



## **A seismic stability of soil structure considering soil parameter uncertainty**

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### **ABSTRACT**

Soil parameters disperse in a great range so that they are treated as random variables in general. In order to obtain the analytical results, the stochastic finite element method has been introduced. This method presented an accurate solution under short CPU time in earthquake engineering practices. This paper examined the effect of uncertainties of soil parameters under horizontal static loading in earthquake. The uncertainty of soil strength influences on soil behavior.

### **KEYWORDS**

Seismic stability; Soil structure; Soil parameter uncertainty; Stochastic finite element method

### **Introduction**

The past severe earthquakes caused large damages to various soil structures such as embankment and slope. Soil structure failures in a part of the Sasana area near the northern edge of the aftershock zone in 1994 Northridge earthquake in Southern California were reported<sup>1)</sup>.

The various uncertainties were caused by subjective or objective reasons when the soil structure was analyzed. Most of analysis, concentrate on parameter variability and ignore estimation errors. However, in principle, Å@later sources of uncertainty can be incorporated with existing techniques. The uncertainty, actually due to

ignorance, is divided into two artificial components, i.e. random fluctuation and estimation error in observation. Soil parameters disperse in a great range so that they are in general treated as random variables. In order to obtain the analytical results, Monte Carlo simulation has been frequently used. However in the simulation, the greater the number of parameters involved, the more the CPU time is needed. Recently, a new method, a stochastic finite element method(SFEM) has been introduced<sup>2)</sup>. This method presented an accurate solution under short CPU time in earthquake engineering practices. The coefficient of variation of soil parameter is less than 10 percent.

The one-dimensional models address variation along vertical lines in the subsurface. Soil behavior such as settlement is calculated by integrating one-dimensional deformations induced by a deterministic stress field. Similar models were developed by Diaz and Vanmarcke<sup>3)</sup>. Stochastic finite element technique for two dimensional soil behavior was suggested by Cornell<sup>4)</sup> in a general discussion of the applicability of the second-moment approximation to linear system. Applications of stochastic finite element methods to rock and soil mechanics have been made by Su, et al.<sup>5)</sup> and Cambou<sup>6)</sup>. Cambou applied the second-moment approximation to a linear solution of finite element method that include autocorrelation among the soil properties. Existing probabilistic soil behavior models, including stochastic finite element method and so on, are the extensions of deterministic techniques in which input parameters are allowed to vary. Thus, these models have the same limitations as deterministic models and should be regarded as refinements of existing techniques.

The present study is to propose a procedure to estimate the displacement and stress of soil structure with the elasto-plastic problem in consideration of uncertainty by using SFEM. In this paper, the results of two-dimensional elasto-plastic are analyzed with the first-order second-moment method using finite elements. The effect of uncertainties of soil parameters under horizontal loading in earthquake is also examined.

### **General formulation**

The two-dimensional model developed here uses finite element discretization of elasto-plastic soil, and uses the first terms of Taylor series expansions to calculate mean and variance of nodal displacements and stresses in elements.

Nodal displacements are calculated by solving the following system of equations,

$$ku = P, \quad DBu = \sigma$$

in which  $k$  = the stiffness matrix,  $u$  = a vector of nodal displacements and  $P$  = the load vector,  $D$  = the stress-strain matrix,  $B$  = the strain-displacement matrix,  $\sigma$  = the vector of stress .

As a nonlinear problem, equations of this type can be solved by a suitable iterative technique, and any one of the many standard methods can be applied. We do not discuss details of such procedures of iteration in this paper, but we only discuss the obvious methods, in which the systems of equations are repeatedly solved with successively improved values of  $k$ . In order to evaluate the stress conditions the finite element analysis with

elasto-plastic is appropriated. The soil behavior of the nonlinear system can be obtained efficiently through equivalent linearization.

Assuming that the stiffness matrices change with force increments, the performance function consists of  $k^{-1}$  in the problem. It is therefore necessary in the search algorithm to repeat the computation of  $k^{-1}$  and  $\partial k^{-1} / \partial \gamma_k$  ( $\gamma_k$ =random variables), so that we reach the rupture point on the failure surface.

### **Probabilistic approximation**

A multidimensional Taylor-series expansion is necessary before expectations are taken. If  $x_i$  is a function of several random variables, that is

$$x_i = g(\gamma_1, \gamma_2, \dots, \gamma_n)$$

in which  $\gamma$ = random variables. We obtain an approximate mean and a variance of  $x_i$  similarly as follows. Expanding the function  $g(\gamma_1, \gamma_2, \dots, \gamma_n)$  in the Taylor-series about the mean values  $(\mu_{\gamma_1}, \mu_{\gamma_2}, \dots, \mu_{\gamma_n})$ , we have

$$x_i = g(\mu_{\gamma_1}, \mu_{\gamma_2}, \dots, \mu_{\gamma_n}) + \sum_{k=1}^n (\gamma_k - \mu_{\gamma_k}) \frac{\partial g}{\partial \gamma_k} + \frac{1}{2} \sum_{k=1}^n \sum_{l=1}^n (\gamma_k - \mu_{\gamma_k})(\gamma_l - \mu_{\gamma_l}) \frac{\partial^2 g}{\partial \gamma_k \partial \gamma_l}$$

in which the derivatives are evaluated at  $(\mu_{\gamma_1}, \mu_{\gamma_2}, \dots, \mu_{\gamma_n})$ . In particular, the first-order approximation to the moment of

$$E[u_i] = g(\mu_{\gamma_1}, \mu_{\gamma_2}, \dots, \mu_{\gamma_n})$$

$$Var[u_i] = \sum_{k=1}^n \sum_{l=1}^n \frac{\partial g}{\partial \gamma_k} \frac{\partial g}{\partial \gamma_l} Cov[\gamma_k, \gamma_l]$$

in which  $u_i$  = the displacements at  $i$  node and  $Cov$  = covariance of variables. The moment of stress is obtained as

$$E[\sigma_i] = f(\mu_{\gamma_1}, \mu_{\gamma_2}, \dots, \mu_{\gamma_n})$$

$$Var[\sigma_i] = \sum_{k=1}^n \sum_{l=1}^n \frac{\partial f}{\partial \gamma_k} \frac{\partial f}{\partial \gamma_l} Cov[\gamma_k, \gamma_l]$$

in which  $\sigma_i$  = the stress in  $i$  element.

### **Failure conditions**

The strength of a soil is usually defined in terms of the stresses developed at the peak of the stress-strain curve. It considers that there are some rupture diagram of the soil, for example, Mohr rupture and so on. The condition for failure corresponds to Mohr's rupture diagram in which the failure envelope is a straight line. In the study, we use Mohr's rupture diagram for condition and Coulomb's equation.

## Analytical model

The data of soil properties are estimated as the Young's modulus  $E$  and Poisson's ratio  $\nu$ . Each value and the correlation of  $E$  and  $\nu$  are calculated from actual field test data. As a probabilistic analysis, Young's modulus and poisson's ratio are assumed to be normal distributed, and the other parameters are assumed to be deterministic. The finite elements mesh used in the present study is shown in Fig.1. The stress-strain curve for soil behavior is approximated shown in Fig.2. This curve shows strain-softening property. The coefficient of variation of statistical soil parameters is to be 0.1. The assumed properties of the soil are listed in Table 1.

The model consists of 44 plane triangular elements with 33 nodes. The nodes along the lower edge of the model are fixed and at the right side fixed boundary is excited by the static loading in the horizontal direction. The displacement field is defined in terms of two components  $u_x$  and  $u_y$  in this example. The field include the body forces of gravity and the external boundary force due to earth pressure from the backfill.

## Analytical results

The computed displacements show large difference among the node. Fig.3 shows that the computed nodal displacements in both the horizontal and the vertical directions increase with the incremental loading step. Calculations of the means of nodal displacements are the same as in the deterministic case. Fig.4 shows that the calculated coefficients of variation of that are relatively large, i.e., more than 0.2 at the all step of incremental loading.

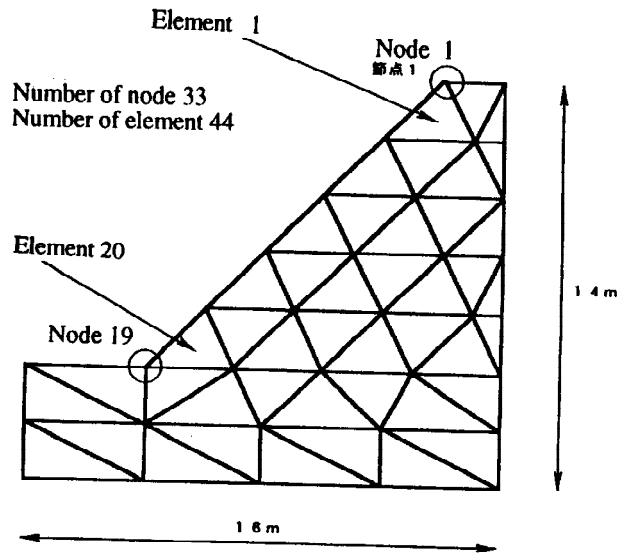


Fig.1 Finite element mesh for the analysis of soil structure.

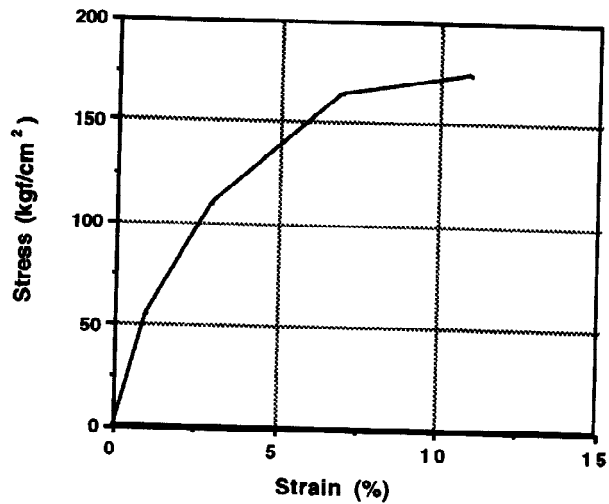
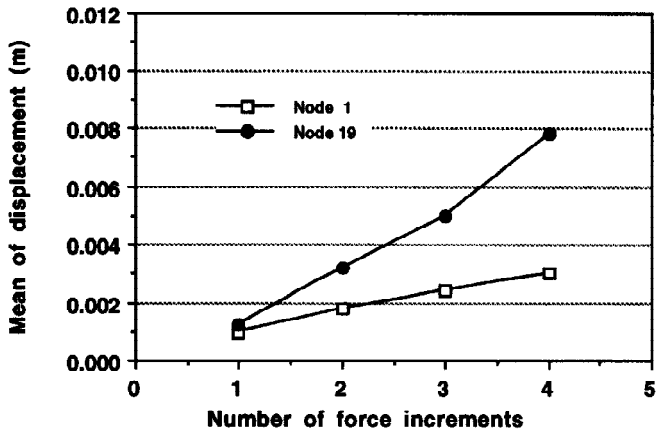


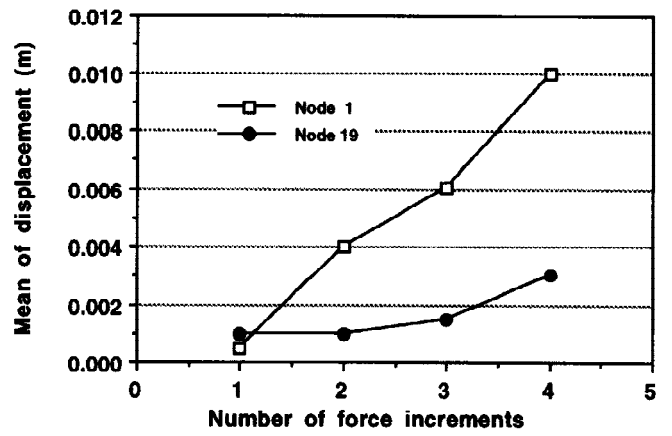
Fig.2 Stress-strain curve for soil.

Table 1 Soil properties and statistical parameter.

Mean of Young's modulus $E$	540 MPa
Mean of Poisson's ratio $\nu$	0.3
Coefficient of variation of $E$ and $\nu$	0.10
Unit weight	22.5 kN/m <sup>3</sup>
Compression critical strain	11%

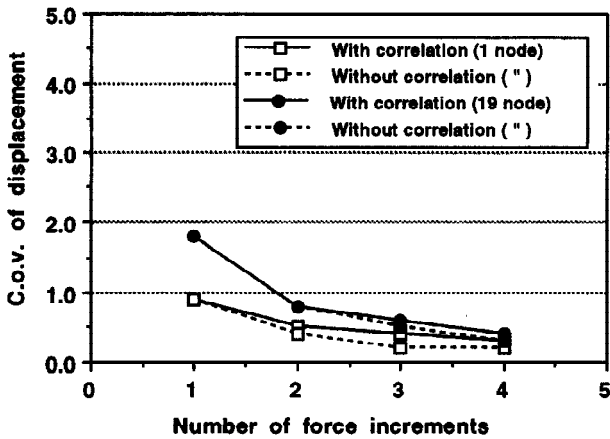


(a) x-direction

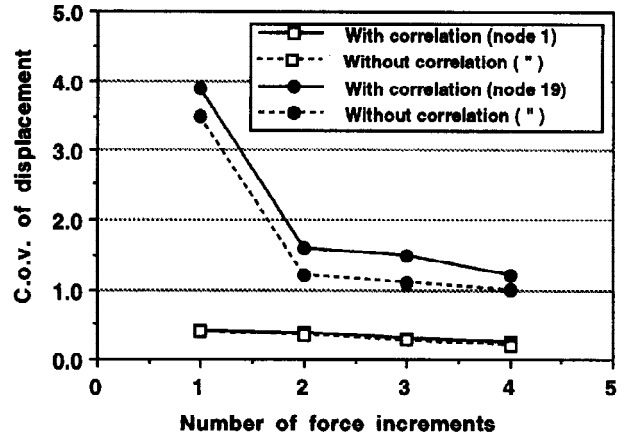


(a) y-direction

Fig.3 Mean of nodal displacement under force increments.

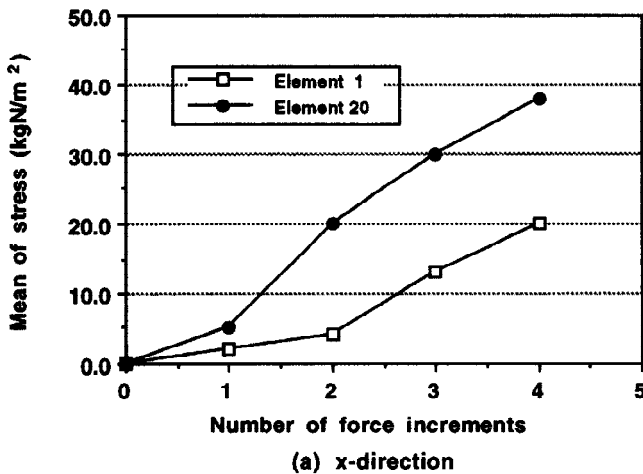


(a) x-direction

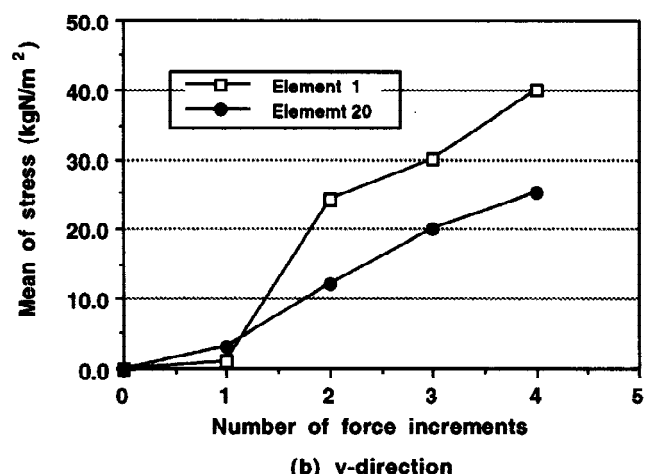


(b) y-direction

Fig.4 C.o.v. of nodal displacement under force increments.



(a) x-direction



(b) y-direction

Fig.5 Mean of stress in element under force increments.

The solid lines denote the coefficient of variance of nodal displacements at the nodes 1 and 19 when the variables of soil are correlated each other, and the segmented lines denote those when the variables of soil are not correlated. The correlation of variables only influence on the results. At the first loading step, the coefficient of variance of nodal displacement is relatively large. At the other step, the coefficients of variance of nodal displacements become small because the means become large.

The means of compressive stresses in the elements 1 and 20 are shown in Fig.5. It shows that the mean of stress in element 20 in the horizontal direction is greater than that in vertical direction. This can be attributed to the increased pressure values as a results of deformation of backfull soil under the loading. The stochastic finite element method is useful analysis of actual soil structures where the information are available.

### **Conclusions**

The method of reliability analysis of soil structure considering soil parameter uncertainty is introduced. The coefficient of variation of soil strength influences on soil behavior. First-order second-moment stochastic finite element techniques allow predictions in random problems based on more realistic analysis than current techniques, and allow the incorporate of variability of soil properties which the deterministic finite-element techniques do not. In principle, stochastic finite element methods can be applied to soil structures with elasto-plastic behavior at reasonable computational costs.

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