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LIQUEFACTION ANALYSIS: A PROBABILISTIC APPROACH

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The phenomena that occurs within cohesionless saturated soils during earthquakes due to the loss of strength or stiffness of a soil is called liquefaction. When performing liquefaction analysis of cohesionless soils (silts, sands, and some gravels) the factors of importance include the earthquake which that the soil profile is located (which affects the magnitude, frequency content, peak ground acceleration, and the duration of the ground acceleration); the previous stressing history of the soil, the topography and the level of the water table of the soil, and the characteristics of the soil grains; their geometry, size and other soil material properties including the rate of flow of water through the voids in the soil skeleton. The occurrence of earthquakes and their intensity is a random phenomena; as well as the physical and material characteristics of the soil. Therefore, in order to obtain realistic results for liquefaction analysis of a soil profile, probabilistic ground characterization should be performed.

The challenge for probabilistic ground characterization is obtaining sufficient soil data in order to perform a full scale study. Research dating back to the 1940's demonstrate that the soil characteristics such as its permeability is lognormally distributed. Other parameters such as the soil relative density, void ratio, and porosity depict non-Gaussian behavior. However, obtaining the distribution function of the soil parameters is based on in-situ tests that give these parameters directly; such as the triaxial test, grain size test, and atterberg limit tests. As these in-situ tests are costly to perform in large numbers , it is difficult to attain reliable probabilistic results. Therefore, the standard penetration resistance, commonly available in large numbers, is utilized in the probabilistic characterization of the ground. The distribution function of the standard penetration resistance is easily obtained, and correlated to other soil parameters including the relative density, friction and dilation angle. Upon determining the distribution functions for the soil parameters, random field depicting the soil profile are generated based on the Spectral Representation Method. Upon attaining these stochastic fields, liquefaction analysis is performed utilizing finite element methodology. The goal of the research presented, is to find an appropriate method to determine the distribution functions of soil parameters.

It is important to determine the probability density function, or distribution of soil parameters in order to generate their soil parameters stochastic fields. Ideally, the distribution functions of soil parameters can be determined numerically. The availability of ample amounts of data is necessary to determine the distribution. The knowledge of the spatially exact locations of data acquisition is essential to estimate the power spectral density function of soil properties. In the absence of the availability of data, the distribution functions are estimated and verified by statistical tests. The data is depicted by a frequency histogram, from which a distribution function is hypothesized. The hypothesis is then validated through a goodness-of-fit test. First, a methodology to determine the distribution function for the soil parameter of relative density is examined.

Probability Density Function for Relative Density of Soils :

A (skewed) distribution function is proposed for the relative density, which is studied since it is one parameter that has boundary limitations when modeling all soils. The procedure to obtain the probability density function for the relative density is outlined below:

1) Let $F(x,y)$ be a Gaussian Field with zero mean and unit standard deviation which is generated by the Spectral Representation Method. Given the mean value and standard deviation for the relative density, $X(x,y)$ is the resulting normalized field. "x" and "y" correspond to the length and depth of the soil profile of interest, respectively. $\mu(y)$ is the Mean Relative Density, and; $\sigma(y)$:Standard Deviation of Relative Density with respect to depth, y.

$$X(x,y) = \mu(y)+\sigma(y)F(x,y) \qquad [1]$$

2) From previous statistical studies performed [Ref.2, 3], it is assumed that the relative density of a soil has a skewed distribution, and the following transformation is performed:

$$X(x,y) = \ln Y(x,y) \qquad [2]$$

$X(x,y)$ is a normally distributed field, and $Y(x,y)$ is a lognormally distributed field. By definition, a random variable that is lognormally distributed has a lower bound of 0. In order to comply with an upper bound of 1, (as the relative density is bounded between 0 and 1), the following is proposed for the field $Z(x,y)$, considering a free parameter of *a*.

$$Z(x,y) = \tanh \frac{Y(x,y)}{a}, \quad a>0. \qquad [3]$$

The hyperbolic tangent is bounded between -1 and 1, therefore equation [3] fits the requirements as the lognormal distribution has a lower limit of 0.

3) Given the probability density function of Y $f_Y(y)$, the distribution function for z, $f_Z(z)$ can be determined by:

$$f_Z(z) = \frac{dy}{dz} f_Y(y) \qquad [4]$$

The probability density function of the lognormal distribution, $f_Y(y)$ is computed by the expression given below where μ_x, and σ_x are the mean and standard deviation of Gaussian Field, F respectively.

$$f_Y(y) = \frac{1}{y\sigma_x\sqrt{2\pi}} \exp\left\{-\frac{1}{2}\left(\frac{\ln y - \mu_x}{\sigma_x}\right)^2\right\}, \quad y \geq 0. \qquad [5]$$

The following equation is deduced from the transformation of field Y(x,y) to Z(x,y) :

$$y = \frac{a}{2}\ln\left(\frac{1+z}{1-z}\right). \qquad [6]$$

First, the lower boundary for z is checked, given that $y \geq 0$. For this transformation, $z \geq 0$, and $a>0$.

4) The distribution of z denoting (the relative density) is determined as:

$$f_Z(z) = \frac{\left[\frac{1}{1+z} + \frac{1}{1-z}\right]}{\sigma_x\sqrt{2\pi}\ln\left(\frac{1+z}{1-z}\right)} \exp\left\{-\frac{1}{2}\left(\frac{\ln\left[\frac{a}{2}\ln\left(\frac{1+z}{1-z}\right)\right]}{\sigma_x}\right)^2\right\}. \qquad [7]$$

$$z \in (0,1)$$

This distribution is depicted in figure 1 for standard deviations of 1. and varying values of *a*. The peak values, and their locations are determined from the first derivative of the distribution of z:

$$f'_Z(z) = \frac{4\left(-\sigma_x^2 + \sigma_x^2 z\ln\left(\frac{1+z}{1-z}\right) - \ln\left[\frac{a}{2}\ln\left(\frac{1+z}{1-z}\right)\right]\right)}{\sigma_x^3\sqrt{2\pi}(z-1)^2(z+1)^2\ln\left(\frac{1+z}{1-z}\right)^2} \exp\left\{-\frac{1}{2}\left(\frac{\ln\left[\pi\ln\left(\frac{1+z}{1-z}\right)\right]}{\sigma_x}\right)^2\right\}. \qquad [8]$$

The peak for standard deviation of 1 are as depicted below:

a	z	f(z)
4	0.0933181	2.653686
6	0.061704	3.961436
8	0.0461486	5.273139
10	0.0368714	6.586376

The mean of the obtained density function f(z) is as follows:

$$\mu_Z(z) = \int_0^1 \frac{z\left[\frac{1}{1+z} + \frac{1}{1-z}\right]}{\sigma_x \sqrt{2\pi}\ln\left(\frac{1+z}{1-z}\right)} \exp\left\{-\frac{1}{2}\left(\frac{\ln\left[\frac{a}{2}\ln\left(\frac{1+z}{1-z}\right)\right]}{\sigma_x}\right)^2\right\} dz \qquad [9]$$

The mean values for the distribution, considering unit standard deviation and varying values of "a" are:

a	μ_Z
4	0.320159
6	0.234578
8	0.184588
10	0.151925

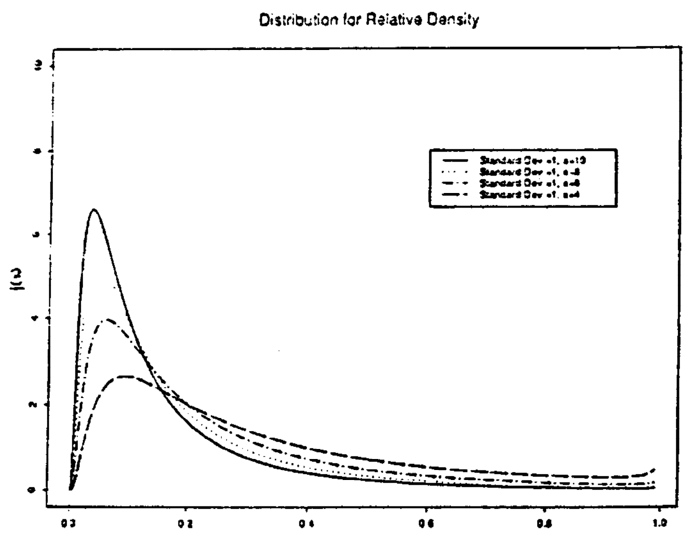

Figure 1 Distribution for Relative Density : *Standard Deviation = 1*

The variance of the obtained density function f(z) is as follows:

$$\sigma_Z(z) = \int_0^1 \frac{(z-\mu_z)^2\left[\frac{1}{1+z} + \frac{1}{1-z}\right]}{\sigma_x \sqrt{2\pi}\ln\left(\frac{1+z}{1-z}\right)} \exp\left\{-\frac{1}{2}\left(\frac{\ln\left[\frac{a}{2}\ln\left(\frac{1+z}{1-z}\right)\right]}{\sigma_x}\right)^2\right\} dz. \qquad [10]$$

Standard Penetration Resistance Probability Density Function:

Soil properties are described by their distribution functions. The distribution function is necessary in generating their stochastic fields. In this study, a procedure to obtain the distribution function of the standard penetration resistance is given. A Non-Gaussian distribution must be considered to fit the data, and then tested with a goodness-of-fit method. The distributions that appropriately depict a positive skew include the lognormal, gamma, and beta distributions [Benjamin and Cornell].

The lognormal distribution may have a shift, "a", and the distribution function becomes:

$$f_T(t) = \frac{1}{\sigma_{\ln Y}(t-a)\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{\ln(t-a)-\mu_{\ln Y}}{\sigma_{\ln Y}}\right)^2\right] \quad \text{for } t > a. \qquad [11]$$

The gamma distribution has a probability density function $f_T(t)$, with a shift of "a":

$$f_T(t) = \frac{\lambda[\lambda(t-a)]^{k-1} e^{-\lambda(t-a)}}{\Gamma(k)} \quad \text{for } t \geq a. \qquad [12]$$

The beta distribution (with lower bound, "a" and upper bound, "b"), has a probability density function of:

$$f_Y(y) = \frac{1}{\frac{\Gamma(r)\Gamma(t-r)}{\Gamma(t)}(b-a)^{t-1}}(y-a)^{r-1}(b-y)^{t-r-1}, \quad a \leq y \leq b. \qquad [13]$$

Amongst the three distributions described above, the beta distribution has 4 parameters that define the shape of the distribution curve, which is comparatively greater than the lognormal (one parameter) or gamma distribution (two parameters). The beta distribution has the capability to fit nearly any histogram (any skew, or shape) with the aid of the shape parameters (q and r), lower bound, a, and upper bound b. Therefore the beta distribution is chosen as the distribution to fit the standard penetration resistance (histograms). Under the hypothesis that the data is beta distributed (skewed to the left : $q < r$), a goodness-of-fit hypothesis test is conducted. A goodness-of-fit test does not choose among contending models, it is a tool used to verify that a given hypothesis is appropriate. For probability distribution models, it is a good comparison between the shape of the distribution, to the shape of the histogram.

For these reasons, the goodness-of-fit tests have certain limitations. Variation is inherent if samples have a small size, and therefore, more confidence is made on conclusions based on larger sample sizes. The goodness-of-fit test selected for this study is the Kolmogorov-Smirnov Test. It is based on the deviations between the hypothesized cumulative distribution function $F_X(x)$ and the cumulative histogram. The Kolmogorov-Smirnov Test is strictly valid for continuous distributions. In the situation that only one value, the mean value is available for each layer of depth, the next section describes a method to hypothesize the distribution function.

Three Gaussian Fields, with varying correlation distances are generated by Spectral Representation, and transformed into Fields of Relative Density by the method proposed above. The effect of the "free parameter" a, is studied. All of the generated fields are correctly bounded between 0 and 1. The fields also realistically depict the values of the relative density to increase with depth of the soil. The fields display that as the parameter a (*note*: c= a/2) increases, the relative density decreases.

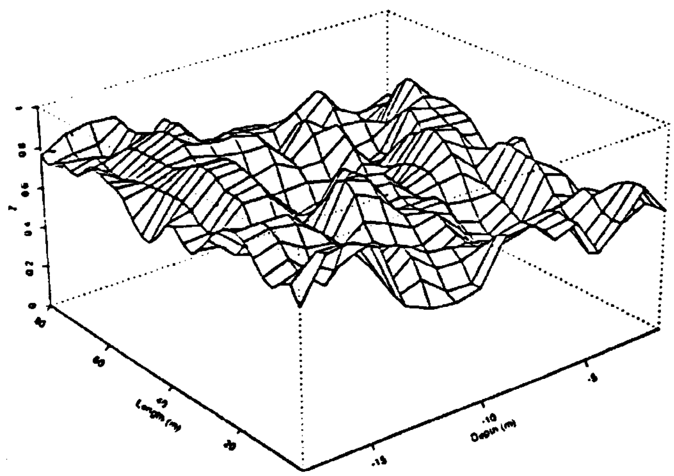

Figure 2. Relative Density Field: $c = a/2 = 2$

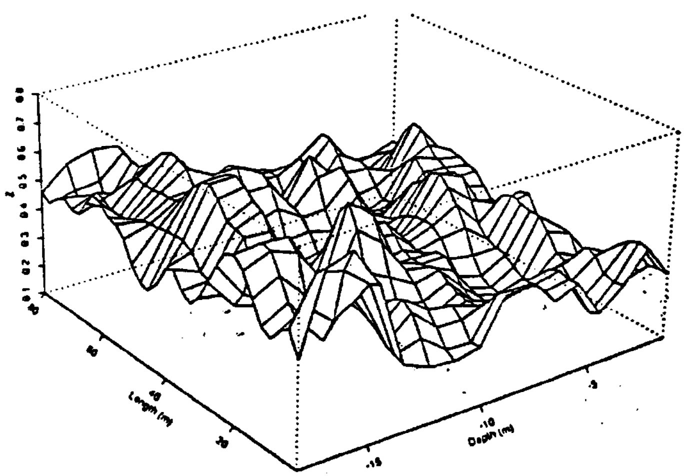

Figure 3. Relative Density Field: $c = a/2 = 4$

Distribution Functions Based on Single Values:

In order to conduct statistical goodness-of-fit tests with reliable results, large numbers of data are required. Since obtaining numerous soil data is costly, it is probable to have single values of soil parameters with depth. Under this circumstance, a procedure to determine the distribution, and its shape parameters are outlined below. Previously the stochasticity of the standard penetration number, N is depicted, and all soil parameters of interest to this study are stochastic as they are correlated to N. From the distributions obtained based on large number of standard penetration resistance the following trends are observed:

- Soil layers with small values of standard penetration resistance have small standard deviations.
- The soil layers closer to the ground surface have smaller standard deviations.
- In general, the skew is to the left, therefore shape parameters Beta1(q) < Beta2 (r).

Validations and Challenges of Model

This model, and distribution function [equation 7] for Relative Density determined, is a model that will fit real data that have distributions with a strong skew to the left. With the aid of the free parameter, the distribution function has flexibility of fitting data, since the standard deviation and mean shift. However, the mean cannot be controlled for dense soils directly, and therefore, the model does not have the capability of modelling all soils. A model based on *any* distribution function is introduced to model all soils. Since the standard penetration resistance is correlated to numerous soil parameters, including the relative density, it is of greater interest to determine the distribution for N. Therefore, a procedure to attain the probability density function for the standard penetration resistance has also been examined.

Conclusions :

In order to perform accurate and realistic analysis for liquefaction potential of soils by finite element analysis, their material random field must be generated, so that the material input data resembles reality. In order to generate random soil parameter fields, their distribution functions must be determined. Therefore, methods to obtain distribution functions are presented in this study. The probably density functions for soil parameters are skewed, (i.e. Non-Gaussian) and generation of fields for the standard penetration resistance based on the beta distribution function, which can be correlated to many soil parameters, is possible by the spectral Representation Method.

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