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SIMPLIFIED PUSH-OVER ANALYSIS OF BUILDING STRUCTURES

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ABSTRACT

In the paper a simplified method for nonlinear static analysis of building structures subjected to monotonicaly increasing horizontal loading (push-over analysis) is presented. The method is based on an extension of the pseudo three-dimensional mathematical model of the structure into the nonlinear range. By a step-by-step analysis an approximate relationship between the global base shear and top displacement is computed. During the analysis the development of plastic hinges throughout the building can be monitored. The method has been implemented into a prototype of a computer program. In the paper the mathematical model, the base shear - top displacement relationships for different types of macroelements and the step-by-step computational procedure are described. The method is applied for the analysis of a seven-story reinforced concrete frame-wall building. A symmetric and an asymmetric variant of the same structure is analyzed.

KEYWORDS

Seismic analysis; nonlinear analysis; push-over analysis; building structures; frame-wall buildings; mathematical model; plastic mechanisms; asymmetric structures; torsion; computer program.

INTRODUCTION

Recently it has been widely recognized that changes in the existing seismic design methodology implemented in codes, based on the assumption of linear elastic structural behavior, are needed (Krawinkler, 1995). Complex analyses, such as nonlinear time history analysis of Multi-Degree-of-Freedom (MDOF) mathematical models, are not practical for everyday design use. A simpler option to estimate the structural performance is a nonlinear static analysis under monotonicaly increasing lateral loading (push-over analysis). In somewhat different formats the method has been proposed, formalized, and evaluated in several studies (e.g. Saiidi and Sozen, 1981, Qi and Moehle, 1991, Fajfar and Gašperšič, 1996).

A nonlinear push-over analysis can be performed with some of well-known programs for nonlinear analysis. However, in the case of a large and complex asymmetric building structure, the use of a general computer program is time consuming and unpractical even if the analysis is restricted to statics. Therefore, based on the studies of inelastic seismic response of asymmetric building structures, some simplified analysis procedures were proposed (e.g. Bertero, 1995, De la Llera and Chopra, 1995, De Stefano *et al.*, 1995, Duan and Chandler, 1995, Moghadam and Tso, 1995). A list of publications published before the end of 1994 was compiled by Rutenberg *et al.* (1995).

At the University of Ljubljana a method for simplified push-over analysis has been developed, which is intended to achieve a satisfactory balance between required reliability and applicability for everyday design use, and which might contribute to the practical implementation of new trends in seismic design (Kilar, 1995). It is based on an extension of the pseudo three-dimensional mathematical model of a building structure into the inelastic range. The method was implemented into a prototype of the interactive and user friendly computer program NEAVEK. In the paper the method is briefly described and applied for the analysis of a test example.

PSEUDO THREE-DIMENSIONAL MATHEMATICAL MODEL

A pseudo three-dimensional mathematical model consists of assemblages of two-dimensional macroelements (substructures) such as frames, walls, coupled walls and walls on columns that may be oriented arbitrarily in plane. Each macroelement is assumed to resist load only in its own plane, but the building as a whole can resist load in any direction. The macroelements are connected at each floor level by diaphragms that are assumed to be rigid in their own planes and have no out-of-plane flexural stiffness. The model has three degrees of freedom for each floor level (two horizontal translations and one rotation about the vertical axis). All other degrees of freedom are eliminated by static condensation on the macroelement level, or by assuming rigid links, or they are ignored. Masses are lumped at the floor levels. The compatibility of axial deformations in columns common to more than one frame or in intersecting shear walls is neglected.

The advantages of a pseudo three-dimensional model over a fully three-dimensional model are easier data preparation, easier interpretation of results and higher computational efficiency. In spite of its obvious limitations the pseudo three-dimensional model has been in different forms implemented in computer codes for linear analysis (e.g. the TABS family of programs) and widely used by the engineering profession in design and in teaching worldwide. In Slovenia, the pseudo three-dimensional model was implemented in the EAVEK program (Fajfar, 1976). The program has become the standard analysis software in Slovenian design offices. In the EAVEK program, the condensed flexibility matrices for "standard" macroelements (i.e. walls, coupled walls, walls on columns and regular orthogonal frames) are determined by close-form analytical formulae, or in the case of some coupled walls, with a static analysis on the macroelement level. The condensed flexibility matrices for "nonstandard" macroelements (e.g. irregular frames) can be computed by any program for the analysis of plane frames and transferred to EAVEK program as input data. The corresponding condensed stiffness matrix of a macroelement is obtained by inversion of the condensed flexibility matrix. This matrix is then transformed from the local (element) coordinate system to the global (structure) coordinate system. The structural stiffness matrix in terms of two translations and one rotation for each floor is determined by summing the transformed matrices of all macroelements. In Fig. 1 the "standard" macroelements used in EAVEK program and their mathematical models are presented.

Only one attempt of the extension of the pseudo three-dimensional model into the nonlinear range, i.e. the program DRAIN-TABS (Guendelman-Israel and Powel, 1977) is known to the authors. Unlike the two "parent" programs (TABS and DRAIN-2D), DRAIN-TABS has been only rarely used. A much more simple extension of the pseudo three-dimensional model into the nonlinear range is applied in the proposed method that is restricted to static analysis and implemented in the NEAVEK program (Nonlinear EAVEK). In the DRAIN-TABS program the nonlinear analysis of individual macroelements is performed in details with the DRAIN-2D program. In the program NEAVEK, however, for each macroelement a bilinear or multilinear base shear - top displacement relationship is determined based on the initial stiffness, strength at the assumed plastic mechanism and assumed post-yield stiffness. So, with the proposed method a step-wise elastic analysis of the structure can be performed. In each step, the stiffness of at least one macroelement and of the whole structure change.

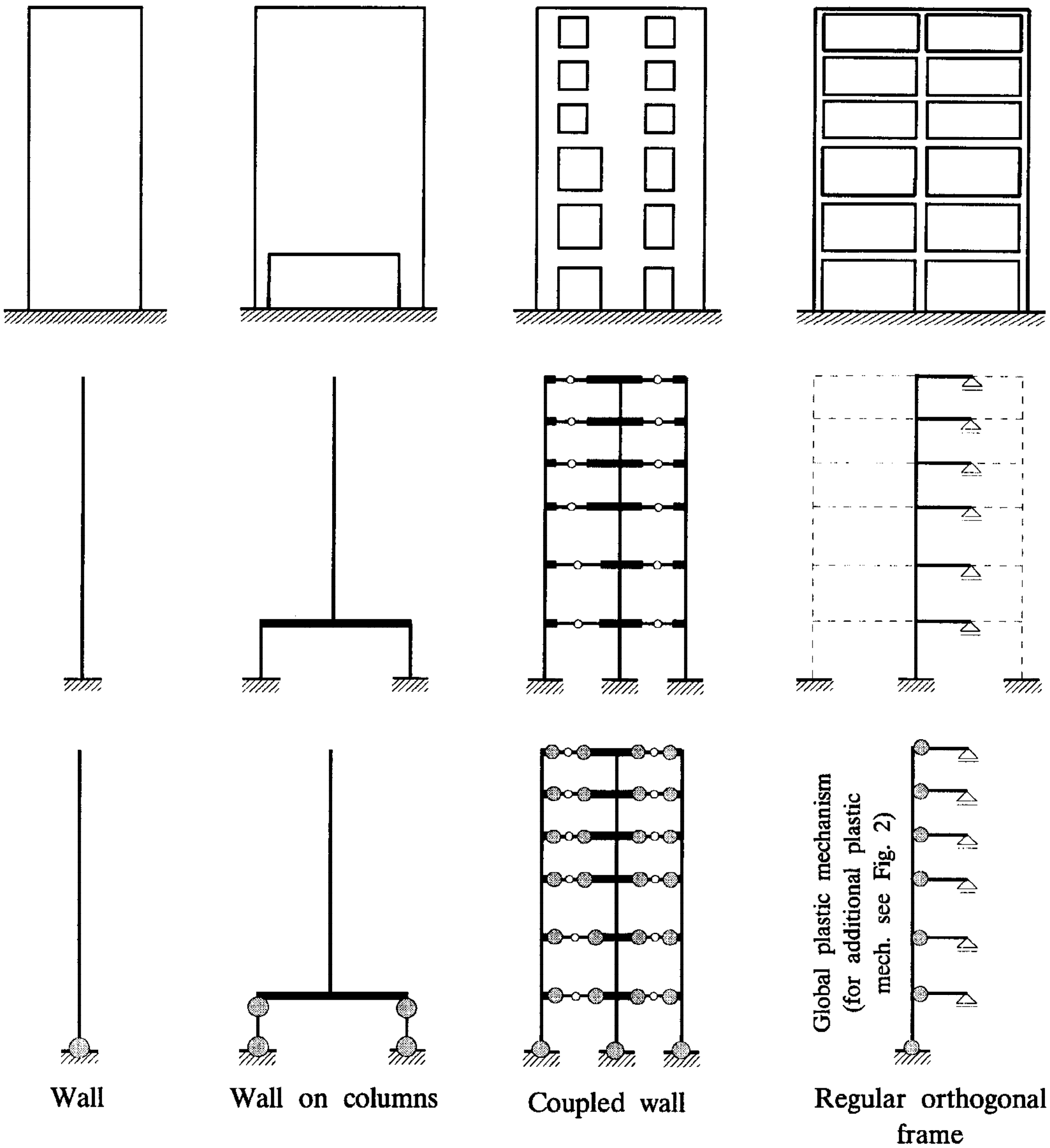

Fig. 1. "Standard" macroelemnts, mathematical models for elastic analysis and assumed plastic mechanisms.

NONLINEAR BASE SHEAR - TOP DISPLACEMENT RELATIONSHIPS FOR MACROELEMENTS

In this section approximate base shear - top displacement relationships are developed for four "standard" macroelements (Fig. 1). It is planned that additional macroelements will be added during the further development of the procedure.

For each macroelement, one or more possible plastic mechanism are assumed. For three macroelement types (wall, wall on columns and frame) elastic behavior is assumed until the plastic mechanism is formed. The elastic stiffness can be based on uncracked, cracked or some average section properties. Base shear V is defined as the sum of horizontal forces F_i over all stories. Top displacement D can be determined as:

$$D = \sum_{i=1}^{n} F_i \, d_{ni} \tag{1}$$

where F_i is the horizontal force in the *i-th* story and d_{ni} are the coefficients of the condensed flexibility matrix that represent top displacement (in the *n-th* story) due to unit horizontal force in the *i-th* story. The formulae for the determination of the condensed flexibility matrices for three macroelement types (wall, wall on columns and frame) are given in (Fajfar, 1975 and 1978). After the formation of the plastic mechanism, the force - displacement relationship is governed by the post-yield stiffness which is arbitrarily assumed on the macroelement level. So, the base shear - top displacement relationship of a macroelement is bilinear, provided that the vertical distribution of lateral loading is constant. If this distribution changes during the loading history (this happens in principle always when one of the macroelements in the structural system yields or changes its stiffness) than the slope of the base shear - top displacement line also changes.

Wall is treated as a cantilever beam element. Only one plastic mechanism, i.e. with the plastic hinge at the bottom of the wall (Fig. 1), is assumed. A plastic hinge at the base appears when the base bending moment M becomes equal to the yield moment M_y. After the plastic mechanism is formed, a flexural spring with a small stiffness is introduced in the mathematical model at the base of the wall. The wall remains in the structural model of the complete structure. The increase of the base shear of the wall due to the increase of external loading depends on the assumed post-yielding stiffness and is typically small. However, the elastic upper part of the wall may substantially influence the distribution of external loading to different macroelements.

Wall on columns is modeled as a cantilever beam supported on two columns that are connected with a stiff horizontal beam. It is assumed that the behavior of the macroelement is elastic until plastic hinges appear simultaneously at the top and bottom of both columns and the plastic mechanism is formed (Fig. 1). At this time the sum of bending moments in all four critical sections is equal to the sum of yield moments in the same cross-sections. The post-yielding behavior of the macroelement is modeled by reducing the flexural stiffness of columns to a small percentage of the initial stiffness.

Frame Only regular frames that form an orthogonal grid of beams and columns can be processed at the present time. Three different main types of plastic mechanisms (Fig. 2) are assumed (Mazzolani and Piluso, 1995). The mechanism of global type is a special case of the type II mechanism.

The frame is loaded with horizontal forces F_i at the levels H_i above the ground level. If ΣMc_i is the sum of yield moments of all columns and ΣMb_i the sum of yield moments of all beams in the story i, the multipliers of the horizontal forces for different types of mechanisms (defined as the ratio between the forces at the plastic mechanism and the forces F_i) can be expressed as (Mazzolani and Piluso, 1995):

Type I mechanism $$\alpha^{I}_{k} = \frac{\Sigma Mc_1 + \Sigma Mc_k + \sum_{i=1}^{k-1} \Sigma Mb_i}{\sum_{i=1}^{k} F_i H_i + H_k \sum_{i=k+1}^{n} F_i} \qquad (2 \le k \le n) \qquad (2)$$

Type II mechanism $$\alpha^{II}_{k} = \frac{\Sigma Mc_k + \sum_{i=k}^{n} \Sigma Mb_i}{\sum_{i=k}^{n} F_i (H_i - H_{k-1})} \qquad (1 \le k \le n) \qquad (3)$$

Type III mechanism $$\alpha^{III}_{k} = \frac{2\, \Sigma Mc_k}{\sum_{i=k}^{n} F_i (H_k - H_{k-1})} \qquad (1 \le k \le n) \qquad (4)$$

In all cases it is assumed that the yield moments of all beams in a story and of all columns in a story are equal. If this is not the case average yield moments are used. For a frame all possible mechanisms are checked (*3n-1* values). The expected plastic mechanism is controlled by the smallest multiplier. After the formation of the plastic mechanism, the stiffnesses of all beams and all columns are decreased to a small percentage of their initial stiffnesses.

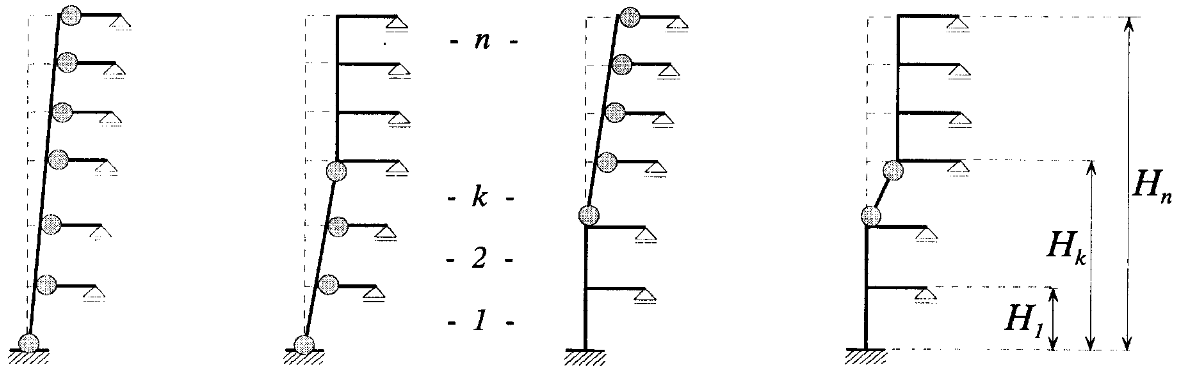

Fig. 2. Assumed types of frame plastic mechanisms.

<u>Coupled wall</u> In the case of a general coupled wall no close-form solutions are available. A numerical static analysis of the mathematical model shown in Fig. 1 is needed. A gradual formation of plastic mechanism is assumed. Consequently, the base shear - top displacement relation is piecewise linear, even if the vertical distribution of lateral loads is constant.

For coupled walls with one row of beams the mechanism according to Fig. 1 (plastic hinges at the base of all walls and in all beams) is assumed. The mechanism is formed in three steps. The behavior of the macroelement is elastic until all beams yield simultaneously at the both ends. For subsequent load increments the coupled wall is divided into two separated walls. Zero post-yield stiffness is assumed for beams. The plastic mechanism occurs when both walls yield at the base (typically not at the same time).

A similar procedure can be applied for a coupled wall with several rows of beams. After the first row of beams yields, such a wall is divided into two separate parts. At least one of them is still a coupled wall. Under increasing lateral loading the procedure is repeated until the coupled wall is disintegrated to individual walls and until each of these walls yields at the base.

METHOD OF ANALYSIS

Analysis is performed as a sequence of linear analyses using an event-to-event strategy. An event is defined as a discrete change of the structural stiffness due to formation of a plastic hinge (or simultaneous formation of several plastic hinges) in a macroelement. Due to stepwise linear force-displacement relationships and "exact" determination of all events, the event-to-event procedure does not produce any unbalanced forces. The computational procedure is as follows:

(1) All structural data have to be known. In addition to the data needed for elastic analysis, yield moments for the critical cross-sections (potential plastic hinges) should be provided. In the cases where the axial force in the critical cross-section is substantially influenced by the horizontal loading, the effect of axial force on the yield moment should be taken into account. Seismic capacity of structural members (including shear strength) is not needed for the analysis. It is needed, however, for the evaluation of the analysis results, for example for checking if a brittle failure mode will occur before the predicted mechanism can be formed.

(2) The distribution of horizontal static loads over the height of the building is chosen (e.g. an inverted triangle) and the increment of the load magnitude is arbitrarily selected. For the asymmetric structures,

where a 3D analysis is performed, the coordinates of the points of load application (usually in the center of masses) are specified, as well as the direction of loading.

(3) For the selected load increment, the elastic static analysis is performed. The global displacement increments, as well as the load distribution, displacements and internal force increments for each macroelement are computed.

(4) Using the approximate nonlinear base shear - top displacement relationships, the event factors for all expected events for all macroelements are calculated. The event factor is defined as the ratio between the load increment that causes an event and the selected load increment. So, the event factor is a scale factor (multiplier) for the selected external load increment. Multipliers defined by eqs. *(2)*, *(3)* and *(4)* represent event factors for the first step of analysis. The smallest event factor defines the event that happens next and the actual load increment that should be added to total external loading in order that the next event will occur.

(5) All response quantities determined as described in (3) are scaled with the minimum event factor and added to the results of the previous step. In this way, the solution advances to the next predicted event.

(6) The mathematical model and/or the stiffness of the macroelement that triggered the event is changed according to the rules described in the previous section.

(7) Using the new mathematical model, the procedure described in (3), (4), (5) and (6) is repeated.

(8) There are several options for the termination of analysis, e.g. the formation of a plastic mechanism for the whole structure, the exceedance of a prescribed maximum allowable top displacement for a macroelement or detection of a brittle failure mode.

The proposed procedure is in principle applicable for building structures of any material. Our applications, however, have been restricted to reinforced concrete buildings so far. Generally, ductile behavior of critical cross-sections was assumed. Yield moments were determined by usual procedures. The effects of axial forces on yield moments were taken into account.

EXAMPLE

Two seven-stories RC building structures were chosen as test examples. The first, symmetric building completely corresponds to the RC frame-wall building (Fig. 3) tested in Tsukuba within the framework of the joint U.S.-Japan research project. More data on this structure are given elsewhere (e.g. Wight, 1985).

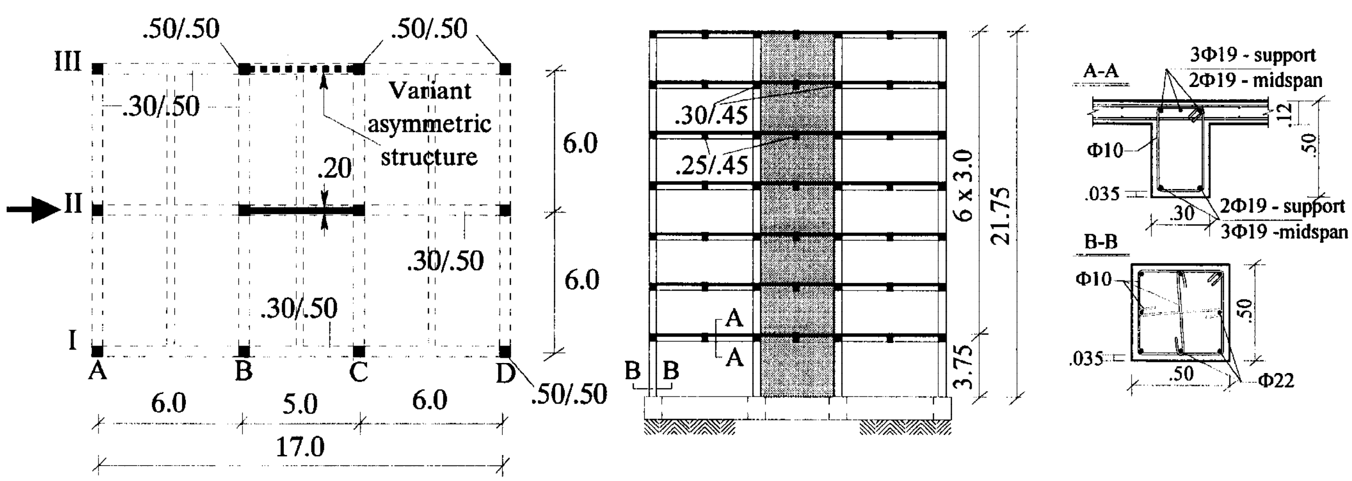

Fig. 3. Floor plan, elevation and details of beam and column reinforcement for the test structure.

In the second, asymmetric building, the structural wall was moved from the middle frame II into the frame III. The initial stiffness of all structural elements was based on gross cross-sections, average measured modulus of elasticity $E=2.5 \cdot 10^7$ kN/m^2 and shear modulus $G=1.0 \cdot 10^7$ kN/m^2. The following yield moments were taken into account (in kNm): outer columns 310, inner columns 384, wall 14250, beams (flange in tension) 87 and beams (flange in compression) 230. The beams of the frames were attached to wall by assuming fully fixed boundary conditions. So, the mathematical model consists of five macroelemens in the longitudinal (X) direction (wall, two three-span frames and two one-span frames) and four identical frames in the transverse (Y) direction. The stiffness of the spring at the base of the wall simulating the post-yield behavior was assumed to be 10 percent of the initial wall stiffness EI. Such a relatively large value was chosen in order to simulate the observed 3-D effects in tests. The post-yield stiffnesses of frame elements was assumed to be 1 percent of their initial stiffnesses. The horizontal loading was applied in X direction in the middle of the building. The inverted triangular distribution of the loading throughout the height of the building was used. The base shear-top displacement relationships are shown in Fig. 4. Top displacements in X direction are monitored at the middle frame. The results, obtained by CANNY program (Li, 1993) and the envelope of pseudo dynamic tests of full-scale symmetric structure in Tsukuba (Wight, 1985) are shown for comparisson.

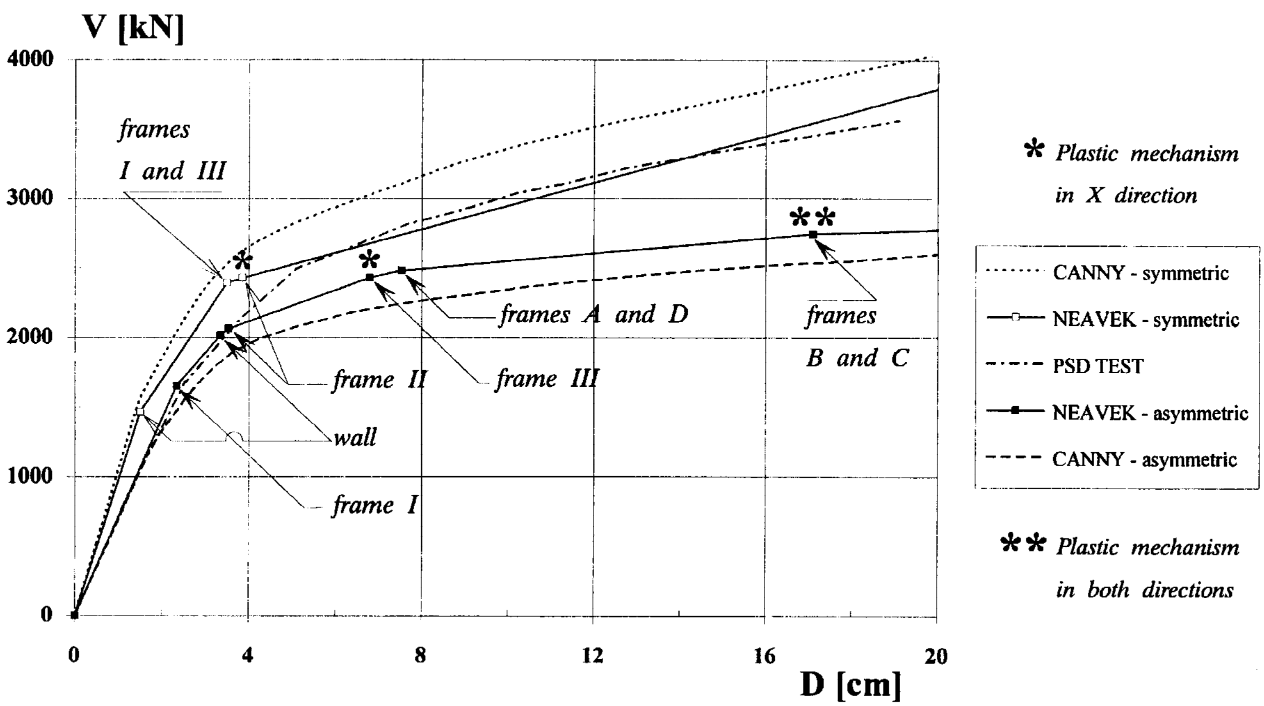

Fig. 4. Base shear-top displacement relationship for symmetric and asymmetric structure.

In the symmetric structure the wall yields first, then the plastic mechanism is formed in frames I and III, and finally in frames in line II. At this moment the global plastic mechanism in X direction is formed at the base shear V=2450 kN and top displacement D=3.9 cm. In the case of asymmetric structure, plastic mechanisms form in the following order: frame I, wall, frame II, frames in line III, frames A and D, and finally frames B and C. The global plastic mechanism in the X direction is formed at V=2430 kN and D=6.8 cm. At this moment the top displacements in frames I and III account to 9.7 cm and 3.8 cm, respectively. The comparison of the results for both structures (Fig. 4) indicates that larger displacements and larger ductilities are required in asymmetric structure in order to develop the same strength as in the symmetric structure, especially at the flexible side of the building. Or, in other words, at the same displacement, the strength of the asymmetric structure is smaller than the strength of the symmetric one. Furthermore, due to different vertical distribution of lateral loading on the macroelements in two structures, the type of the plastic mechanism for the frame I changes from the global type to the type I with plastic hinges in columns in the fifth story.

CONCLUSIONS

The proposed simple procedure for the push-over analysis of building structures is capable to estimate several important characteristics of nonlinear structural behavior, especially the real strength and the global plastic mechanism. The efforts needed for data preparation, computation and interpretation of results are much smaller as in the case of other nonlinear analysis methods. So, the proposed procedure may be appropriate for practical analysis and design of earthquake resistant building structures and for evaluation of existing structures.

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