# SEISMIC RISK ANALYSIS OF BUILDINGS USING SFEM

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## SUMMARY

The safety evaluation of building structures under short duration dynamic loadings, especially seismic loading, is studied. All the load and resistance-related parameters are modeled as realistically as possible. The uncertainty in them is explicitly addressed. A nonlinear time domain reliability analysis procedure, in the context of stochastic finite element concept, is proposed to estimate the associated risk. Both the serviceability and strength limit states are considered. The proposed algorithm integrates the concepts of the response surface method, the finite element method, the first-order reliability method, and the iterative linear interpolation scheme. The unique feature of the algorithm is that actual earthquake loading time histories can be used to excite structures, enabling a realistic representation of the loading conditions. The algorithm has been extensively verified using the Monte Carlo simulation technique. The verified algorithm is then used to study the reliability of a frame structure excited by 13 actual earthquake time histories, 12 of them recorded during the Northridge earthquake of 1994. The reliabilities of the frame are found to be quite different although it is essentially excited by the same earthquake. Some other important observations are made.

## INTRODUCTION

There has been a need for efficient and accurate reliability estimation method for nonlinear structures subjected to time-variant loadings, i.e., seismic and other short-duration loadings. The consideration of uncertainty in the short duration dynamic loadings has been a major challenge. Conceptually, the uncertainty can be introduced in several ways. Two attractive choices are (i) the nonlinear random vibration techniques, and (ii) time history analysis. In the random vibration approach, the uncertainty in the loadings is emphasized but cannot estimate reliability in the time domain. Only a few of the methods in the random vibration approach deal with the uncertainty of system parameters. Therefore, the incorporation of uncertainties in the time history analysis is very desirable for the reliability analysis of nonlinear structures subjected to seismic loading, and is the subject of this paper.

Seismic loading causes a significant amount of damage to structures and is very unpredictable. The safety evaluation of nonlinear structures considering uncertainties in the time history of earthquakes is very desirable. A very limited amount of work has been reported in the literature on the subject. The Monte Carlo simulation method (MCS), the classical Stochastic Finite Element Method (SFEM) [Haldar and Gao, 1997; Haldar and Mahadevan, 1999b], and Response Surface method (RSM) can be used for this purpose. Although a class of Monte Carlo simulation methods may be appropriate, considering their accuracy and computational efficiency, they may be too costly and cumbersome for the reliability analysis of nonlinear dynamic systems. The classical SFEM as proposed by Haldar and Gao (1997) has been proven to be very elegant, economical, and accurate for static loading. However, its use for dynamic loadings has yet to be undertaken. Using this algorithm, it is possible to estimate the reliability at every time increment of a seismic loading. However, it is very difficult and may be too time-consuming for practical structures. The RSM has the potential to consider the appropriate mechanical behavior of nonlinear systems and the uncertainty in the load and resistance-related parameters

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without compromising the efficiency and accuracy to a great extent. The objective of this study is to develop such an algorithm by combining the RSM, FEM, and the first-order reliability method (FORM).

## UNCERTAINTIES IN TIME-VARIANT LOADING

In the past, the uncertainties in the dynamic loadings were grouped into short duration loadings and long duration loadings. For long duration loadings, the works of Bucher, Chen, and Schüller (1989) and Yao and Wen (1996) are noteworthy. Bucher, Chen, and Schüller (1989) converted the time-variant problem to a time-invariant problem by applying the lifetime maximum effect of combined load processes after evaluating the limit state function. Bucher and Bourgund (1990) and Rajashekhar and Ellingwood (1993) considered short duration loadings. They examined the same dynamic problem, which is a nonlinear single degree of freedom (SDOF) oscillator with random system parameters subjected to a rectangular load pulse with random duration and amplitude. Their procedures cannot be used for the reliability analysis of nonlinear structures subjected to general short duration loadings including seismic loading.

There is no guideline for considering uncertainties in both the amplitude and frequency content of an earthquake loading. The uncertainty in the amplitude of an earthquake was successfully considered in the context of RSM [Huh and Haldar, 1999]. They estimated the reliability of structures considering the uncertainty in the amplitude of an earthquake loading. Several earthquake time histories were recorded in close proximity to each other during the Northridge earthquake of 1994. The authors' initial study indicates that the frequency content of these records is quite different. This leads the authors to consider the uncertainty in the frequency content of an earthquake indirectly. Since all the time histories were recorded during the same earthquake and have different frequency content, they can be used to study the effect of uncertainty in the frequency content of an earthquake. An extensive parametric study is conducted in an attempt to quantify the effect of uncertainty in the frequency content of earthquakes on the overall reliability of structures.

## RISK EVALUATION USING LIMIT STATE FUNCTIONS

The reliability analysis using FORM requires that a performance function or limit state function, implicit or explicit, must be available. Commonly used limit state functions can be broadly divided into two groups: the serviceability and strength limit states. In general, the serviceability limit states represent the global structural behavior. The reliability estimation using the serviceability limit states is relatively simple. The strength limit states, on the other hand, generally represent the behavior of local structural elements, and risk evaluation using them is not straightforward. Since a structure can fail due to excessive lateral or interstory deflection, or due to failure of several components in strength forming a local or global mechanism, each limit state needs to be considered separately. The proposed algorithm is capable of calculating risk using both types of limit states. They are briefly discussed below.

### Serviceability Limit State

For structures subjected to seismic loading, the design may be controlled by the serviceability, e.g., inter story drift or the overall lateral displacement (deflection). Limit states corresponding to drift or deflection can be formulated using the recommendations given in current codes. The information on the maximum drift or deflection, $y_{max}$, at a particular location of a structure and the corresponding code-specified allowable value can be used to develop a serviceability limit state. The $y_{max}$ can be represented as ○(**X**), where ○( ) is an explicit polynomial function developed through the RSM and **X** is a vector representing all the variables in the response surface. Then the serviceability limit state becomes,

$$g(\mathbf{X}) = \delta_{allow} - y_{max}(\mathbf{X}) = \delta_{allow} - \hat{g}(\mathbf{X}) \qquad (1)$$

where $\delta_{allow}$ is the allowable drift or deflection specified in codes for the structural system under consideration.

## Strength Limit State

Most of the elements in the structural system considered in this study are beam-columns, i.e., they are subjected to both axial load and bending moment. For design purpose, interaction equations are generally used to consider the combined effect of axial load and bending moment. In the U.S., the interaction equations suggested by the American Institute of Steel Construction's (AISC's) *Load and Resistance Factor Design* (LRFD) manual for two dimensional structures are:

$$\frac{P_u}{\phi P_n} + \frac{8}{9}\left(\frac{M_{ux}}{\phi_b M_{nx}}\right) \leq 1.0; \quad \text{if} \quad \frac{P_u}{\phi P_n} \geq 0.2 \tag{2}$$

$$\frac{P_u}{2\phi P_n} + \left(\frac{M_{ux}}{\phi_b M_{nx}}\right) \leq 1.0; \quad \text{if} \quad \frac{P_u}{\phi P_n} < 0.2 \tag{3}$$

where N and $N_b$ are the resistance factors, $P_u$ is the required tensile/compressive strength, $P_n$ is the nominal tensile/compressive strength, $M_{ux}$ is the required flexural strength, and $M_n$ is the nominal flexural strength. $P_n$ and $M_{nx}$ can be calculated using AISC's LRFD code.

### *Dynamic Effect in the Interaction Equations*

In dynamic analysis, $P_u$ and $M_{ux}$ are functions of time. Thus, the ratio of $P_u$ and $M_{ux}$ is also a function of time, indicating that the interaction equations used for static problems cannot be used for dynamic loadings. Furthermore, the axial and bending strength of the members, $P_n$ and $M_{nx}$, are also functions of time since the load effects change from compression to tension and vice versa in dynamic loading. To address this complex problem, the interaction equations are divided into two parts; the part containing the effect of axial load only, and the part containing the effect of bending moment only. In order to follow the intent of the design guidelines, the effects of axial load and bending moment are represented separately as $\alpha_1(P_u/P_n)$ and $\alpha_2(M_{ux}/M_{nx})$, respectively. The coefficients $\alpha_1$ and $\alpha_2$ represent the time-variant aspects of $P_u/P_n$ and $M_{ux}/M_{nx}$. These modifications do not affect the efficiency or generality of the algorithm since they are obtained simultaneously from an identical design. Only an estimation of the coefficients has to be made for both polynomials.

In general, the maximum values of both $P_u$ and $M_{ux}$ do not occur at the same time. It is difficult to predict which interaction equation [Equation (2) or Equation (3)] is applicable for a given sampling point, since the $P_u/P_n$ ratio is unknown. To address this issue, the responses for each sampling point are tracked, i.e., in a deterministic dynamic analysis, recording the time when the interaction equation [left hand side of the Equation (2) or Equation (3)] reaches a maximum. This will give the necessary information on which interaction equations are to be used. Then the contributions of $\alpha_1(P_u/P_n)$ and $\alpha_2(M_{ux}/M_{nx})$ can be evaluated for a sampling point. Similarly, for other sampling points the contributions of axial load and bending moment can be evaluated. However, these contributions are expected to be different, indicating the dynamic nature of the analysis. Interestingly, these values can be used to formulate the necessary response surface required for reliability analysis using FORM. Since the strength limit state functions are now available in explicit form, the standard FORM is used to estimate the corresponding reliability index. The efficiency of the algorithm can be improved further by using sensitivity index concept [Haldar and Mahadevan, 1999a].

## PROPOSED SFEM

The proposed algorithm intelligently integrates the concepts of RSM, FEM, FORM, and an iterative linear interpolation scheme to search for the most probable failure point (MPFP). First, responses for a limit state function are generated in terms of *k* basic random variables by conducting RSM around the initial center point with saturated design and the second order polynomial without cross terms and FEM. Using the explicit expression for the limit state function thus obtained and FORM, the reliability index $\beta$ and the corresponding design point are obtained. The design point is used to locate a new center point for the next iteration by applying a linear interpolation scheme. The updating of the center point continues until it converges to a predetermined tolerance level. In the final iteration, the information on the most recent center point is used to formulate a response surface using central composite design with full second order polynomials. This gives an explicit expression of the limit state function. The FORM method is then used to calculate the reliability index and the corresponding MPFP.

The accuracy of the proposed methodology depends on the appropriateness of the response surface generated and used in the subsequent analyses, as well as the location of the center point around which the response surface is generated. It is an approximate technique, and its applicability needs to be determined on a case by case basis. Some of the salient features of the proposed algorithm are discussed in the following sections.

**Sequential Response Surface Method**

The response surface approach [Khuri and Cornell, 1996] is a set of statistical techniques designed to obtain the best value of the response considering the uncertainty in the input variables. The primary purpose of applying RSM in reliability analysis is to approximate the original complex and implicit limit state function [Wong, 1984; Rajashekhar and Ellingwood, 1993; Bucher and Bourgund, 1990; Faravelli, 1989; Kim and Na, 1997] using a simple and explicit polynomial. For the type of problem under consideration, at least a second order polynomial is necessary. In this study, two types of second order polynomial, i.e., with and without cross terms, are used to represent the response surface in the intermediate iterations and the final iteration, respectively. They can be expressed as:

$$\hat{g}(\mathbf{X}) = b_0 + \sum_{i=1}^{k} b_i X_i + \sum_{i=1}^{k} b_{ii} X_i^2 \tag{4}$$

$$\hat{g}(\mathbf{X}) = b_0 + \sum_{i=1}^{k} b_i X_i + \sum_{i=1}^{k} b_{ii} X_i^2 + \sum_{i=1}^{k-1} \sum_{j>1}^{k} b_{ij} X_i X_j \tag{5}$$

where $X_i$ ($i = 1,$ □□□□, $k$) is the $i^{th}$ random variable, and $b_0$ , $b_i$, $b_{ii}$, and $b_{ij}$ are unknown coefficients to be determined. The number of coefficients for each polynomial in Equation (4) and Equation (5) are $p_1 = 2k+1$ and $p_2 = (k+1)(k+2)/2$, respectively.

The polynomial can be fully defined from regression analysis for the responses at specific data points called sampling points. The selection of sampling points where responses need to be calculated is known as experimental design. Saturated design and the central composite design could be the two most promising ones among the techniques available to generate sampling points [Huh and Haldar, 1999]. The details of experimental design cannot be given here due to lack of space. However, they can be found in the literature [Bucher and Bourgund, 1990; Faravelli, 1989; Huh and Haldar, 1999; Khuri and Cornell, 1996]. Considering the form of the polynomial and the selection requirements for the experimental design points, the efficiency and accuracy of the proposed algorithm can be increased by applying (1) saturated design using a second order polynomial without cross terms during the intermediate iterations and (2) central composite design using a full second order polynomial in the final iterations.

**Determination of the Center Point**

To accurately estimate the probability of failure, it is important to locate the most probable failure point (MPFP) so that the response surface can be generated around it. Thus, it is necessary to improve on the location of the center point in subsequent iterations. Bucher and Bourgund (1990) suggested an iterative linear interpolation scheme that can be used to locate the center point efficiently and accurately, as discussed below.

In the iterative scheme, the center point is initially selected to be the mean value $\overline{X}_i$ of the random variable $X_i$. Then, using the values of g(**X**) obtained from the deterministic FEM evaluations for all the sampling points around the center point, the response surface $\bigcirc$(**X**) can be generated explicitly in terms of the random variable $X_i$. Once a closed form of the limit state function, $\bigcirc$(**X**), is obtained, the coordinates of the design point $\boldsymbol{x}_D$ can be estimated using FORM and the information about $X_i$, that is, means, standard deviations, and the distribution types. The actual response can be evaluated again at the design point $\boldsymbol{x}_D$, i.e., $g(\boldsymbol{x}_D)$ and a new center point $\boldsymbol{x}_M$ can be selected using linear interpolation from the mean vector $\overline{\mathbf{X}}$ to $\boldsymbol{x}_D$ such that g(**X**) = 0; i.e., for each random variable,

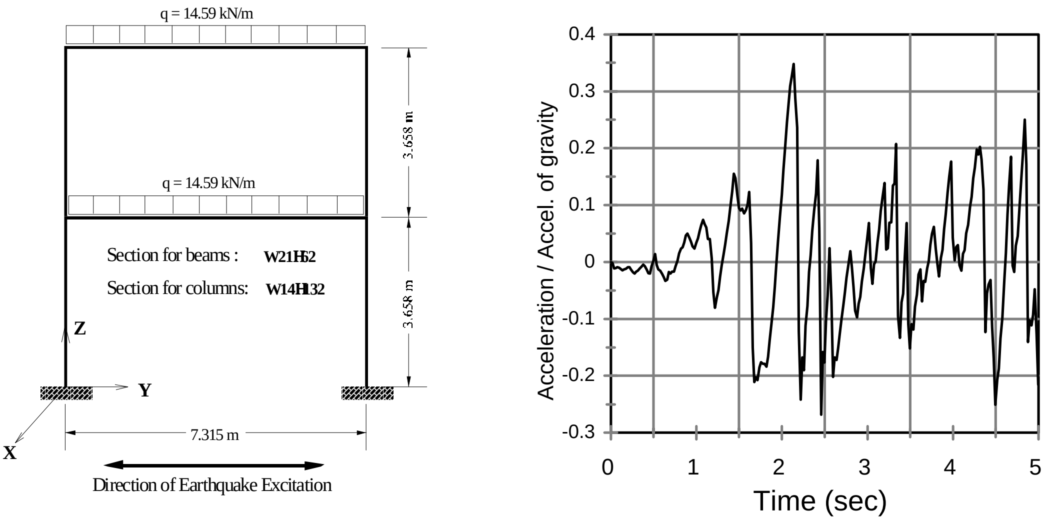
$$\text{IF } g(x_D) \geq g(\overline{X}), \quad x_{M_i} = \overline{X}_i + \left(x_{D_i} - \overline{X}_i\right) \frac{g(\overline{X})}{g(\overline{X}) - g(x_D)} \tag{6}$$

$$\text{IF } g(x_D) < g(\overline{X}), \quad x_{M_i} = x_{D_i} + (\overline{X}_i - x_{D_i}) \frac{g(x_D)}{g(x_D) - g(\overline{X})} \tag{7}$$

The point $\boldsymbol{x}_M$ is now used as a new center point for the next iteration. This iteration scheme can be repeated until a preselected convergence criterion is satisfied. It has been found that the center point selection scheme generally takes only 2 to 3 iterations.

## SEISMIC RISK INVESTIGATION USING ACTUAL EARTHQUAKE RECORDS

To elaborate the algorithm further and to investigate the effect of the frequency content of the seismic loading on the reliability of a structure, a two story steel frame shown in Figure 1(a) is considered. All the beams and columns of the frame are made of W21×62 and W14×132, respectively, and A36 steel is used for this example. The period of the structure is found to be 0.522 sec. The period is well within the upper and lower bounds suggested by Goel and Chopra (1997). The frame is excited for 5.02 seconds by 13 recorded acceleration time histories as identified in Table 1. The first one was recorded during the El Centro earthquake of 1940 and the other 12 were recorded during the Northridge earthquake of 1994. The El Centro records are used for verification purposes, since they are widely used in the research community. Each record contains two lateral components of excitation. They are denoted hereafter as East-West (E-W) and North-South (N-S) components. The time histories of all the earthquakes cannot be shown here due to the lack of space. A representative acceleration time history is shown in Figure 1(b).




**(a) Two Story Steel Frame Structure** **(b) Typical Accelerogram of an Earthquake**

**Figure 1. Two Story Frame Structure and Earthquake Excitation Time History**

Details of the 26 earthquake time history records, in terms of their peak ground acceleration (PGA) and root-mean-square (RMS) values are calculated and are summarized in Table 1. The PGA and RMS values are expressed in the unit of 1 gravity acceleration (9807 mm/sce$^2$). Seismic reliabilities of the frame excited by the 26 records are then estimated for both the serviceability and strength limit states using the algorithm presented here.

For the serviceability limit state, the allowable lateral displacement at the top of the frame is assumed to be $h$/400, where $h$ is the height of the frame. Thus, for this example, $\delta_{allow}$ is equal to 1.829 cm, and using Equation (1), the corresponding serviceability limit state is:

$$g(\mathbf{X}) = \delta_{allow} - y_{max}(\mathbf{X}) = 1.829 - y_{max}(\mathbf{X}) \tag{8}$$

in which $y_{max}(\mathbf{X})$ is the maximum lateral displacement response of the system.

For the strength limit state, the reliability index of the weakest member is reported here. The results are summarized in Table 1.

**Table 1. Identification of 13 Earthquakes, their PGA and RMS values, and Reliability Indexes**

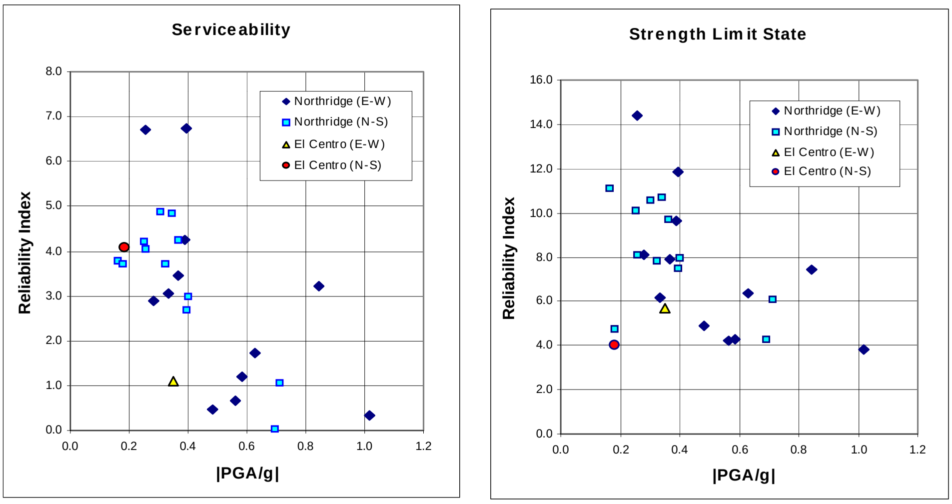
| Earthquake | Direction | PGA ($g$ unit)* | RMS ($g$ unit)* | Reliability Index, β | |
|---|---|---|---|---|---|
| | | | | Serviceability | Strength |
| Uentro | E-W | 0.348 | 0.110 | 1.111 | 5.657 |
| | N-S | 0.182 | 0.073 | 4.068 | 4.405 |
| u923c | E-W | 0.842 | 0.039 | 3.235 | 7.413 |
| | N-S | 0.710 | 0.057 | 1.068 | 6.112 |
| u4979c | E-W | 0.366 | 0.054 | 3.443 | 7.894 |
| | N-S | 0.304 | 0.050 | 4.889 | 10.55 |
| u4253c | E-W | 1.018 | 0.183 | 0.334 | 3.819 |
| | N-S | 0.692 | 0.132 | 0.049 | 4.310 |
| u3822c | E-W | 0.282 | 0.062 | 2.895 | 8.072 |
| | N-S | 0.163 | 0.027 | 3.773 | 11.11 |
| u259c | E-W | 0.628 | 0.216 | 1.715 | 6.336 |
| | N-S | 0.394 | 0.135 | 2.695 | 7.495 |
| u4980c | E-W | 0.256 | 0.030 | 6.701 | 14.38 |
| | N-S | 0.252 | 0.036 | 4.219 | 10.09 |
| u1520c | E-W | 0.333 | 0.049 | 3.041 | 6.162 |
| | N-S | 0.180 | 0.047 | 3.718 | 4.776 |
| u185c | E-W | 0.562 | 0.211 | 0.650 | 4.232 |
| | N-S | 0.255 | 0.067 | 4.056 | 8.087 |
| u931c | E-W | 0.397 | 0.026 | 6.728 | 11.84 |
| | N-S | 0.342 | 0.025 | 4.844 | 10.68 |
| u552c | E-W | 0.387 | 0.074 | 4.252 | 9.621 |
| | N-S | 0.323 | 0.082 | 3.716 | 7.848 |
| u1276c | E-W | 0.585 | 0.118 | 1.192 | 4.304 |
| | N-S | 0.399 | 0.092 | 2.996 | 7.991 |
| u4049c | E-W | 0.481 | 0.206 | 0.452 | 4.872 |
| | N-S | 0.364 | 0.107 | 4.260 | 9.678 |
| * $g$ = 1 gravity acceleration (9807 mm/sce$^2$) | | | | | |

The statistical description of the uncertainties associated with all the random variables required for the reliability analysis are given in Table 2.

**Table 2. Statistical Description of the Random Variables**

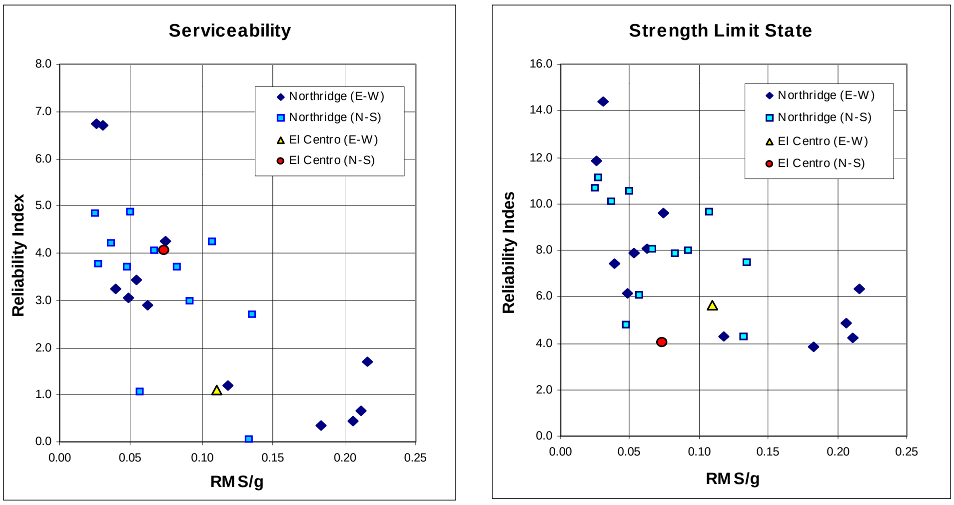
| Random Variable | Mean value | C.O.V | Distribution |
|---|---|---|---|
| E (kN/m$^2$) | $1.9996\times10^8$ | 0.06 | Lognormal |
| $A^b$ (m$^2$) | $1.1806\times10^{-2}$ | 0.05 | Lognormal |
| $I^b_x$ (m$^4$) | $5.5359\times10^{-4}$ | 0.05 | Lognormal |
| $A^c$ (m$^2$) | $2.5032\times10^{-2}$ | 0.05 | Lognormal |
| $I^c_x$ (m$^4$) | $6.3683\times10^{-4}$ | 0.05 | Lognormal |
| > | 0.05 | 0.15 | Lognormal |
| $g_e$* | 1.0 | 0.20 | Type I (EVD) |
| * $g_e$ is a magnification factor for the amplitude of actual seismic acceleration | | | |

In order to investigate the effect of the PGA and frequency content of the earthquake excitations, PGA versus the reliability index and RMS versus the reliability index are plotted in Figures 2 and 3, respectively. The reliability indexes for the serviceability and the strength limit states are considered separately in these figures.




(a) Serviceability (b) Strength Limit State

**Figure 2. Reliability Index vs. PGA for 26 Earthquake Time History Records**




(a) Serviceability (b) Strength Limit State

**Figure 3. Reliability Index vs. RMS for 26 Earthquake Time History Records**

Several observations can be made from Table 1, and Figures 2 and 3. The reliability indexes for the serviceability limit state are much smaller than those for the strength limit state, in almost all cases. This implies that the frame is more vulnerable to failure caused by the lateral displacement. Frames are generally first designed for strength and then studied for serviceability. However, authors and their associates observed that this practice is not desirable. It may also indicate the allowable displacement, $\delta_{allow}$ = ($h$/400), is relatively conservative compared to the strength interaction limit state, particularly for seismic loading. It can also be observed from Figure 2 that the damage potential of an earthquake cannot simply be defined by its PGA. However, a better correlation between the RMS and the reliability index can be observed in Figure 3. The RMS value could be a better predictor of the damage potential of an earthquake than the PGA. The time domain seismic reliability analysis procedure discussed here is feasible and can be used in the future.

## CONCLUSIONS

An efficient and accurate nonlinear SFEM algorithm is proposed to estimate the reliability of structures subjected to seismic loading in the time domain. Both the serviceability and strength limit states can be used for the reliability evaluation. With the help of an example, it is shown that the proposed algorithm can be used to estimate the risk for nonlinear structures subjected to short duration dynamic loadings, including seismic loading. The serviceability limit state appears to be the controlling factor in most cases. The allowable lateral deflection suggested in design codes may need reevaluation, particularly for seismic loading. The PGA is not a good predictor of the damage potential of an earthquake excitation. A better correlation is observed between the RMS value and the reliability index. The proposed algorithm can be used to estimate the seismic reliability of structures.

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