



## INITIAL DYNAMIC STIFFNESS OF MEXICO CITY CLAY FROM FIELD TESTS

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### ABSTRACT

This paper presents expressions that correlate shear wave velocities with CPT strengths for soils in Mexico City. The correlations were derived using cavity expansion theory and hyperbolic stress-strain models. Results of field experiments are used to calibrate the correlations, point out their limitations and propose a method which is useful in practical problems dealing with the seismic response of soft clay deposits.

### KEYWORDS

Shear moduli; shear wave velocity; field tests; correlations; CPT soundings

### INTRODUCTION

Stress-strain relationships to model the seismic behaviour of soils often specify the functions that relate shear modulus with strain and are expressed, on many occasions, in terms of the value of the shear modulus,  $G$ , at small strains. This value is usually denoted by  $G_{\max}$  since shear moduli generally adopt their maximum value when shear strains are smaller than about  $10^{-3}$  %. Field tests to measure shear wave velocities,  $V_s$ , are particularly well suited for obtaining values of  $G_{\max}$ , as they commonly induce even smaller strains in the soil mass.

Approximations to  $V_s$  and consequently to  $G_{\max}$ , can be obtained from correlations with the results of geotechnical soundings like the standard or the cone penetration tests (SPT or CPT). These correlations are extremely useful in regional analyses, preliminary studies or in less-than-desirable situations where it is simply not possible to perform field or laboratory tests to obtain dynamic soil properties. This paper presents the derivation of correlations between shear wave velocities and cone penetration resistance,  $q_c$ , for soils in Mexico City by interpreting a CPT sounding with cavity expansion theory and, also, by assuming that soil behaviour is adequately modelled with hyperbolic stress-strain relationships.

## FIELD TESTS

Shear wave velocities were determined from the results of down-hole and suspension logging tests performed in 15 sites within the lake zone in Mexico City. The former type of test is well known and the latter, developed by Oyo Corp during the late 70's and early 80's, is described *in extenso* elsewhere (e. g. Kitazunesaki, 1980). CPT soundings were performed at each of the test sites so that profiles of  $V_s$  and  $q_c$  against depth were available in each of them, as exemplified by the graphs presented in figs 1 and 2.

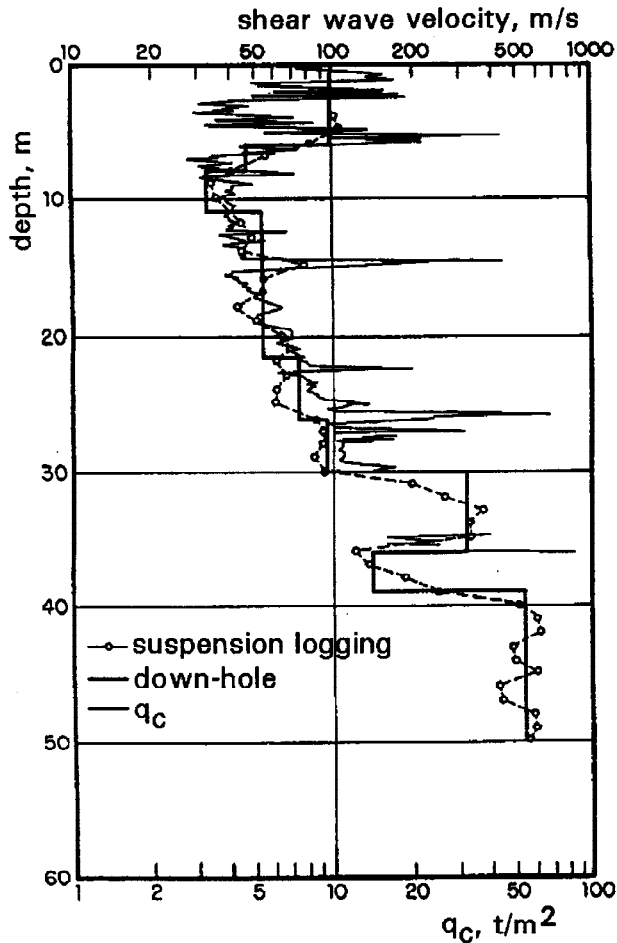


Fig 1 CPT sounding and shear wave velocity profiles in a site in the pre-loaded lake bed

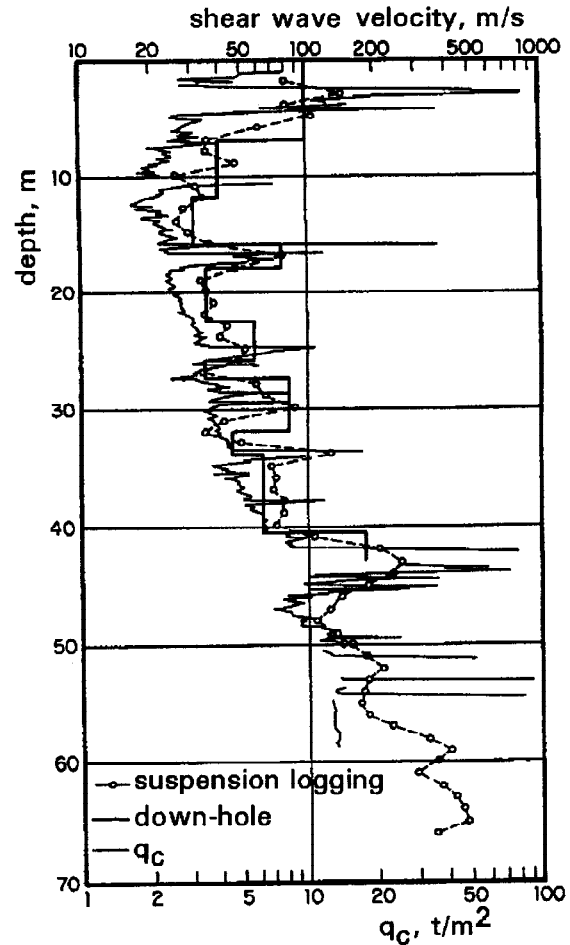


Fig 2 CPT sounding and shear velocity profiles for a site in the former Xochimilco-Chalco lake

Stratigraphy at the test sites is characterized by the presence of highly compressible lacustrine clays. Two main clay strata were identified in each of the soundings. The first one extends from about 3 to 5 m, down to depths that vary between 30 to 40 m. The second one is separated from the upper clays by a much harder silts and silty sands that constitute the first hard layer, usually 2 to 3 m thick. The lower clay stratum is less compressible and has been subjected to increases in effective stresses brought about by deep well pumping; its base is located at depths that vary between 40 and 50 m. Numerous studies have demonstrated that most of the seismic amplification effects for which Mexico City is so notorious occur within these two clay strata (e. g. Romo and Ovando, 1995).

## DERIVATION OF CORRELATIONS

### *Penetration resistance and undrained shear strength*

For the Mexico City clays, the following empirical relationship relates undrained shear strength,  $c_u$ , with  $q_c$ :

$$c_u \approx \frac{q_c}{N_k} \quad (1)$$

where  $N_k$  is a correlation coefficient that depends on soil type and on the shape and size of the penetrating tip. This expression has been used in the Mexico City area for several decades now and is backed by the results of large amounts of CPT and static unconsolidated undrained triaxial tests from which it has been found that  $N_k$  varies between 10 and 14 (Santoyo *et al*, 1989). In establishing a relationship between  $V_s$  and  $c_u$ , undrained strength must be interpreted as a dynamic parameter and consequently, the value of the coefficient parameter can be expected to be lower.

### *Relationships between $q_c$ and $V_s$ from cavity expansion theory*

Classical plasticity has been used to study the longitudinal expansion of a cylindrical cavity within an elasto-plastic medium (Hill, 1950). The same theory has been applied to estimate the point bearing capacity of piles (Ladanyi, 1967) and can also be used to interpret a CPT test. The internal pressure required to produce a continuous longitudinal expansion of such a cavity,  $p_i$ , is related to the stress state within the soil mass before the expansion of the cavity and to soil properties. Assuming that the soil behaves like a perfect elasto-plastic solid,  $p_i$  can be expressed as (Ovando and Romo, 1992):

$$p_i = p_0' + \frac{2}{3} q_p \left[ 1 + \ln \left( \frac{2E_p}{3q_p} \right) \right] \quad (2)$$

where  $p_0'$  is the mean effective stress before the expansion of the cavity;  $E_p$  stands for undrained Young's modulus at half the deviator stress at failure ( $= q_p$ ). In the case of a CPT test,  $p_i$  is related to tip penetration resistance and the soil's undrained strength.

$$q_c = p_i + q_p \quad (3)$$

with the usual values of  $N_k$ ,

$$q_c = c_u N_k \approx p_i \quad (4)$$

It is convenient to express the mean effective stress in terms of  $q_c$ . To this end,  $p_0'$  must first be expressed as a function of the vertical effective stress,  $\sigma_v'$ . Next, it is assumed that the ratio between undrained strength and vertical effective stress is constant,  $c_u/\sigma_v' = \beta$ . Hence,

$$p_0' = \frac{(1+2K_0)}{3\beta} \frac{q_c}{N_k} \quad (5)$$

where  $K_0$  is the coefficient of earth pressure at rest.

Introducing  $p_0'$  into equation (2), and taking into account that  $E_p = 2\rho(1+\nu)V_s^2$ , an expression for  $V_s$  is obtained:

$$V_s = \sqrt{\frac{3\rho q_c}{2N_{kc}(1+\nu)} \exp\left[\frac{3N_{kc}-4}{4} - \frac{1}{2\beta}\right]} \quad (6)$$

Non-defined terms are mass density,  $\rho$ , and Poisson's ratio,  $\nu$ ; it was also assumed that  $K_0 = 0.5$ .

### Use of hyperbolic functions

Relationships between shear wave velocity and penetration resistance can also be obtained assuming that the stress-strain behaviour can be represented by a hyperbolic model. Take, for example, the Ramberg-Osgood model:

$$\tau = \frac{G_{max}\gamma}{1 + \alpha \left| \frac{\tau}{c_u} \right|^{r-1}} \quad (7)$$

where  $\tau$  and  $\gamma$  are shear stresses and strains, respectively;  $\alpha$  and  $r$  are experimentally determined parameters. An equivalent expression to equation (7) is:

$$\alpha \frac{\tau^r}{c_u^r} + \frac{\tau}{c_u} = \frac{\gamma}{\gamma_r} \quad (8)$$

where  $\gamma_r$  is the reference strain;  $\gamma_r = c_u/G_{max}$ . If  $\gamma \rightarrow 0$  in the equation above, the left hand side of the equation will also tend to zero. However, since the value of  $r$  for the Mexico City clay is about 2 (Jaime, 1987), the term containing this exponent will approach zero faster. Hence,

$$\frac{\tau}{c_u} \approx \frac{\gamma}{\gamma_r} \quad (9)$$

Making the appropriate substitutions one finds that

$$V_s = \sqrt{\frac{q_c}{\rho N_{kh} \gamma_r}} \quad (10)$$

$N_{kh}$  is another coefficient of correlation.

## COMPARISON WITH FIELD MEASUREMENTS

Penetration values obtained from formulas (6) and (10) were used to estimate shear wave velocities and the results were then compared with measured values of  $V_s$ . In using equation (6), it was assumed that  $\beta = 0.26$ , which appears to be a reasonable hypothesis for normally consolidated or lightly overconsolidated soils, like those found in Mexico City. In order to use equation (10), the value of  $\gamma_r$ , which depends strongly on soil plasticity, must be specified first. Results of cyclic triaxial and resonant column tests performed previously were used to assign values to  $\gamma_r$  (Romo, 1995).

Soil type and stress history will influence shear wave velocity and consequently, the field data were grouped following two criteria. The first one takes into account the location of the test site and whether it has been subjected to significant external overburdens. Accordingly, it was possible to separate shear

wave velocity data from three different zones: the virgin lake Texcoco bed (sites with no significant external loads), data from the heavily urbanized core of the city (pre-loaded lake Texcoco bed) and, finally, data from the southernmost portion of the Mexico City basin (Xochimilco-Chalco lake bed). The other criterion is stratigraphical and it distinguishes between the highly plastic clay formations and the harder non plastic materials that form the first hard layer and the dessicated surficial crust.

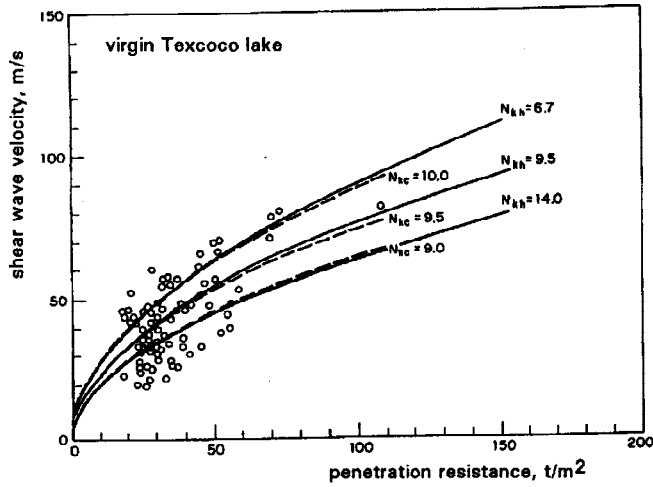


Fig 3 Shear wave velocity against penetration resistance. Virgin Texcoco lake

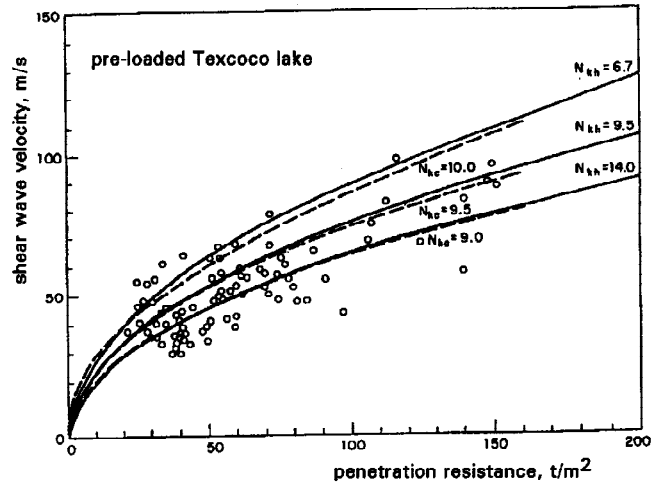


Fig 4 Shear wave velocity against penetration resistance. Pre-loaded Texcoco lake

The graphs in figs 3 to 5 show the results obtained for the clayey soils from the three zones mentioned previously. In the case of non-plastic materials, the experimental data can be accommodated in a single graph, irrespective of the location of the test sites, fig 6. Mean values obtained with either equation (6) or (10) are roughly equivalent. As expected, field values show a rather large scatter due to several factors, like experimental errors in the measurement of  $q_c$  and  $V_s$ . The hypotheses assumed for deriving the correlations may not necessarily simulate actual soil behaviour in the field and will also contribute to the observed dispersion; the same is true in regard to the uncertainties involved in assigning values to some of the parameters. However, the correlations do follow the expected trends and for practical applications the scatter can be reduced making an additional consideration.

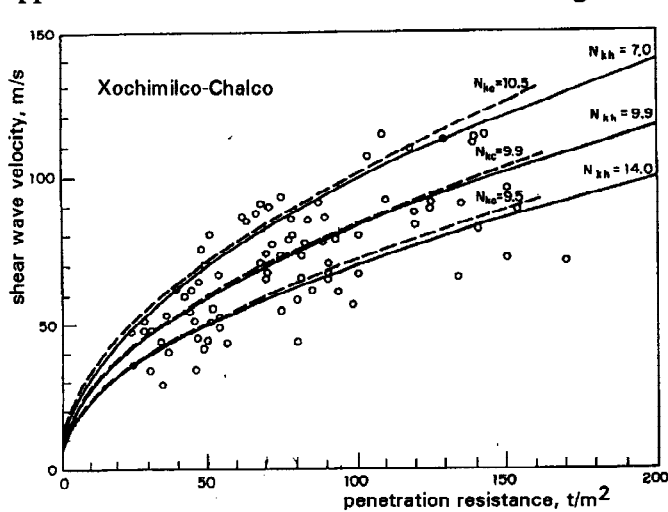


Fig 5 Shear wave velocity against penetration resistance. Xochimilco-Chalco lake

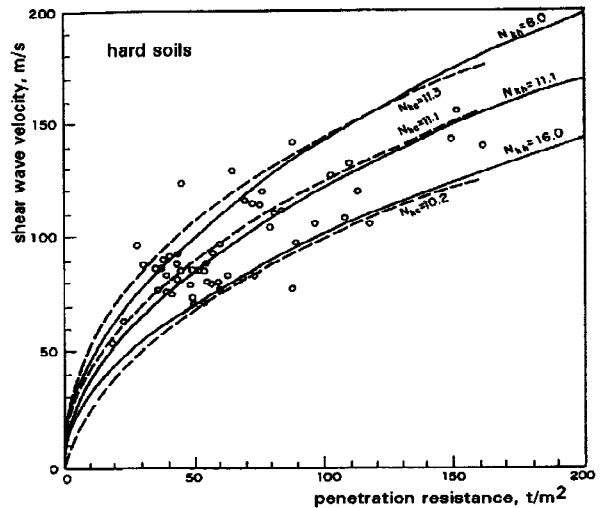


Fig 6 Shear wave velocity against penetration resistance. Non plastic soils in the totality of the ancient lake beds in the Valley of Mexico

The elastic relationship between the dominant period,  $T_0$ , of a soil deposit of depth  $H$  and mean shear wave velocity,  $\hat{V}_s$  is  $T_0 = 4H/\hat{V}_s$ . The value obtained thus agrees well, in general, with dominant periods determined experimentally in many sites in Mexico City from ambient vibration studies or from surficial movements recorded during earthquakes, even for large magnitude events like those that occurred in 1985 (Lermo *et al*, 1990) due to the fact that Mexico City clay exhibits nearly elastic behaviour at rather large strains levels, as has been shown and discussed previously (e. g. Romo and Ovando, 1995). An expression for  $\hat{V}_s$  is:

$$\hat{V}_s = \frac{H}{\sum_{i=1}^n \frac{h_i}{V_{si}}} \quad (11)$$

where  $h_i$  and  $V_{si}$  are the thicknesses and the shear wave velocities of the strata that constitute the soil deposit.

The procedure for using equation (6) or (10) is to first determine the dominant period from ambient vibration measurements, which is cheap and pretty straight forward or, alternatively, from maps of equal period contours like those included in the Mexico City Building Code. Secondly, to find a combination of  $V_{si}$  values obtained from either of these equations, within the bounds of dispersion indicated in figs 4 to 6, such that it satisfies (10) and (11). In doing so, due regard must be given to stratigraphical variations observed in the results of any CPT sounding. This procedure reduces the scatter in the values of shear wave velocity.

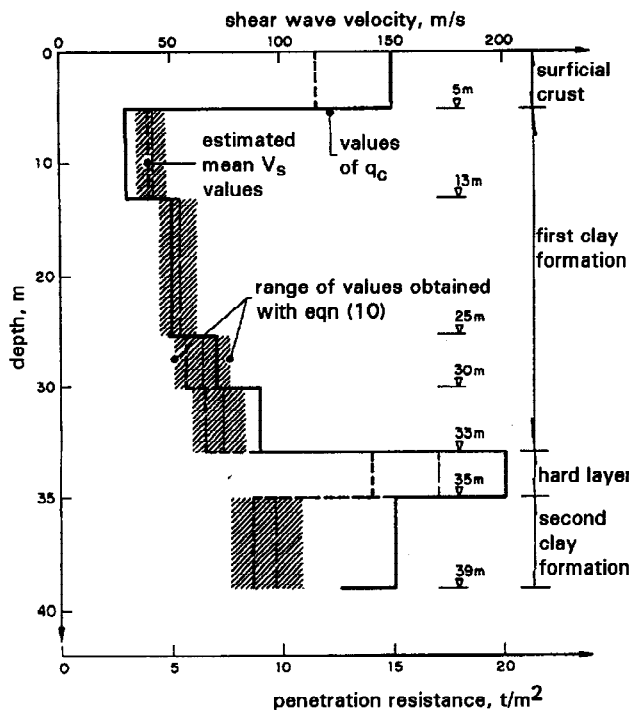


Fig 7 Simplified CPT and shear wave velocity profiles for a site in the pre-loaded Texcoco lakebed

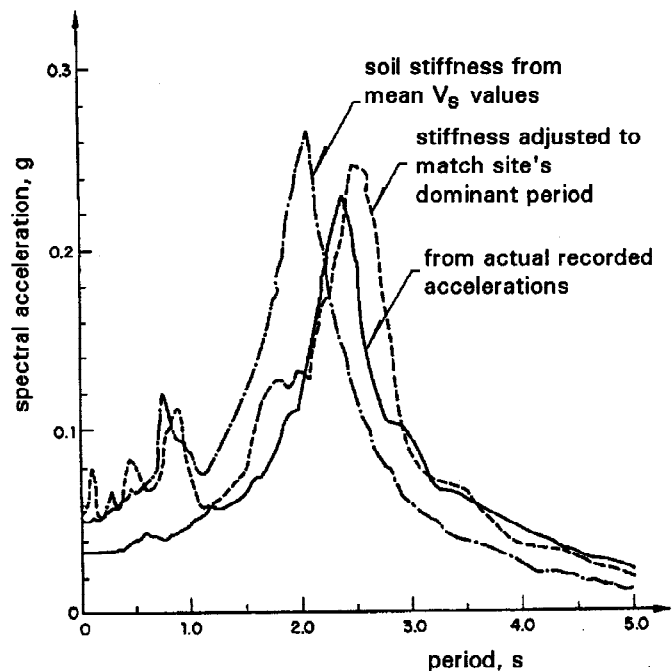


Fig 8 Response spectra from recorded accelerations and from 1-D analyses. The site is located in the pre-loaded Texcoco lakebed

### EXAMPLE

The use of the proposed correlations and the suggested procedure for applying them is illustrated with results of a CPT sounding performed at a site within the old lake bed in Mexico City. The simplified

profile is shown in fig 7, together with the shear wave velocities estimated with equation (10). Surface acceleration records obtained during the 25 April, 1989, earthquake were available at that site. The data given in figure 7 were used to model the stratigraphy. Surficial movements along one of the horizontal components were calculated using a one dimensional wave propagation model (Schanbel *et al*, 1972), using as input motion a record obtained at a hard soil site in the hills of the city. The response spectra shown in fig 8 (5 % damping) were obtained from the actual accelerograms and from the movements calculated with the 1-D model. As indicated there, one of the response spectra was obtained from a model in which shear wave velocities are the mean values predicted by equation (10), i. e.  $N_{kh} = 9.5$ , whilst the other one was obtained adjusting the values of shear wave velocity within the bounds indicated in fig 7, until the site's natural period was matched. This spectrum closely agrees with the one obtained from the actual surficial accelerations.

## CONCLUSIONS

Cavity expansion theory can be used to interpret CPT soundings and to relate soil stiffness and strength. Other relationships between stiffness and strength can be obtained postulating that dynamic soil behaviour can be adequately represented with hyperbolic stress-strain models. Both these approaches were used to justify the correlations between penetration resistance, obtained from CPT soundings, and shear wave velocity.

Shear wave velocities obtained from down-hole and suspension logging tests, together with the results of CPT soundings were used to calibrate the proposed correlations. Experimentally determined data follow the expected trends but show considerable scatter and the correlations must therefore be used judiciously. A site's dominant period reduces the uncertainty involved in using the proposed correlations and may lead to very satisfactory estimations of actual response spectra in Mexico City.

The correlations derived in this paper provide useful approximate estimates of the shear wave velocity profiles from CPT soundings and also give an idea of their range of variation but must never be used to substitute field or laboratory tests to obtain dynamic soil properties.

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