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MODELING OF VOLUME CHANGE CHARACTERISTICS OF SAND UNDER CYCLIC LOADING

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ABSTRACT

Stress dilatancy relationship of sand under cyclic loading including cyclic mobility and that of liquefied sand is examined. Post liquefaction Stress dilatancy characteristic is one of the key information required to estimate earthquake induced displacement. Specifically, sand response, when it undergoes excursions through states of zero effective stress, is needed when modeling the progress of liquefaction. Therefore, sand response at cyclic mobility and after liquefaction where excursions through zero effective stress occurs, is examined. On the other hand, response at both cyclic mobility and that after liquefaction is examined in order to identify the continuos change of behavior of the material from onset of liquefaction to that after liquefaction. Finally, an attempt is made to model the behavior of sand during cyclic loading specially when it undergoes excursion through states of zero effective stress.

KEYWORDS

Liquefaction; cyclic mobility; stress dilatancy; sand; bulk modulus; shear modulus; simple shear.

INTRODUCTION

Dilatancy characteristics play an important role in the liquefaction behavior of sand. Many researches have been conducted to develop stress-strain models (Dafalias, 1994; Towhata, 1989). Yoshida (1991) pointed out that many constitutive models based on the plasticity theory have its own difficulties in expressing the behavior of sand during and after the liquefaction. One is that, in the model, hysteritic behavior becomes stable fairly quickly after phase transformation whereas in the laboratory test, shear strain increases with loading. The other is that volume change due to dissipation of excess pore water pressure derived from models is nearly constant and much smaller than that of test in which volume change depends on the loading applied after liquefaction (Nagase *et al.*, (1988)).

Development in this field comes from the researches focused on the behavior after liquefaction including liquefaction-induced large permanent displacement. Yasuda *et al.*, (1994) conducted laboratory test in which sand is subjected to monotonically increasing strain after liquefaction. Yoshida *et al.*, (1994a) processed this test data and found the existence of an unstable region near the liquefaction, the extent of this region depends on the loading cycles. Since their investigation focused on the behavior after liquefaction, transient phenomena at liquefaction was not clear. In this paper, we investigate the dilatancy behavior in transient to liquefaction based on the cyclic load test result and show that it can be expressed in a continuous way from transient to the behavior after liquefaction.

BEHAVIOR DURING CYCLIC LOADING

Stress parameters

$$p = (\sigma'_a + \sigma'_t)/2$$
$$q = \sqrt{[(\sigma'_a - \sigma'_t)/2]^2 + \sigma'^2_{at}} \tag{1A}$$
$$\tan 2\beta_\sigma = 2\sigma'_{at} / (\sigma'_a - \sigma'_t)$$

Strain parameters

$$d\varepsilon_v = \sqrt{d\varepsilon_a + d\varepsilon_t}$$
$$d\bar{\varepsilon} = \sqrt{(d\varepsilon_a - d\varepsilon_t)^2 + (2d\varepsilon_{at})^2} \tag{1B}$$
$$\tan 2\beta_{de} = 2d\varepsilon_{at} / (d\varepsilon_a - d\varepsilon_t)$$

where β_σ and $\beta_{d\varepsilon}$ are the angles made by principal stress and principal strain increment direction with vertical respectively.

Stress-Dilatancy relationship

Consider the plastic shear work which is used as primary form of stress dilatancy relation;

$$dW^p = \sigma_{ij} d\varepsilon^p_{ij}$$

In two dimensional form;

$$dW^p = \sigma_{ij} d\varepsilon^p_{ij} = \sigma_a d\varepsilon^p_a + \sigma_t d\varepsilon^p_t + 2\sigma_{at} d\varepsilon^p_{at} \tag{2}$$

In terms of stress and strain invariant as given in Eq. 1,

$$dW^p = p d\varepsilon^p_{vd} + q d\bar{\varepsilon}^p \cos(2\beta_\sigma - 2\beta_{d\varepsilon}) = p d\varepsilon^p_{vd} + q d\bar{\varepsilon}^p \cos 2\psi \tag{3}$$

where $\psi = \beta_\sigma - \beta_{d\varepsilon}$ is the angle of non-coaxiality; the angle by which the principal stress direction and principal strain direction vary (Gutierrez, 1989). ε_{vd} is the volumetric strain due to shearing. Superscript *'p'* stand for plastic component of the corresponding strain.

The stress dilatancy relationship is derived assuming that normalized shear work can be expressed as a function of accumulated plastic shear strain,

$$d\Omega^p = dW^p / p = d\varepsilon^p_{vd} + (q/p) d\bar{\varepsilon}^p \cos 2\psi = \mu d\bar{\varepsilon}^p$$

Yielding,

$$d\varepsilon^p_{vd} / d\bar{\varepsilon}^p = \mu - c.q/p \tag{4}$$

where $c=cos2\psi$, μ ($=\mu_c$; sine of phase transformation angle) is assumed to be a constant.

Drained behavior.

Peiris *et al.*, (1994) has shown that the dilatancy relation given by Eq. 4 must be modified in accurate estimate of volumetric strain development. Modification for the stress-dilatancy relationship is mainly by assuming a varying m, given as follows.

$$\begin{aligned} \mu &= \mu_c \left[\frac{c.q/p-(c.q/p)_i}{\mu_c-(c.q/p)_i} \right] && \text{if} \quad c.q/p \le \mu_c \\ \mu &= \mu_c. && \text{if} \quad c.q/p \ge \mu_c \end{aligned} \tag{5}$$

Where $(c.q/p)_i$ is the value of $(c.q/p)$ at the beginning of each loading step. Constant μ ($=\mu_c$) is assumed during virgin and/or monotonic loading.

Torsional simple shear test data (by Pradhan *et al.*, (1989)) is re-examined in assessing the validity of the modified dilatancy relationship. Comparison of predicted (based on Eqs. 4 and 5) and experimental data is given in Fig.1.

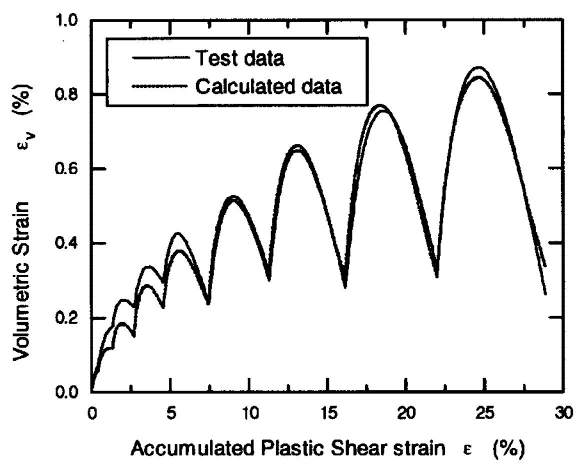

Fig. 1 Estimated and observed plastic volumetric strain due to shear.

Undrained behavior.

As far as the liquefaction is concerned the behavior under undrained condition is of prime important. Therefore, undrained cyclic behavior of Toyoura sand under torsional simple shear is examined. Test results are shown typically in Figs. 7a and 7b. Cyclic mobility is observed as the onset of initial liquefaction.

It is common to derive the stress-dilatancy relationship for drained test since the volume change due to shearing can be directly measured in these tests. Since it has been observed that the modified stress-dilatancy relation (given by Eq.5) is valid for cyclic drained test, the same relationship is employed in undrained cyclic loading. Thus, volumetric strain due to dilatancy can be computed from Eqs. 4 and 5. Under the undrained condition, however, this volume change does not occur, but effective mean stress changes by the amount

$$dp' = K(-d\varepsilon^p_{vd}) \tag{6}$$

where K is the bulk modules.

On the other hand, conceptual change of volumetric strain due to change of effective mean stress (or volumetric strain due to consolidation) is given by

$$d\varepsilon_{vc} = -d\varepsilon_{vd} \qquad (d\varepsilon_v = d\varepsilon_{vd} + d\varepsilon_{vc} = 0) \tag{7}$$

Figure 2 illustrates the variation of volumetric strain due to consolidation (given by Eq.7) with the change of mean effective stress as obtained from the torsional simple shear test data. In the same figure conventional consolidation characteristics are given by a dotted line.

$$d\varepsilon_{vc} = \frac{1}{K_0 (p'/p_0)^{1.0}} dp' \tag{8}$$

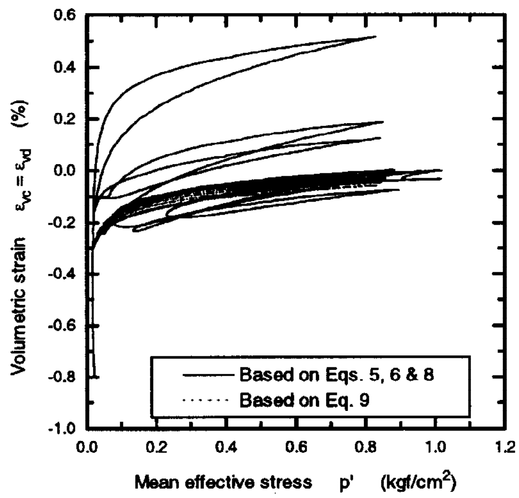

Fig. 2 Fictitious volumetric strain characteristics during the onset of cyclic mobility in undrained torsional simple shear test, estimated using stress dilatancy relation (Eq. 5) and Eq. 7.

BEHAVIOR AFTER LIQUEFACTION

Yoshida *et al.*, (1994b) measured volume change due to change in effective confining pressure, in the process of excess pore water pressure dissipation after liquefaction, as shown in Fig.3. They pointed out that large volume change occurs just at the beginning of drainage, and its amount depends on the amount of cyclic loading applied until the drainage. They also pointed out that in the conventional expression of bulk modules in which bulk modules is proportional to some power of effective confining pressure can express the behavior at higher effective confining pressure, but cannot express the behavior at low effective confining pressure close to zero. Yoshida and Finn (Peiris *et. al,* (1995)) proposed an equation expressing this behavior in the form,

$$\frac{p'}{p'_0} = \frac{e^{\varepsilon_v/c} - 1}{e^{\varepsilon_{v0}/c} - 1} \tag{9}$$

where c is a constant and ε_{v0} is volumetric strain at reference mean stress. As shown in Fig. 3, agreement between test result and Eq. 8 is good.

Yasuda *et al.*, (1994) conducted laboratory test of sand in which cyclic shear load is applied first even after the liquefaction and then shear strain is increased monotonically. Figure 4 shows an example of test result. They pointed out that there appears a low stiffness region at the beginning of monotonic loading. Safety factor against liquefaction, F_L, is used as an index of total amount of loading applied until the monotonic loading; F_L value decreases as the amount of this loading increases. The range of low stiffness region depends on F_L value as seen in Fig.4.

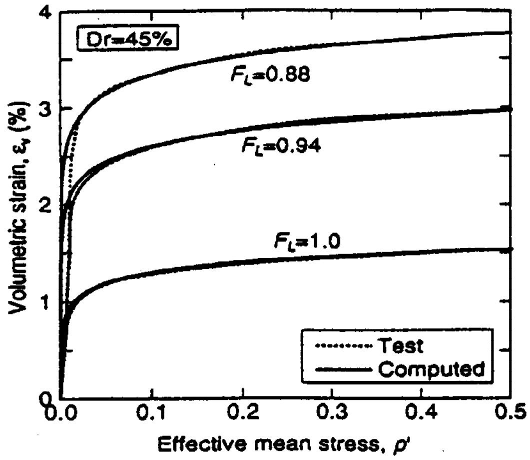

Fig. 3 Volumetric strain characteristics during the excess pore water pressure dissipation after liquefaction

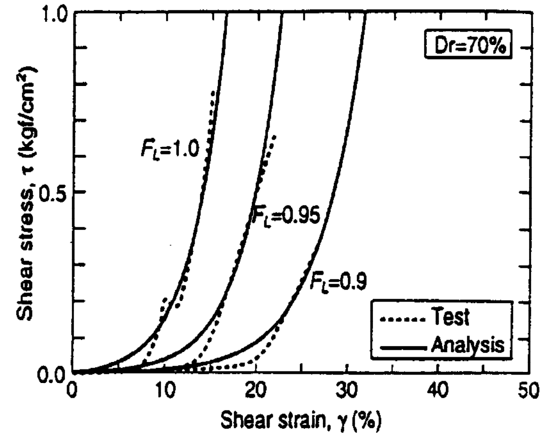

Fig. 4 Stress-strain relationship of sand subjected to monotonic loading after liquefaction.

Yoshida *et al.*, (1994a) analyzed this test and showed that the behavior can be simulated if shear modules at small strain also has the shape given by Eq. 8. Their analysis is also shown in Fig. 3. The agreement between the test and the analysis is good. They also pointed out that various material properties such as internal friction angle change with the amount of load applied after liquefaction.

In summary, after the occurrence of liquefaction, material property changes, because structure of soil skeleton is disturbed very much due to the occurrence of liquefaction at which inter granular force is very small. At low confining pressure, a new structure is not well developed, therefore stiffness is very small. When confining pressure increases, new structure is formed. Conventional theory seemed to be based on latter stage where material behavior is somewhat stable.

COMPARISON OF THE BEHAVIOR OF SAND BEFORE AND AFTER LIQUEFACTION

In order to make qualitative comparison of behavior of sand before and after liquefaction, the cyclic undrained test data is presented in different form. As shown in Figs. 5, behavior only during the increase of effective confining stress is illustrated. This figures is in accordance with the Fig. 3 which illustrates the behavior of sand after different degree of liquefaction.

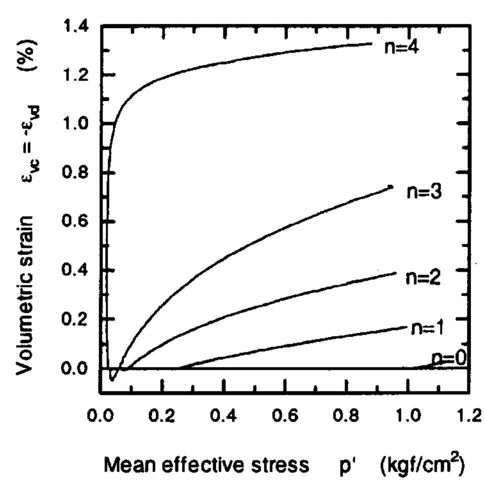

Fig. 5 Fictitious volumetric strain due to change of mean effective stress in undrained torsional simple shear

It is interesting to note that variation of volumetric strain due to consolidation at low effective confining pressure is very much differ from the conventional consolidation characteristics, but very much in comply with the consolidation and shear deformation characteristics of the sand after different degree of liquefaction (F_L) as reported by Yoshida *et al.*, (1994a) and Yasuda *et al.*, (1994). However, consolidation characteristics of sand at liquefaction is similar to conventional consolidation characteristics as implied by the gradient of solid and dotted lines at stress levels other than low effective confining stresses close to zero, in Fig. 2. Thus indicating the effective use of conventional liquefaction analysis at stress levels other than low effective confining stresses close to zero.

MODELING OF BEHAVIOR OF SAND UNDER CYCLIC LOADING

The stress dilatancy relationship derived above and volume change characteristics given by Yoshida and Finn (Peiris *et. al,* (1995)) is combined with a constitutive model. The constitutive model developed by Tobita and Yoshida (1993) has been employed. This elastic-plastic model is an isotropic bounding surface model with Mohr-Culomb yield criteria and hyperbolic hardening rule, capable in identifying the non-coaxiality which is somewhat important in foregoing discussion. Detailed explanation on the model is given by Tobita *et al.*, (1993), hence the discussion is limited to the modifications addressed in this paper, *i.e.* Stress dilatancy relationship and the volume change characteristics (or the evolution of bulk modulus).

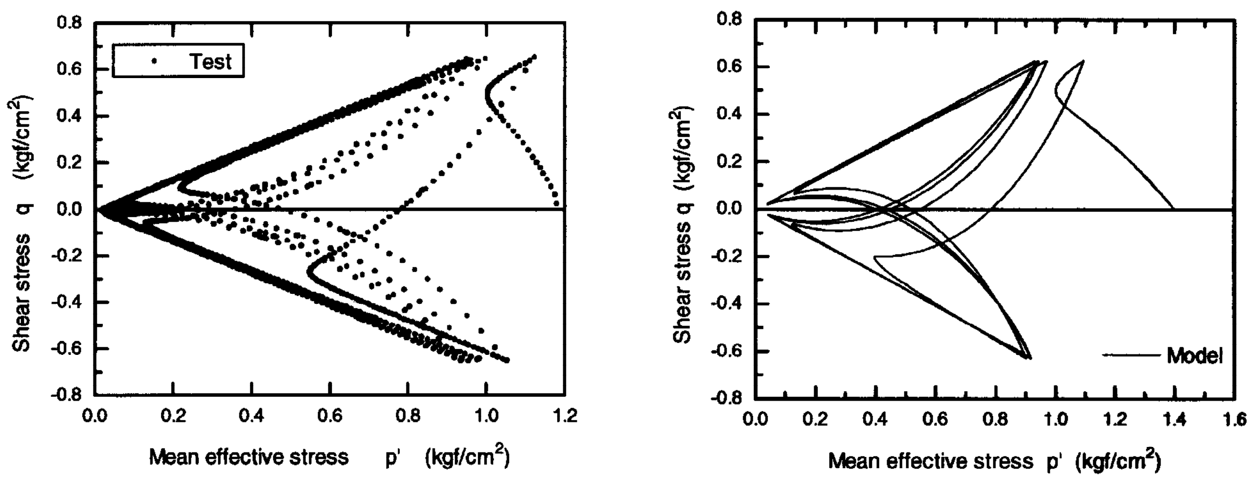

Fig. 6a Stress path of undrained torsional simple shear; Tested and Modeled data

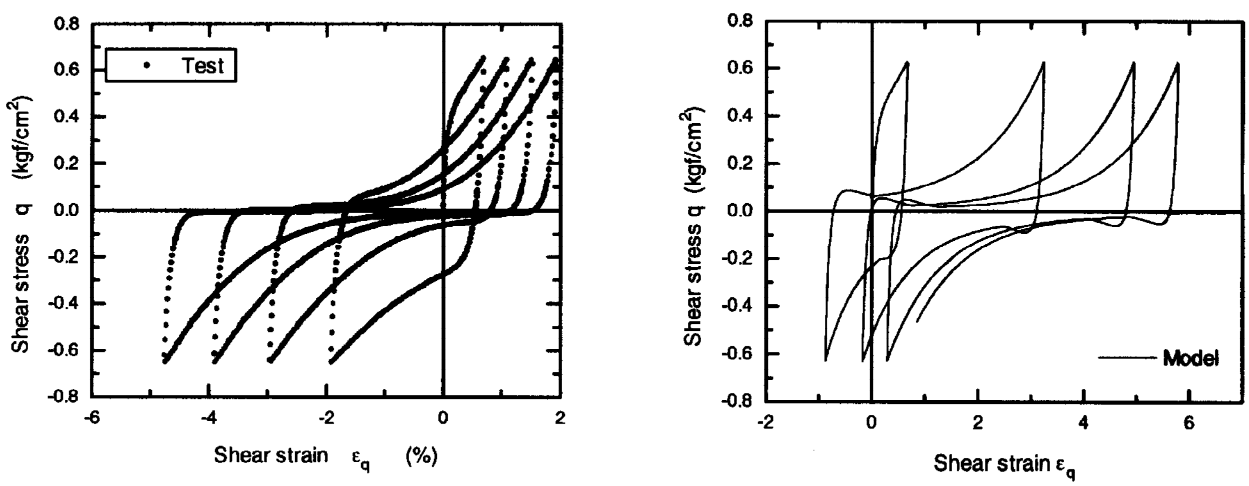

Fig. 6b Stress-strain relationship for undrained torsional simple shear; Tested and Modeled data

Equations 4 and 5 is incorporated as dilatancy relationship. The evolution of bulk modulus is derived from Eq. 9, which yields;

$$K = \frac{1}{c} \cdot \frac{p'_0 + p'(e^{\varepsilon_{v0}/c} - 1)}{e^{\varepsilon_{v0}/c} - 1} \tag{10}$$

where $c=0.053+0.0007\varepsilon_{v0}$ and ε_{v0} is assumed to be a linear function of maximum shear strain $(\varepsilon_q)_{\max}$.

Note that the $\bar{\varepsilon}$ as given in Eq. 1b is the accumulated shear strain, whereas the ε_q is the normal shear strain.

Model predictions are shown in Figs. 6 and 7. Figure 6 compares the predicted and observed behavior of sand in terms of normal shear strain, while Fig. 7 is for accumulated shear strain. Although the predictions look less accurate in terms of normal shear, it is interesting to identify that corresponding loops in predicted and observed behavior are very much similar even quantitatively, resulting better prediction in terms of accumulated shear strain.

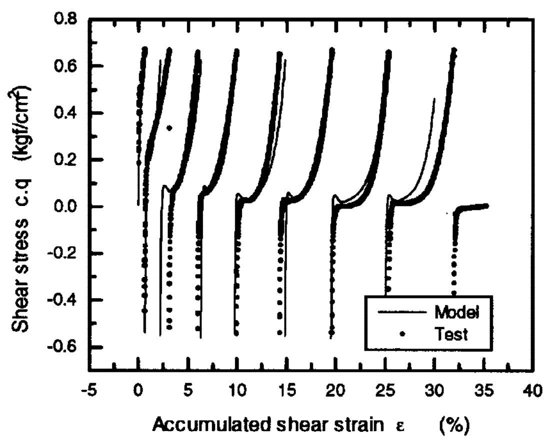

Fig. 7a Stress-strain behavior of sand under undrained torsional simple shear; in terms of accumulated shear strain

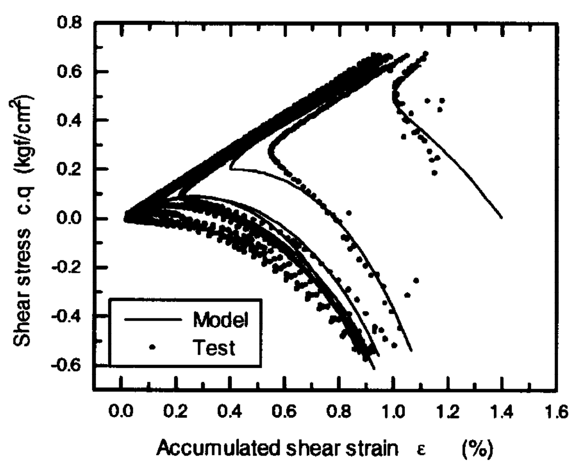

Fig. 7b Stress path of sand under undrained torsional simple shear; in terms of accumulated shear strain

CONCLUDING REMARKS

Investigation on behavior after liquefaction indicates that the use of new expression for bulk modules instead of conventional expression of bulk modules is necessary at low confining effective stress levels close to zero. This is well observed in Fig. 2 where consolidation characteristics of sand after liquefaction is illustrated.

Undrained test data produce the conceptual volumetric strain due to change of effective confining stress which is very much differ from conventional consolidation characteristics. On the other hand, the conceptual volumetric strain calculated as above show somewhat similar relation to the consolidation characteristics of sand after liquefaction. Further, the undrained test includes the cyclic mobility which can be considered as the onset of initial liquefaction or the transient to liquefaction. In deriving the conceptual volumetric strain in undrained test data, modified stress-dilatancy relation established and verified by drained test result is used.

The Tobita-Yoshida model in which the stress dilatancy component and bulk modulus component have been modified, produces better predictions quantitatively. Although the assumption of ε_{v0} as a function of maximum shear strain is supported by previous studies (Ishihara *et al.*, (1992)), further study is necessary in deriving a well defined function.

Furthermore, it can be deduced that behavior observed at transient to liquefaction and after liquefaction is a continuos process. In other words, the behavior continuously change from cyclic loading including on set of liquefaction to behavior after liquefaction.

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