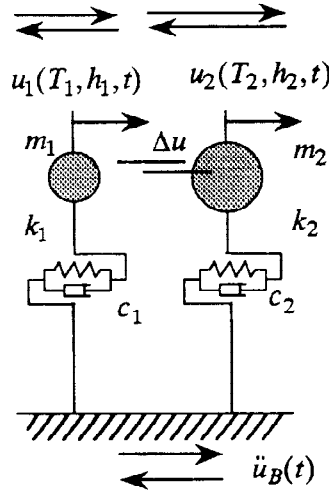


Fig. 1 Idealization of relative displacements between two structural segments

Displacements of the oscillators relative to their bases for a ground motion are denoted as $u_1(T_1, h_1, t)$ and $u_2(T_2, h_2, t)$. The relative displacement between the two oscillators is then defined as

$$\Delta u(T_1, T_2, h_1, h_2, t) = u_2(T_2, h_2, t) - u_1(T_1, h_1, t) \quad (1)$$

The peak values of the displacements are written as

$$S_D(T_1, h_1) = \max |u_1(T_1, h_1, t)|; \quad S_D(T_2, h_2) = \max |u_2(T_2, h_2, t)|; \quad \Delta S_D(T_1, T_2, h_1, h_2) = \max |\Delta u(T_1, T_2, h_1, h_2, t)| \quad (2)$$

in which $S_D(T, h)$ = displacement response spectrum, and $\Delta S_D(T_1, T_2, h_1, h_2)$ = relative displacement response spectra.

Representing $T_2 = \alpha T_1$, difference of two natural periods can be written as $\Delta T = (\alpha - 1)T_1$. Thus, $\alpha - 1$ represents $\Delta T / T_1$. Using this expression, $\Delta S_D(T_1, T_2, h_1, h_2)$ may be expressed as $\Delta S_D(T_1, \alpha T_1, h_1, h_2)$. Normalizing $\Delta S_D(T_1, \alpha T_1, h_1, h_2)$ by $S_D(T_1, h_1)$, one obtains

$$R_D(T_1, \alpha T_1, h_1, h_2) = \Delta S_D(T_1, \alpha T_1, h_1, h_2) / S_D(T_1, h_1) \quad (3)$$

where $R_D(T_1, \alpha T_1, h_1, h_2)$ is called *relative displacement ratio response spectra*.

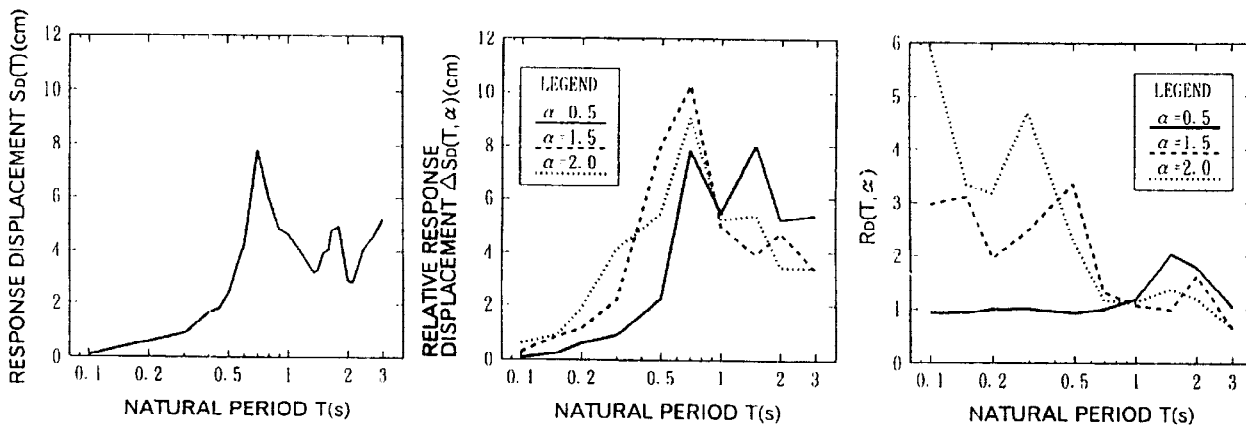
In the following analysis, it is assumed that $h_1 = h_2 = 0.05$ for the simplicity of analysis. Damping ratio of 0.05 may be the typical value for most of structures with moderate size. Damping ratio h and subscripts 1 and 2 are then dropped from Eq.(3); hence, $R_D(T, \alpha) = R_D(T, \alpha T, 0.05, 0.05) = \Delta S_D(T, \alpha) / S_D(T)$.

RELATIVE DISPLACEMENT RESPONSE SPECTRA

The relative displacement response spectra $\Delta S_D(T, \alpha)$ and the relative displacement ratio response spectrum $R_D(T, \alpha)$ were evaluated for a ground motion recorded near Itajima bridge during the 1968 Hyuga-

nada-oki earthquake with magnitude 7.5. The epicentral distance was 103 km. Response displacement $u(T, 0.05, t)$ and relative displacement $\Delta u(T, \alpha T, 0.05, 0.05, t)$ were computed for various natural periods and α . It is interesting to note that the relative displacement between two oscillators is sometimes larger than the response displacement of the oscillators depending on natural period and ground motion. For example, the peak displacement of an oscillator with $T_1=0.5$ s is 2.39 cm, while the peak relative displacement is 8.0 cm for $\alpha=1.5$ and 5.48 cm for $\alpha=2$. The relative displacement is 3.3 and 2.3 times larger than the displacement of an oscillator.

Computing the relative displacement for various natural period, one can obtain the relative displacement response spectrum $\Delta S_D(T, \alpha)$ and the relative displacement ratio response spectrum $R_D(T, \alpha)$ for the Itajima record as shown in Fig. 2. The displacement response spectrum $S_D(T)$ is also presented in Fig. 2 for comparison. It is seen in Fig. 2 that $\Delta S_D(T, \alpha)$ takes peaks at about 0.7 s. This reflects the fact that 0.7 s is one of the predominant periods of the ground motion. It should be noted in Eq. (2) that $\Delta S_D(T_1, T_2, 0.05, 0.05) = \Delta S_D(T_2, T_1, 0.05, 0.05)$. For example, $\Delta S_D(T, \alpha)$ for $T = 0.5$ s and $\alpha = 2$ and $\Delta S_D(T, \alpha)$ for $T = 1.0$ s and $\alpha = 0.5$ are the same. $R_D(T, \alpha)$ is large at small natural period, because $S_D(T)$ is smaller than $\Delta S_D(T, \alpha)$ at short natural period.



(a) Displacement response spectra $S_D(T, 0.05)$ (b) Relative displacement response spectra $\Delta S_D(T, \alpha)$ (c) relative displacement ratio response spectra $R_D(T, \alpha)$

Fig. 2 Relative displacement response spectra $\Delta S_D(T, \alpha)$, relative displacement ratio response spectra $R_D(T, \alpha)$ and displacement response spectra $S_D(T, 0.05)$ for the Itajima record

Figure 3 shows $R_D(T, \alpha)$ vs. $\alpha - 1 (= \Delta T / T)$ relation from Fig. 2. $R_D(T, \alpha)$ is always zero at $\alpha - 1 = 0$ from the definition. At $\alpha - 1$ smaller than about -0.4, $R_D(T, \alpha)$ approaches to 1.0. This is because as α becomes smaller, $u_2(T_2, h_2, t)$ becomes negligible small as compared to $u_1(T_1, h_1, t)$. At $\alpha - 1 \geq 0$, $R_D(T, \alpha)$ seems to take its peaks at a certain value of $\alpha - 1$.

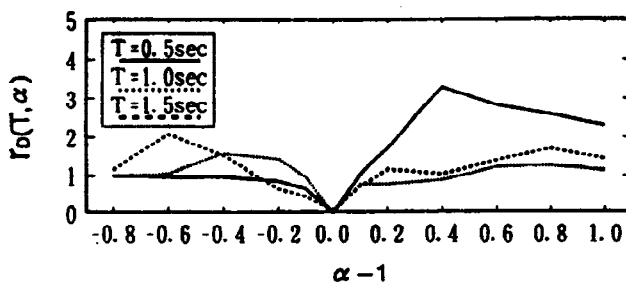


Fig. 3 $R_D(T, \alpha)$ vs. α relation for the Itajima record

The same analysis was made for 63 components of strong motion records which were obtained in Japan by earthquakes with magnitude larger than or equal to 6.5 and with focal depth less than 60 km. The classification was made in accordance with the Seismic Design Specifications of Highway Bridges (JRA 1990, Kawashima and Hasegawa 1994 b), and is based on natural period of the sites. Number of records at Type I (stiff), Type II (moderate) and Type III (soft) sites is 13, 37 and 13, respectively.

Fig. 4 shows how $R_D(T, \alpha)$ depends on the earthquake magnitude M and the epicentral distance Δ . Because other cases show the similar results, $R_D(T, \alpha)$ with $\alpha - 2$ is presented in Fig. 4. Although scattering of $R_D(T, \alpha)$ depending on M and Δ is significant, consistent change of $R_D(T, \alpha)$ depending on M and Δ is not observed. It may be considered that $R_D(T, \alpha)$ is almost independent of the earthquake magnitude and the epicentral distance.

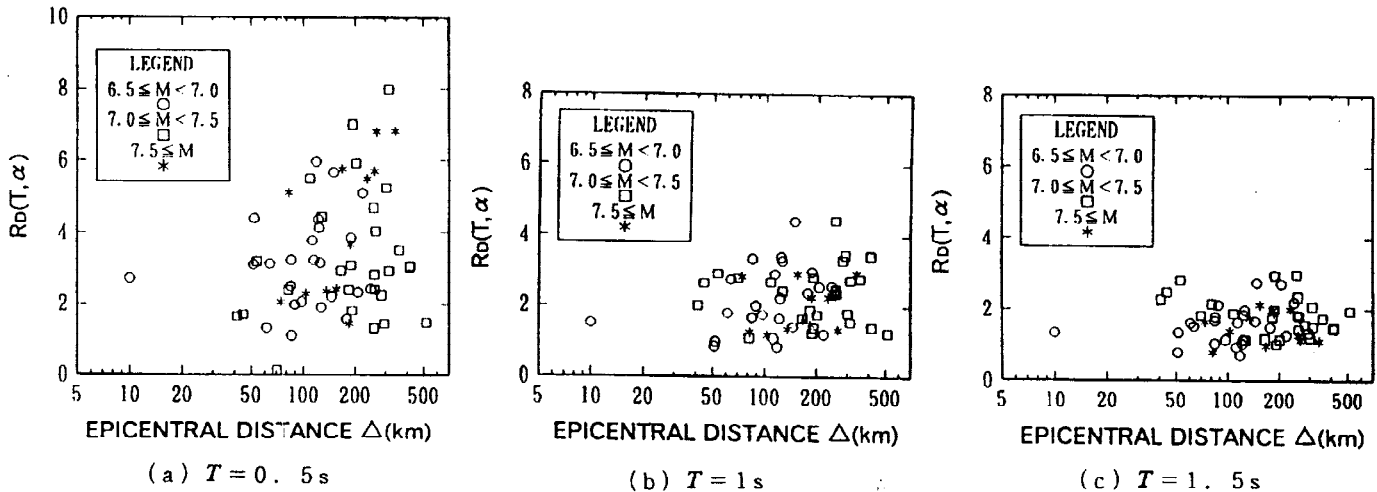


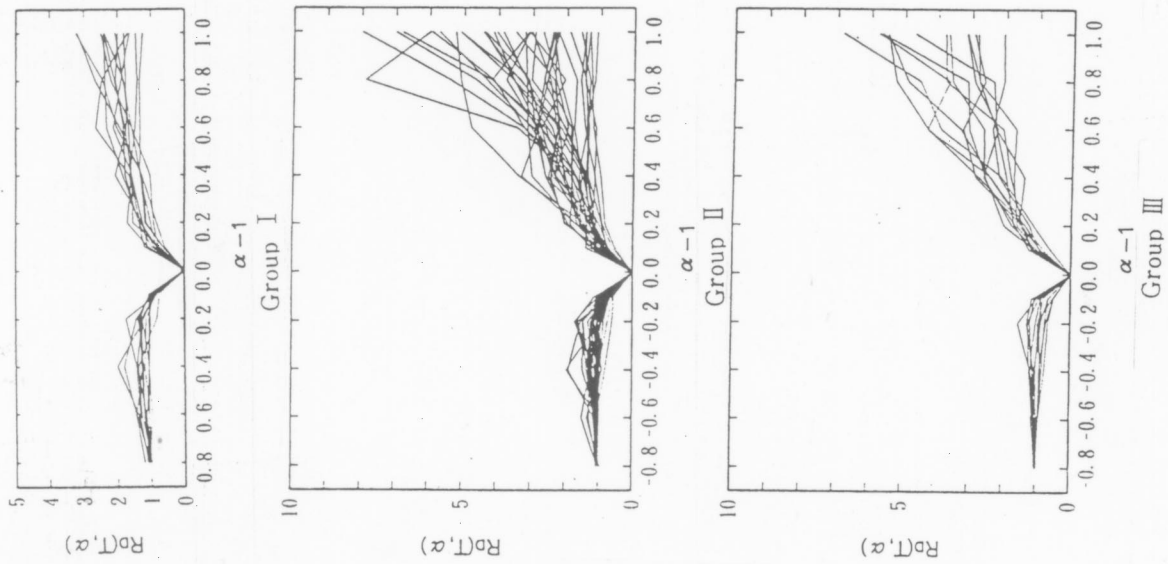
Fig. 4 Effect of earthquake magnitude and epicentral distance on $R_D(T, \alpha)$ vs. α relation

Fig. 5 shows the effect of site condition on $R_D(T, \alpha)$ vs. $\alpha - 1$ relation for various natural periods. Because scattering of $R_D(T, \alpha)$ is quite large, the mean values of $R_D(T, \alpha)$ were computed as shown in Fig. 6. They are similar between three site conditions although there is some differences at $\alpha - 1 > 0$ in $T = 0.5$ s. It may be therefore considered that the effect of site condition on $R_D(T, \alpha)$ is less significant.

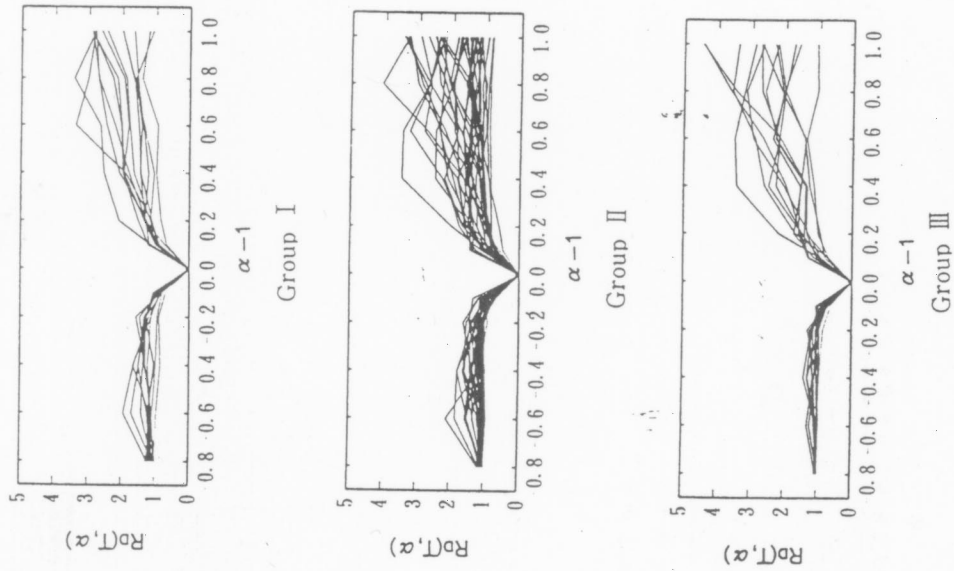
$R_D(T, \alpha)$ PROPOSED FOR DESIGN PURPOSE

Because the effect of earthquake magnitude, epicentral distance and the site condition on the relative displacement ratio response spectra is less significant, the mean value and the standard deviation of $R_D(T, \alpha)$ were evaluated as shown in Fig. 7. Based on the definition, the mean value of $R_D(T, \alpha)$ is zero at $\alpha - 1 = 0$, and approaches to 1.0 as $\alpha - 1$ becomes smaller than about 0.6. $R_D(T, \alpha)$ increases as $\alpha - 1$ increases. At natural period longer than about 1.5 s, $R_D(T, \alpha)$ tends to have their peak values at a certain $\alpha - 1$. For example, when T is 1.5 s, $R_D(T, \alpha)$ takes its peak value at $\alpha - 1 = 1.6 \sim 1.8$. This may be due to the fact that as T increases, the natural period αT becomes longer than the predominant period of ground motions.

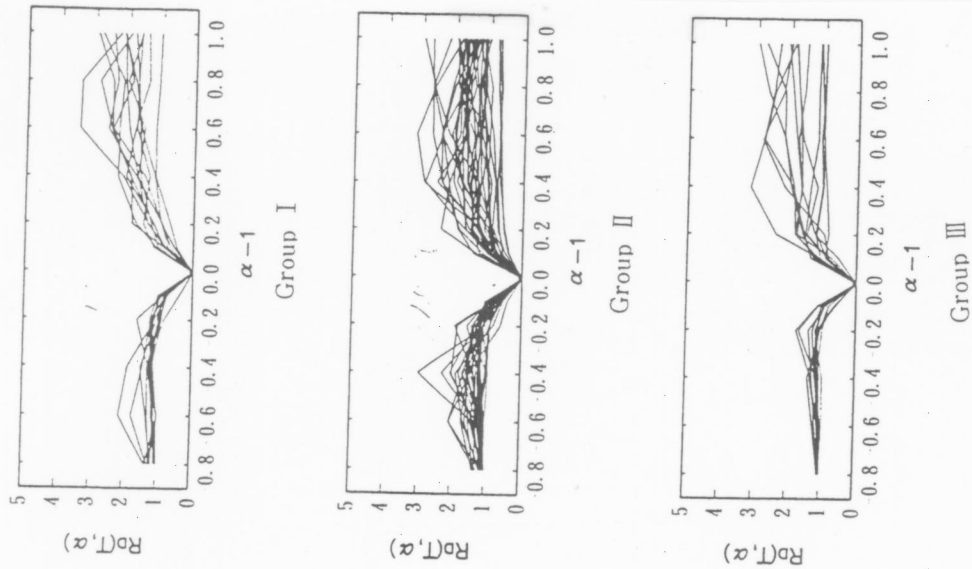
Because the scattering of $R_D(T, \alpha)$ is very large, it is proposed to include one standard deviation from the mean value in evaluating the relative displacement ratio response spectra $R_D(T, \alpha)$ for design purpose. Fig. 8 shows $R_D(T, \alpha)$ thus obtained.



(a) $T = 0.5$ s

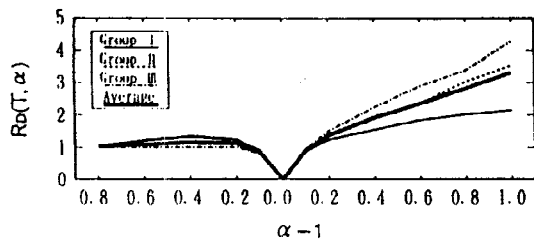


(b) $T = 1$ s

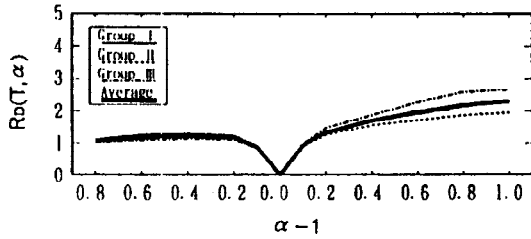


(c) $T = 1.5$ s

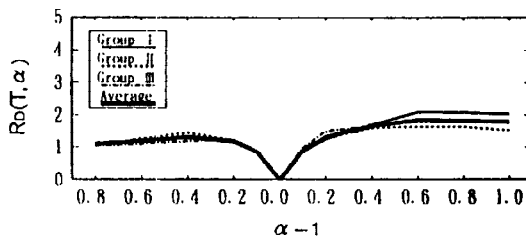
Fig. 5 Effect of site condition on $R_D(T, \alpha)$ vs. α relation



(a) $T = 0.5 \text{ s}$

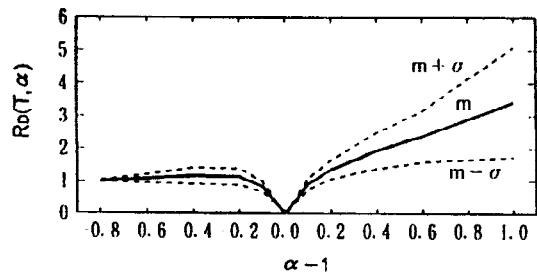


(b) $T = 1 \text{ s}$

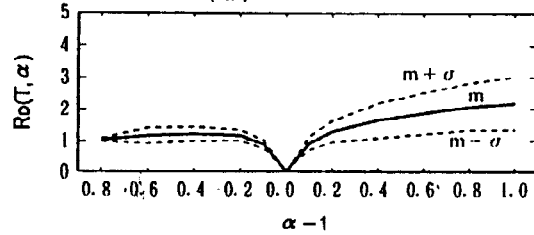


(c) $T = 1.5 \text{ s}$

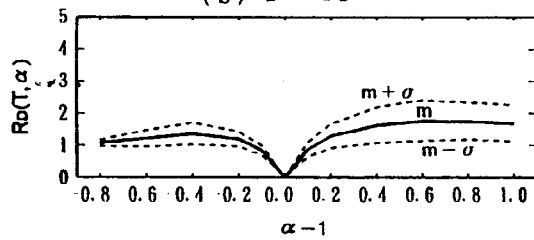
Fig. 6 Mean value of $R_D(T, \alpha)$



(a) $T = 0.5 \text{ s}$



(b) $T = 1 \text{ s}$



(c) $T = 1.5 \text{ s}$

Fig. 7 Mean value and standard deviation of $R_D(T, \alpha)$

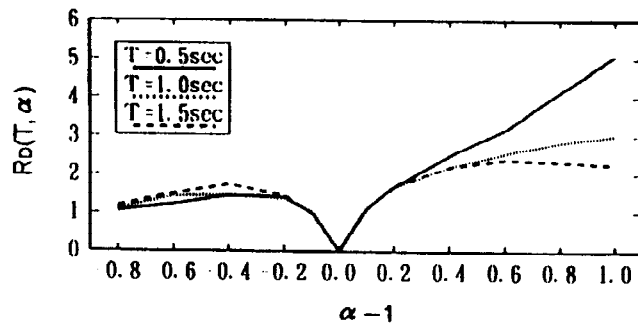


Fig. 8 Relative displacement ratio response spectra $R_D(T, \alpha)$ proposed for design

APPLICATION OF $R_D(T, \alpha)$ TO EVALUATE SEAT LENGTH OF BRIDGES

The relative displacement response spectra was applied to evaluate relative displacement of bridges at hinge joints. Let us consider a bridge consisting of two structural segments which are divided by an intermediate hinge. It is assumed that natural period of the two structural segments is T and αT , and that damping ratio h is 0.05. We assume that T is 0.5 s and 1 s and that α is from 0.2 to 2. This combination covers wide range of typical bridges with moderate size. Because mass of bridges concentrate at the deck, they can be idealized with reasonable accuracy by single-degree-of-freedom oscillators.

The displacement response spectra $S_D(T,0.05)$ may be evaluated by $S_D(T,0.05) = (T / 2\pi)^2 \cdot S_A(T,0.05)$ where $S_A(T,0.05)$ represents the acceleration response spectra with 5% damping ration. $S_A(T,0.05)$ may be evaluated as (Kawashima *et al.*, 1984 a, Kawashima and Aizawa, 1984 b),

$$S_A(T,0.05) = a(T,GC) \times 10^{b(T,GC)M} \times (\Delta + 30)^{-1.178} \quad (4)$$

where, $a(T,GC)$ and $b(T,GC)$ = coefficients depending on natural period T and site condition GC , M = earthquake magnitude and Δ = epicentral distance (km). It should be noted that Eq. (4) represents the mean value of the attenuation, and if we consider one standard deviation from the mean value, we have to multiply 1.7 to the value evaluated by Eq. (4). Fig. 9 shows the mean value of $S_D(T,0.05)$ thus evaluated for $M = 8$ and $\Delta = 50$ km. The response displacement is generally larger at soft soil site. The relative displacement response was then obtained from Fig. 8 and Fig. 9 as shown in Fig. 10. As $\alpha - 1$ increases $\Delta S_D(T, \alpha)$ increases. When $T = 1$ s and soil condition is Type III (soft soil), $\Delta S_D(T, \alpha)$ reaches 68.9 cm at $\alpha = 2$.

On the other hand, in the "Part V Seismic Design" of the "Design Specifications of Highway Bridges" (JMA, 1990, Kawashima and Hasegawa, 1994 b), the minimum seat length S_E (m) at hinges is provided as

$$S_E = 0.7 + 0.005l \quad (5)$$

where l is span length (m). Eq. (5) may be interpreted as a linear combination of relative displacement due to response of bridges and the relative displacement associated with ground strain between the span length l . The second term has to be evaluated by the ground strain induced during earthquakes (Okubo *et al.*, 1983). 5,000 micro strain is assumed in Eq. (5). Comparing the relative displacement in Fig. 12 to 0.7 m in Eq. (5), it may be said that 0.7 m is conservative for most of bridges with moderate natural period T and moderate difference of natural period. However at Type III soil site (soft soil site), 0.7 m may be underestimation of the relative displacement for the bridges with long natural period T and large difference of natural period.

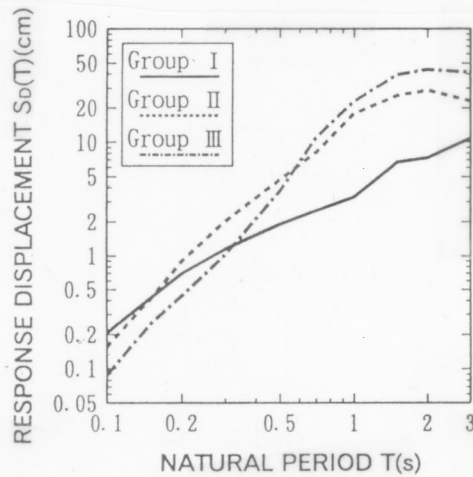
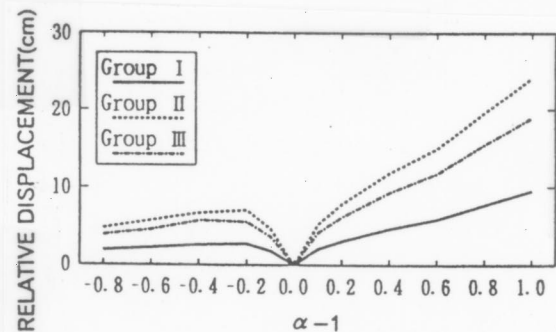
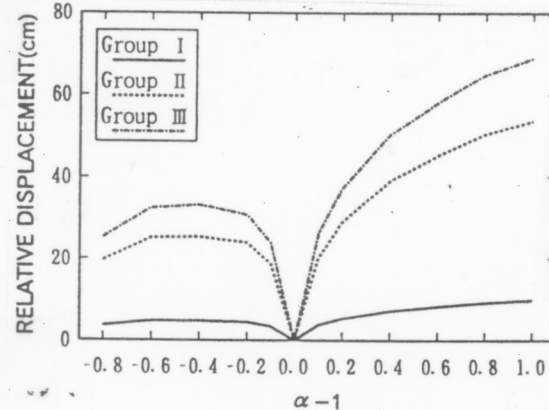


Fig. 9 Displacement response spectra $S_D(T,0.05)$ estimated by Eq. (4) for $M = 8$ and $\Delta = 50$ km



(a) $T = 0.5$ s



(b) $T = 1$ s

Fig. 10 Relative displacement $\Delta S_D(T, \alpha)$ estimated from Figs. 8 and 9

CONCLUSIONS

For evaluating relative displacement between two structural segments with different natural period and damping ratio, the *relative displacement response spectra* and the *relative displacement ratio response spectra* was proposed, and their characteristics were clarified based on 63 acceleration records. A numerical example for its application was also presented. Based on the results presented herein, the following conclusions may be deduced:

(1) Relative displacement between two structural segments is often larger than the displacement of structural segments. The relative displacement response spectrum is effective to represent overall characteristics of the relative displacement between two structural segments.

(2) Relative displacement depends on earthquake magnitude, epicentral distance and ground condition. However, effect of those parameters on relative displacement response ratio spectra $R_D(T, \alpha)$ vs. $\alpha - 1$ relation is less significant. Based on this, $R_D(T, \alpha)$ for design purpose was proposed as shown in Fig. 8.

(3) The proposed relative displacement ratio response spectra was applied to evaluate the seat length S_E at hinges of highway bridges. It was seen that the hinge seat length S_E specified in the current Japanese highway bridge code is conservative for most of bridges with moderate natural periods T and moderate difference of natural period (α). However, it may provide some underestimation at soft soil site when T and α increase.

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