EFFECTS OF MAGNITUDE UNCERTAINTIES ON SEISMIC HAZARD ESTIMATES

David A RHOADES[1] And David J DOWRICK[2]

SUMMARY

Magnitude uncertainties affect different components of an estimate of seismic hazard in a variety of ways, but methods are available for countering such effects.

Uncertainties in measured magnitudes cause bias in estimates of the Gutenberg-Richter activity-rate parameter and, if magnitude uncertainties are correlated with magnitude, in the b-value also. The combined effect can easily amount to a factor of two in estimating the frequency of occurrence of large earthquakes. The biases can be corrected if standard errors of magnitude estimates are known.

In attenuation modelling, ignoring magnitude uncertainties can inflate the residual variance and lead to spurious terms being included in the model. Explicit treatment of magnitude uncertainties formally involves an extension of the usual random effects regression model.

The maximum magnitude that is assumed to be possible in a given source region is highly influential on seismic hazard estimates, but also subject to much uncertainty. A substantial part of this uncertainty can be quantified from relations between earthquake magnitude and source dimensions, and used to adjust estimates of the frequency-magnitude relation.

The effects of magnitude uncertainties on seismic hazard estimates are potentially large. Allowance for such uncertainties should be a standard part of seismic hazard assessment.

INTRODUCTION

The standard probability tree methodology of probabilistic seismic hazard assessment (PSHA) allows for the explicit treatment of many kinds of uncertainty. These include both epistemic uncertainty (e.g., uncertainty in the parameters of the frequency-magnitude relation for a given source region, and in the form and parameters of relations for attenuation of strong shaking) and aleatory uncertainty (i.e., natural variability about the fitted relations). However, the analysis of uncertainty seldom extends to a formal treatment of uncertainties in the data on which the relations are based. Such uncertainties are not readily captured by the probability tree approach; rather they affect the quality of the fitted relations, and as shown below, may be a cause of bias or of spurious variability.

Earthquake magnitude estimates play a crucial part in the estimation of both the frequency-magnitude relation and attenuation relations. The standard methods for fitting both types of relation ignore magnitude uncertainties. The data sets used to estimate these relations are seldom homogeneous, and include magnitudes with different levels of uncertainty. This is because the quality of magnitude determination varies spatially for a given seismograph network, and seismograph networks themselves are subject to frequent changes due to station outages and less frequent changes due to major upgrading. Modern catalogues usually contain an estimate of the standard error of individual magnitude determinations. For early magnitude data, an average standard error applicable to the era of the catalogue can usually be adduced.

[1] *Institute of Geological and Nuclear Sciences, P.O. Box 30-368, Lower Hutt, New Zealand. Email: d.rhoades@gns.cri.nz*
[2] *Institute of Geological and Nuclear Sciences, P.O. Box 30-368, Lower Hutt, New Zealand. Email: d.dowrick@gns.cri.nz*

Seismic hazard analysts are often faced with the need to select from the available data to ensure that data of low quality (i.e., high uncertainty) will not bias the results. Maintaining data quality thus necessitates discarding some information. An alternative approach is to use all the available information, and to accord to each data point the weight that is due to it, given its uncertainty. This is the approach that is pursued here. It is made possible by using methods which allow for the uncertainty in individual data values.

MAGNITUDE UNCERTAINTIES IN THE GUTENBERG-RICHTER LAW

Let us consider an earthquake catalogue with observed magnitudes $(x_i, i = 1, \cdots, n)$ and corresponding standard errors $(\sigma_i, i = 1, \cdots, n)$. Suppose that the catalogue magnitudes are free from any systematic bias. If they are determined as the average over a number of stations, then the central limit theorem assures the approximate normality of the distribution of each catalogued magnitude, i.e.,

$$x_i \sim N(m_i, \sigma_i^2) \tag{1}$$

where m_i is the (unknown) true magnitude. Rhoades (1996) noted that, in light of the Gutenberg-Richter frequency-magnitude relation $\log N = a - bM$ (Gutenberg and Richter, 1944), the prior distribution of m_i given b, or, equivalently, given β, where $\beta = b \log_e 10$, has density

$$f(m_i \mid \beta) \propto \exp(-\beta m_i) \tag{2}$$

and showed that the posterior distribution, given x_i, σ_i, and β is

$$m_i \mid x_i, \sigma_i, \beta \sim N(x_i - \sigma_i^2 \beta, \sigma_i^2) \tag{3}$$

Thus an observed magnitude x_i has an associated bias that depends on its uncertainty; the larger the uncertainty, the larger the bias. Frequency-magnitude relations estimated from real catalogues without allowing for uncertainties are therefore biased also. Tinti and Mulargia (1985) showed that if all the earthquakes have a common standard deviation, then the bias in the a-value of the Gutenberg-Richter relation can be corrected by

$$a = a_{GR} - \frac{1}{2}\sigma^2 \beta^2 \log_{10} e \tag{4}$$

where a_{GR} is the estimate obtained from the observed magnitudes. In this case, observation error does not cause bias in estimates of β. Standard methods appropriate to exact magnitude data may be used, e.g., the maximum likelihood method of Aki (1965)

$$\frac{1}{\beta} = \overline{m} - m_0 \tag{5}$$

where $\overline{m}$ is the average magnitude exceeding some threshold of completeness m_0, or the refinement of this method which allows for magnitude rounding (Utsu, 1966).

In the case where the standard deviations differ between earthquakes, the activity rate for earthquakes exceeding a given magnitude m is estimated by

$$\hat{\lambda}(m) = \frac{1}{T} \sum_{j=1}^{n} [1 - F_j(m)] \tag{6}$$

where T is the the time period of the catalogue and F_j is the posterior cumulative distribution of $m_j, j = 1, \cdots, n$. Rhoades (1996) showed that this estimate has variance

$$\mathrm{Var}[\hat{\lambda}(m)] = \frac{1}{T^2}\sum_{j=1}^{n}[1 - F_j(m)^2] \qquad (7)$$

In this case there is a potential for bias in the estimate of β. It is apparent that a bias will occur if the standard error is correlated with magnitude, as illustrated in Figure 1.

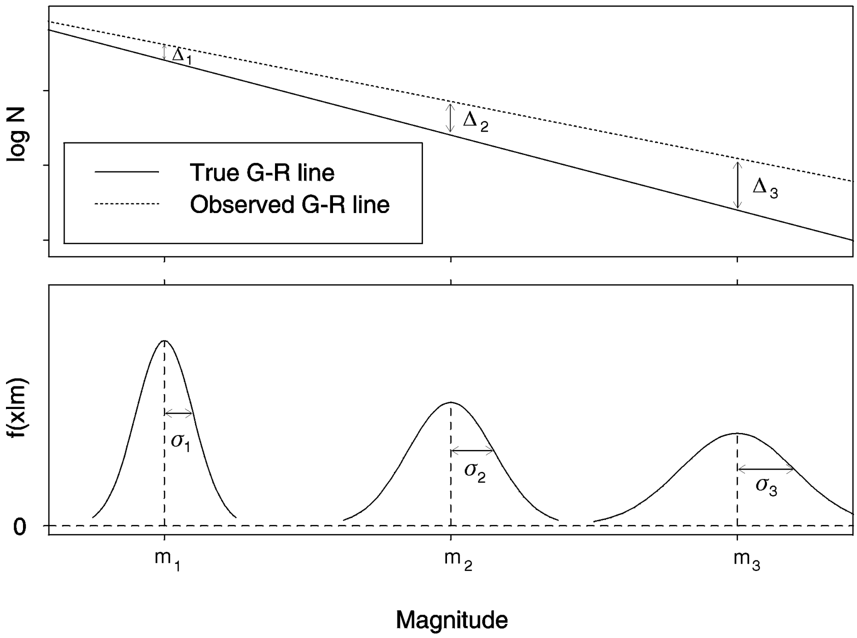

Figure 1. Schematic plot of the bias in the estimated Gutenberg-Richter relation in the presence of magnitude uncertainties. From equation (4), $\Delta_i = \tfrac{1}{2}\sigma_i^2\beta^2 \log_{10} e$ **.**

Rhoades (1996) showed how to correct the bias in β by employing an iterative backfitting procedure, based on the relation

$$\frac{1}{\beta} = \frac{\sum_{j=1}^{n}\int_{m_0}^{\infty} m f_j(m)dm}{\sum_{j=1}^{n}[1 - F_j(m_0)]} \qquad (8)$$

For the distribution of equation (3), the individual terms in the numerator of (8) are given by

$$\int_{m_0}^{\infty} m f_j(m)dm = (x_j - \sigma_j^2\beta)\left[1 - \Phi\left(\frac{m_0 - x_j + \sigma_j^2\beta}{\sigma_j}\right)\right] + \sigma_j\phi\left(\frac{m_0 - x_j + \sigma_j^2\beta}{\sigma_j}\right) \qquad (9)$$

where ϕ and Φ denote the standard normal density and cumulative distribution function, respectively. A more complex, but nevertheless computable, formula applies if magnitude rounding is allowed for (Rhoades, 1996). In either case, the backfitting procedure is to use an initial estimate of β to get an intial estimate of $f_j(m)$ by equation (3), and then to apply equations (8) and (3) alternately until the estimate of β converges, usually in just a few iterations.

Equation (4) suggests an alternative approximate procedure that still involves iteration, but avoids the need to evaluate normal integrals. Note that the corrected *a*-value in equation (4) is the value that would be obtained by applying the usual *a*-value estimate to observed magnitudes that have each been reduced by $\frac{1}{2}\sigma^2\beta$. This suggests that a simple correction

$$m_j = x_j - \frac{1}{2}\sigma_j^2\beta \qquad (10)$$

to each observed magnitude x_j, and the application of the standard maximum likelihood procedures for estimating *a*- and *b*-values, should correct the bias in both parameters, when the σ_j vary. In the case of *b*-value estimation, this means alternating between equations (5) and (10) instead of equations (3) and (4). This proposed approximate method involves much less computation.

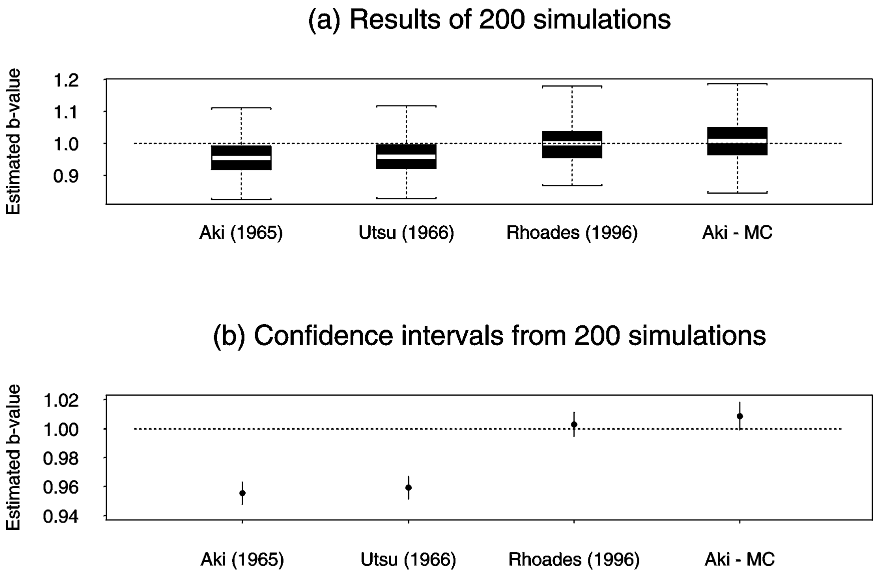

Figure 2. Estimates of *b*-value from simulated catalogues with magnitude uncertainties positively correlated with magnitude using: standard maximum likelihood (Aki, 1965), maximum likelihood corrected for rounding (Utsu, 1966), maximum likelihood corrected for rounding and magnitude uncertainties (Rhoades, 1996), and standard maximum likelihood with magnitude correction (10) (Aki - MC). The true *b*-value is 1. The estimates are presented as (a) box plots of 200 simulations and (b) the mean and 95% confidence limits from 200 simulations.

The calculation of *b*-values using the proposed approximate method of standard maximum likelihood after magnitude correction (Aki - MC) is compared with the procedure of Rhoades (1996) with correction for rounding and magnitude uncertainties, and the procedures of Aki (1965) and Utsu (1966), in 200 simulated catalogues. In the simulated catalogues, magnitude uncertainties are positively correlated with magnitude, and, following Rhoades (1996), each catalogue has 5000 earthquakes conforming to the Gutenberg-Richter relation with *b*-value 1, $m_j \geq 2.7$ and $\sigma_j = 0.1(1+u_j m_j)$, where u_j is uniformly distributed on the interval (0,1). Observed magnitudes are rounded to 1 decimal place. For calculation of *b*-values the magnitude threshold is taken as 3.95. The simulation results are given in Figure 2 in the form of (a) boxplots which show the median, quartiles and extremes of the distribution of b-values and (b) approximate 95% confidence intervals for the mean b-value. It can be seen that the Aki (1965) procedure, which ignores magnitude uncertainties, and the Utsu (1966) procedure, which allows for rounding but otherwise ignores magnitude uncertainties, give estimates that are significantly biased. The Rhoades (1996) method corrects the bias, and the Aki - MC method is only marginally biased. Given its relative simplicity, it has much to recommend it as a practical method.

UNCERTAINTIES IN MAXIMUM MAGNITUDE

Estimates of maximum regional magnitude $m_{\max}$ are subject to much uncertainty. Kijko and Sellevoll (1989) and Kijko and Graham (1998) have given and compared methods for estimating $m_{\max}$ by extrapolation of earthquake frequency-magnitude data. However, since magnitude determinations are usually adequate over only the last few decades and are not necessarily a good guide to maximum possible regional magnitudes, $m_{\max}$ must in practice often be estimated independently of historical seismicity data. For this purpose, statistical relationships between source dimensions and earthquake magnitudes (e.g., Wells and Coppersmith, 1994) are useful. A recent example of such a relationship is shown in Figure 3.

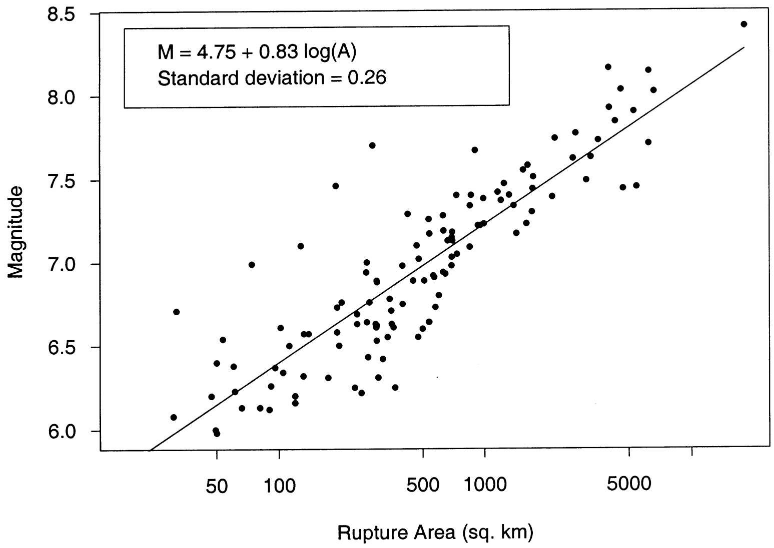

Figure 3. Regression of earthquake magnitude against rupture area using worldwide data, in which rupture area is the product of the surface rupture length and the downdip width of the rupture. After Stirling et al. (1998).

Suppose that on the basis of such a relationship, $m_{\max}$ is estimated to be normally distributed with mean μ and standard deviation σ. Let us consider the estimation of the whole frequency-magnitude relation, including the tail-off at the high magnitude end, using both a seismicity catalogue and imperfect information on the maximum magnitude. Suppose that equation (10) has already been applied to adjust for individual observed magnitude uncertainties. The frequency-magnitude relation is assumed to be (negative) exponential between the threshold of completeness m_0 and the unknown maximum magnitude $m_{\max}$. Let m_x denote the largest magnitude in the catalogue. Then we find that the conditional density of magnitudes exceeding m_0 is

$$f(m \mid \beta, \mu, \sigma) = \begin{cases} c\exp[-\beta(m - m_0)] & m \le m_X \\ c\exp[-\beta(m - m_0)]\dfrac{1 - \Phi\left(\dfrac{m-\mu}{\sigma}\right)}{1 - \Phi\left(\dfrac{m_x-\mu}{\sigma}\right)} & m > m_X \end{cases} \tag{11}$$

Integrating over (m_0, ∞), the constant c can be shown to satisfy

$$\frac{1}{c} = \frac{1}{\beta}\left[1 - \frac{\Phi\left(\dfrac{\mu - m_x - \beta\sigma^2}{\sigma}\right)\exp\left[\tfrac{1}{2}\sigma^2\beta^2 - (\mu - m_0)\beta\right]}{1 - \Phi\left(\dfrac{m_x - \mu}{\sigma}\right)}\right]. \tag{12}$$

The log likelihood of the earthquake catalogue is

$$\log L = \sum_{i=1}^{n} \log f(m_i \mid \beta, \mu, \sigma) \tag{13}$$

which can be optimised numerically to estimate β and hence c. The effect of uncertainty in $m_{\max}$ on the magnitude distribution so obtained is illustrated in Figure 4, in which the density of equation (11) has been estimated from a simulated catalogue of 100 earthquake magnitudes exceeding $m_0 = 5.0$ with μ=7.5 and a range of values of σ.

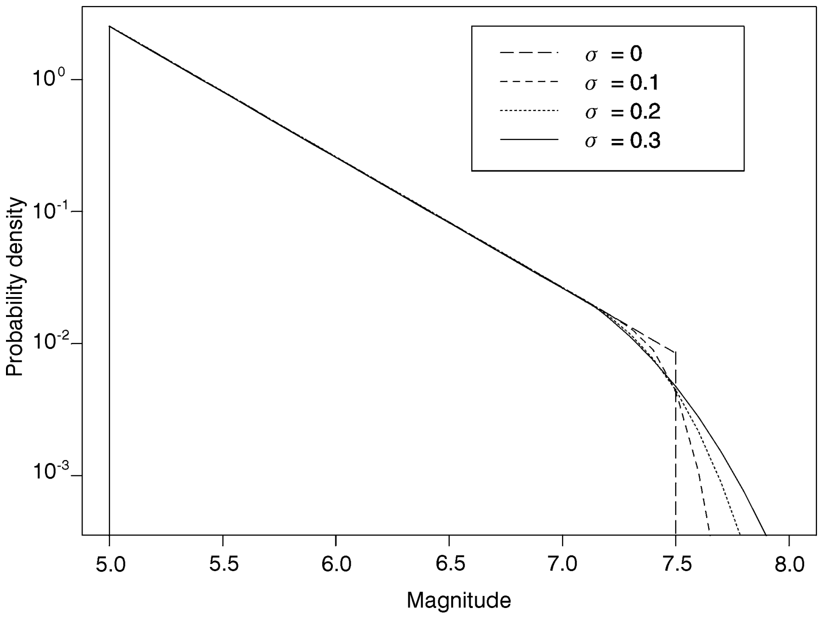

Figure 4. Effect of uncertainty in maximum magnitude $m_{\max}$ on estimation of frequency-magnitude relation. Probability density fitted to 100 earthquakes of magnitude 5.0 and above, with $m_{\max} \sim N(\mu, \sigma^2)$, where μ=7.5 and σ=0, 0.1, 0.2 and 0.3.

MAGNITUDE UNCERTAINTIES IN ATTENUATION RELATIONS

Estimation of attenuation relations for strong-motion data requires careful treatment of uncertainties because of the structure of the data. Strong-motion data sets typically consist of a large number of observations generated by a much smaller number of earthquakes. The between-earthquake and within-earthquake components of variation have to be treated separately. The partitioning of the error variance into the two components is accomplished in the random effects regression model (e.g. Abrahamson and Youngs, 1992).

Earthquake magnitude uncertainty is one factor that contributes to the apparent random earthquake effect, and hence to the between-earthquake component of variance. If uncertain magnitudes are treated as exact in the random effects model, then the between-earthquake component of variance is inflated by the magnitude uncertainty. Rhoades (1997) introduced explicit allowance for magnitude uncertainties into the random effects attenuation model. He proposed the following model:

$$y_{ij} = \alpha + \beta M_i + f(r_{ij}, \boldsymbol{\theta}) + \eta_i + \varepsilon_{ij} \tag{14}$$

for $i = 1, \cdots k;\ j = 1, \cdots, n_i$, where the y_{ij} are observations of some strong motion parameter, the M_i are uncertain earthquake magnitudes, the r_{ij} are distances of the observations from the earthquake source, and α, β and the vector **θ** represent unknown parameters. The between-earthquake variations η_i and within-earthquake variations ε_{ij} are assumed to be independently and normally distributed with zero mean and unknown variances τ^2 and σ^2, respectively. The M_i are assumed to to be normally distributed with known means m_i and known variances s_i^2. Equation (14) can then be recast as

$$y_{ij} = \alpha + \beta m_i + f(r_{ij}, \boldsymbol{\theta}) + \xi_i + \varepsilon_{ij} \tag{15}$$

where now $\xi_i \sim N(0, \beta s_i^2 + \tau^2)$. This model can be fitted by an extension to the procedure proposed by Abrahamson and Youngs (1992) for the random effects model. Rhoades (1996) showed that for the Joyner and Boore (1981) peak horizontal acceleration attenuation data, 57% of the random effects component could be explained by magnitude uncertainties alone, and in particular by the large uncertainty associated with using local magnitude M_L as a surrogate for moment magnitude M_w.

The model (15) has been applied by Dowrick and Rhoades (1999) to estimating attenuation relations for Modified Mercalli intensities in New Zealand earthquakes. In that study, the modelling of magnitude uncertainty allowed earthquake magnitudes of four different types with widely varying uncertainties to be included in the study, without fear of contaminating the between-earthquake component of variance.

The removal of the magnitude uncertainty from the random effects, as is accomplished by model (15), improves estimation of the between-earthquake component of variance and facilitates further modelling to explain this component of variance by fitting other physically meaningful variables such as tectonic setting and focal mechanism.

EFFECT ON HAZARD ESTIMATES

Modelling of magnitude uncertainties will not necessarily make a big difference to the assessed hazard in every case. However, the effects are not always trivial either. It does offer both quantitative and qualitative improvements in earthquake hazard estimation and sometimes the effects may be substantial. For example, using the New Zealand catalogue of local magnitudes and associated standard errors for the period 1987-1992, Rhoades (1996) showed that allowing for magnitude uncertainties gave consistently higher b-value estimates than the standard maximum likelihood method. The increases were as high as 0.068 for some subsets, which amounts to a 40% reduction in the estimated rate of occurrence of earthquakes of magnitude 7 and above, when the lower magnitude threshold is m_0 =4.0. This reduction is increased to 60% if the bias in *a*-value determination is also allowed for.

In the case of maximum magnitude uncertainties, it is clear from Figure 4 that the uncertainty on an assumed maximum magnitude has the potential to markedly affect estimates of the rate of occurrence of earthquakes at the high end of the magnitude scale. Since it is the large earthquakes that are the most important from a hazard point of view, realistic estimation of maximum magnitude uncertainty is a matter of great importance in most seismic hazard assessments.

The attenuation uncertainty often makes a significant contribution to the overall uncertainty in seismic hazard studies. Allowing for magnitude uncertainties here, by reducing reducing the between-earthquake component of variance, can be expected to bring about a moderate reduction in the overall attenuation uncertainty.

CONCLUSION

The proper handling of uncertainties of all kinds is now an essential part of best practice in probabilistic seismic hazard analysis. Uncertainty in magnitude determination is one of the contributing factors that needs to be considered. The methods that are now available for dealing with this factor, in frequency-magnitude relations, maximum magnitudes and attenuation relations, eliminate potential and actual biases and are not difficult to implement. They should become a standard part of practical seismic hazard assessment.

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