INFLUENCE OF THE STEEL PROPERTIES ON THE DUCTILITY OF R.C. STRUCTURES

Antonio GRIMALDI[1] And Zila RINALDI[2]

SUMMARY

This work aims at highlighting the steel properties influence on the behaviour of reinforced concrete structures, by means of a simplified beam model loaded by bending moment and axial force. The relationship between average strength and mean deformation is derived. Furthermore the phenomenon of strain localization in the steel is pointed out.

New kinds of steels, now widespread in the whole Europe, and mainly the ones made with the Tempcore process, are characterised by a low ratio between the ultimate and yield strengths, by a reduced ultimate strain, and by yield strength values higher than the nominal ones. The use of these steels can increase the local strength, but can reduce the local ductility, and the lack of plastic deformation spread in the reinforcement can produce a global brittle behaviour of the structure. The interest this problem has arisen is witnessed by the recent literature available on this subject, and by the experimental studies, still in development, aiming at the definition of the minimum ductility values to require for the reinforcing steels, with special reference to r.c. structure in seismic zones.

INTRODUCTION

The quoted references show the scientific interest about the influence of the steel properties on the global behaviour of r.c. structures, which has been caused, in these last years, by the need to state the minimum requirement on the ductility characteristics of rebars. The hardening ratio is defined as the ratio between the ultimate (f^u_s) and yield (f^y_s) steel strengths, the ultimate strain (ε^u_s) is a conventional mean strain at the maximum testing load. These parameters define the ductility properties of the steel.

In particular a new process, named Tempcore, allows to obtain reinforced bars characterised by high strengths (grade 500), with low carbon levels, and then weldable, but with reduced hardening ratio and ultimate uniform strain values. This kind of production has been accepted by the more recent European codes [EC2 1993, EC8 1994, ENV 10080 1995] in which a steel degree named B500B has been introduced, with yield strength equal to 500 N/mm^2. For structures in seismic zone these codes require the use of high ductility steels corresponding to a minimum hardening ratio of 1.2 and a minimum ultimate strain ε^u_s = 9%

In this paper the steel properties influence on the ultimate behaviour of beam elements and simple framed structures is pointed out. The strength and ductility characteristics of reinforced concrete members are strictly related to the cracking phenomenon and to the materials behaviour. In fact, when in a cracked section the steel reaches the yield strength, further load increments lead to an increase of the steel deformation as higher as lower is the hardening ratio. Near the cracked zones localization of deformation in the rebars can occur, and then the ultimate deformation and the failure of the bar can be quickly achieved. In order to evaluate the rotation capacity of r.c. beams, many models, even much sophisticated, are available in literature and, for the great number of parameters involved in the problem, quite often a numerical solution is used. In this paper a model is proposed that allows to obtain an approximate, but simple evaluation of the mean curvature and of the plastic rotation. The

[1] *Department of Civil Engineering, University of Rome "Tor Vergata", Rome, Italy. Email: Grimaldi@ing.uniroma2.it*

[2] *Department of Civil Engineering, University of Rome "Tor Vergata", Rome, Italy. Email: Rinaldi@ing.uniroma2.it*

model is applied to simple statically determined and undetermined structures, aiming at analysing the influence of the steel properties on the structural behaviour.

BEAM MODEL BEHAVIOUR

The evaluation of the ductility capacity has been firstly developed with reference to a beam element, with a length (l) equal to the cracks distance, subjected to tensile load. A similar analysis has been performed for a beam model under bending and axial forces. The mean strain applied at the element is the loading parameter and the behaviour is analysed up to the failure.

After the initial elastic behaviour (Fig. 3 - Phase 1), cracking in the concrete and slip occur with a stress and strain redistribution (Fig. 3 - Phase 2). The mean value of the axial strain or the mean curvature is considered as the deformation parameter. By increasing the applied strain, the steel yield is reached in the cracked middle section firstly, then the plastic deformations spread along the element (Fig 3 - Phase 3) up to the ultimate strain in the steel or in the concrete. Simple material constitutive relations are used in the analysis, as given in Fig. 1 in order to point out the parameters which influence the behaviour of the model.

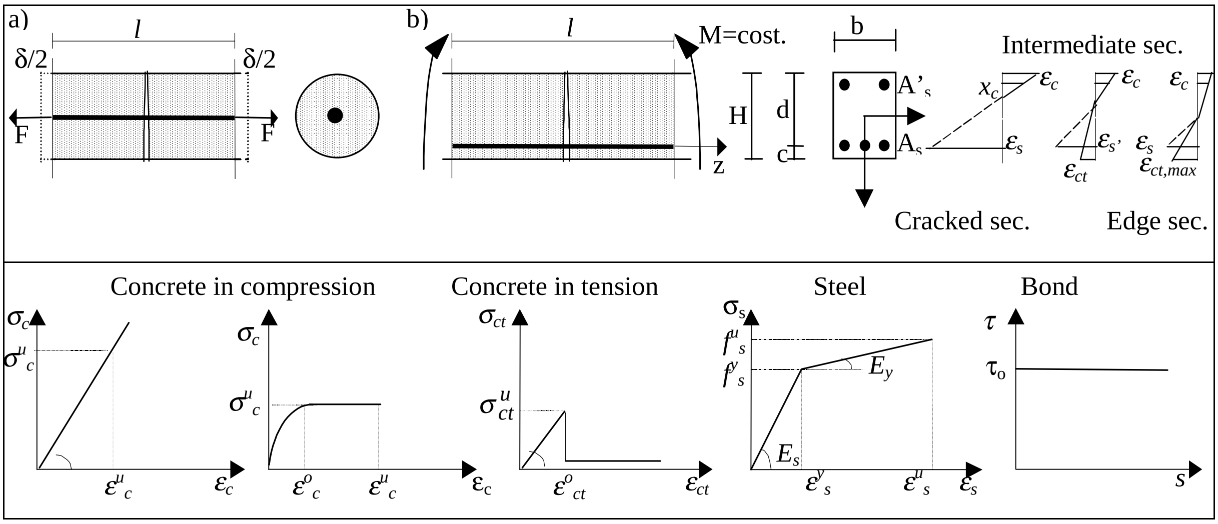


Fig. 1 - Beam model and constitutive relations

The tensile behaviour of the element (Fig 1 – a) is analysed by increasing the applied mean deformation $\varepsilon_m=\delta/l$, and by taking into account the slip between the concrete and the steel. This model has been already studied both theoretically than experimentally [Marti et al. 1998]. At the first crack, in the middle section the tensile strength in concrete vanishes and a slip between the two material occurs. In this phase the model assumes a constant value of the bond stress along the bar, equal to τ_o. The equilibrium condition gives the linear stress distribution in the steel and in the concrete.

After the stabilization of the cracks formation, the bond strength and the cracks distance are related to the tensile concrete ultimate strength (σ^u_{ct} in Fig. 1) reached in the edge section of the element. This condition corresponds to the hypothesis of maximum cracks spacing. As well known a minimum cracks distance can be evaluated according to the fracture energy criteria [Bazant et al., 1983]. It is pointed out that in the model equation the product τl appears as a parameter depending only on the ultimate tensile concrete strength.

The model allows to highlight the localization of the steel deformation near the cracks, during the yielding phase. Such phenomenon leads to a considerable increase of the strain in the rebar in the cracked zone, as higher than the average one, as lower is the hardening ratio of the steel. At the rebar yielding, firstly reached in the cracked section, small increments of the applied deformation can lead to the failure of the steel bar due to the reaching of its ultimate strain. The phenomenon is particularly relevant when the steel behaviour is quite an elastic-perfectly plastic one, i.e when $(f^u_s/f^y_s) \approx 1$. In this case when the yielding is achieved, the steel strain in the cracked zone rises instantaneously up to the ultimate one. The relation between the local strain increment in the cracked zone ($\Delta\varepsilon_{s,1}$) and the average one ($\Delta\varepsilon_m$), at the steel yielding (ε^y_s) is [Rinaldi, 1998]:

$$\left(\frac{\Delta\varepsilon_{s,1}}{\Delta\varepsilon_m}\right)_{\varepsilon_s^y} = \frac{E_s}{E_y} \quad [1]$$

The steel properties also affect the ultimate behaviour of the analysed r.c. element. The evaluation of the ultimate plastic strain, in fact, can be expressed by [Rinaldi, 1998]:

$$\varepsilon^u{}_{m,p} = \frac{\phi}{2\tau l}\left(f_s^u - f_s^y\right)\left(\varepsilon_s^u - \frac{f_s^u}{E_s}\right) \quad [2]$$

being ∅ the bar diameter. The equation points out the influence of the steel characteristics on the plastic ultimate deformation and furthermore the localization phenomenon is confirmed. In particular, due to the lack of plasticization spread along the element, the plastic deformation vanishes when the steel behaviour is elastic perfectly plastic.

The analysis developed for the element loaded by increasing the tensile deformation, has been then applied to a beam member loaded by constant bending and axial actions. For the evaluation of the stress and strain distribution inside the element, and for then the definition of the local and mean curvatures, a beam element with rectangular cross-section and a length l equal to the cracks distance, is considered. The classical hypothesis of plane sections is used only for the middle cracked section, where the tensile concrete stress vanishes; the assumption of perfect bond between steel and concrete is removed, and the slip between the two materials is taken into account. In all the cross-sections of the element the concrete strain pattern is assumed as a bilinear one, (Fig.1), as also proposed in [Gambarova et al., 1998], and as shown by experimental works, [Giuriani et al., 1979] and numerical analyses [Plauk et al., 1981, Ngo et al., 1967]. Also in this case the cracks are supposed to be diffused at constant spacing. The ultimate concrete strength has assumed at the edge of the element. The constitutive relations for the materials are given in Fig. 1.

The element behaviour is analysed increasing the mean curvature up to the failure, defined as the achieving of the limit strain in the steel or in the concrete. The aim is the formulation of a simple relationship between the bending moment and the mean curvature. At the concrete cracking a slip along the whole element occurs and according to the bond rigid-plastic behaviour the bond stress distribution is constant and the steel one is linear from the middle to the edge section of the element. The neutral axis (x_c) and concrete stress patterns, instead, are not linear with z along the element. The first step of the procedure is the solution of the equilibrium equations in the cracked section, where the tensile concrete stress is assumed to vanish. The parameter τl can be derived in a simple way by the equilibrium condition of the rebar, at the first crack formation. The value of this parameter is assumed to be constant, increasing the mean deformation, and this allows to solve the equilibrium equations of the edge section. This analysis is then repeated in all the intermediate sections and the stress and strain distributions in steel and concrete are obtained. After the yielding, the steel stress is always linear along the rebar, while the strain pattern is bilinear along the element according to the steel constitutive model.

Of particular interest is the analysis of the intermediate section where the rebar strain is equal to the yield one. The distance (m) between this section and the middle cracked one defines the length of the yielded steel, and is related to the deformation level and to the steel and concrete properties, according to the relation:

$$\frac{m}{l} = \frac{\phi}{4\tau l}E_y\left(\varepsilon_s^1 - \varepsilon_s^y\right) \qquad \frac{m}{l} = \frac{\phi}{4\tau l}\frac{f_s^u - f_s^y}{\varepsilon_s^u - \varepsilon_s^y}\left(\varepsilon_s^1 - \varepsilon_s^y\right) \quad [3]$$

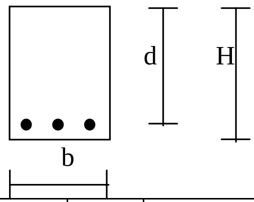
In this expression the parameter τl/∅ is related to the ultimate concrete tensile strength. The steel ultimate strain affects the plastic zone length only when the collapse occurs with concrete failure. On the contrary, the steel deformation in the cracked section $\varepsilon^1{}_s$ is equal to the ultimate steel strain $\varepsilon^u{}_s$.

The procedure can be summarised as follows:

1) Analysis of the cracked middle section (Known: ε_c; ε_{ct}): evaluation of the strain distribution (ε_s; x_c) by solving equilibrium equations;
2) Evaluation of the τl/∅ parameter by equilibrium condition along the steel bar;
3) Analysis of the edge cross section (known ε_s; $\varepsilon_{ct} = \varepsilon^o{}_{ct}$): - evaluation of the strain distribution (ε_c; x_c) by solving equilibrium equations;
4) Analysis of the yielded cross section (known $\varepsilon_s = \varepsilon^y{}_s$; x_c): - evaluation of strain distribution (ε_c; ε_{ct}) by solving equilibrium equations.

The local curvature is defined, according to the European Model Code 90 [CEB-FIP, 1991] as the ratio between the sum of the deformation in the tensile steel and compressed concrete and the effective depth (d). The mean curvature has been evaluated assuming the curvature $\rho(z)$ as a linear function from the middle cracked section to the intermediate yielded one and still linear to this section to the edge one.

An example of moment-mean curvature (ρ_m) relationship is plotted in the Fig. 3. The cross section geometry and the materials parameters are given in the Fig. 2; different values of steel hardening ratios but constant ultimate steel stress ad strain are considered.



b	H	σ_c^u	ε^u_s	σ^u_s	E_s
[cm]	[cm]	[N/mm^2]		[N/mm^2]	[N/mm^2]

Fig. 2 – Geometrical and materials characteristics

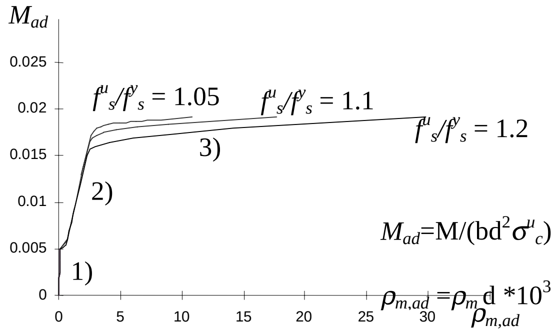


Fig. 3 - Moment – mean curvature

It is worth noting the relevant influence of the hardening ratio on the element behaviour; the reduction of this parameter from 1.2 to 1.05, leads to small increments of the bending moment, but produces a large ductility reduction.

Finally, we observe that the mean curvature ρ_m can be evaluated with reference to the steel strain according to the approximate relation:

$$\rho_m = \int_o^{l/2} \frac{\varepsilon_s(z)}{d - x_c(z)} dz \, . \qquad [4]$$

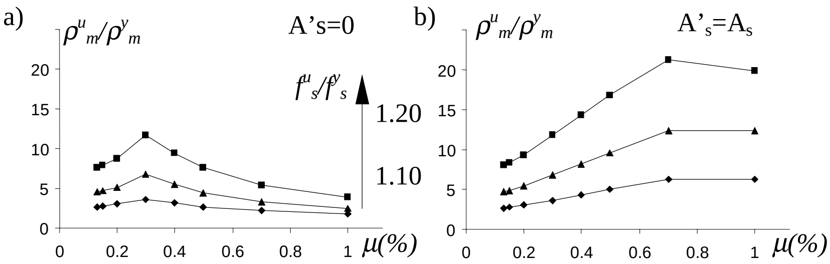
When the element failure corresponds to the ultimate strain in steel bar, then the mean ultimate curvature can be simply expressed as:

$$\rho_m^u = \frac{1}{d-\bar{x}} \left\{ \varepsilon_s^y \left(\frac{f_s^u}{f_s^y} - 1 \right) \left[\frac{\phi E_s}{4\tau l} \left(\varepsilon_s^u - \varepsilon_s^y \right) \left(1 - \frac{E_y}{E_s} \right) + 1 \right] + \varepsilon_s^y - \frac{\tau l}{\phi E_s} \right\} \qquad [5]$$

where ($\bar{x}$) is a mean value of the neutral axis depth inside the element The difference between this simplified formulation and the more exact procedure is practically negligible.

STRUCTURAL DUCTILITY EVALUATION

The proposed formulation has been applied first to a single beam element under constant bending moment, then to simple models subjected to bending moment with linear distribution. Finally the frame models in Fig. 10 have been examined. The results concerning the beam element under constant bending moment, given in Fig. 4 and Fig. 5, are discussed. The hardening ratio (varying from 1.05 to 1.20), the tensile steel percentage μ (from 0.13% to 1%) and the steel amount in compression, expressed as a percentage of the tensile one, are the model parameters. In the next pictures the ratio between the ultimate and the yield mean curvatures (curvature ratio ρ^u_m/ρ^y_m) is plotted versus the steel tensile percentage for two values of the compression steel; the curvature ratio is then represented versus the steel ultimate strain, in the case of rebar failure, for two values of the hardening ratio. The different behaviour of the element is well highlighted in Fig. 5. A sharp tip ($\mu_{cr} \approx 0.3\%$) separates the range corresponding to the steel failure ($\mu < \mu_{cr}$), to the one related to the concrete collapse ($\mu > \mu_{cr}$). These results agree with the experimental and theoretical ones [Eligehausen et al., 1987]. The influence of the hardening ratio is relevant particularly in the case of steel failure, when a variation of f^u_s/f^y_s from 1.05 to 1.2 gives a very large curvature ratio decrease.



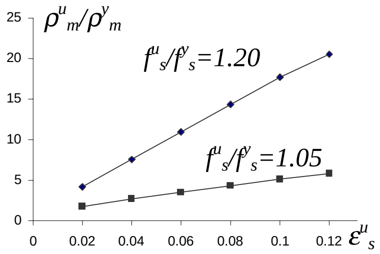


Fig. 4 Mean curvature versus steel percentage

Fig. 5 – Mean curvature versus ultimate steel strain

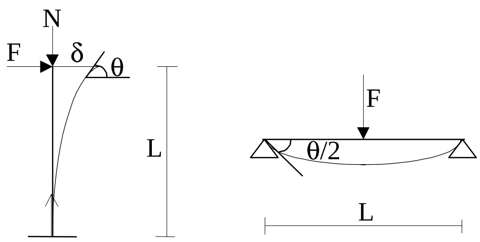
In the case of compression steel bars the structural behaviour is modified (Fig 5.b), and the hardening ratio effects are relevant also for tensile steel percentage of about 1%. In Fig. 5 the influence of the ultimate deformation on the curvature ratio is shown. The relation between ρ^u_m/ρ^y_m and ε^u_s is almost linear, and the ultimate steel deformation effects are particularly evident for steels with higher hardening ratio. A ultimate steel strain variation from 2% to 12% gives a ductility increase of about 200% for steel hardening ratio f^u_s/f^y_s=1.05, and of about 400% if f^u_s/f^y_s=1.20.

The procedure, then, has been applied to a beam element subjected to combined bending moment and axial force and the obtained results have been compared with the available experimental and numerical data, with satisfying agreements. The model formulation and the related examples are given in [Rinaldi, 1998].

Next the models in Fig. 6 have been considered, i.e. a simply supported beam subjected to linear bending moment, and a cantilever beam subjected to bending and compression. The plastic rotation of the end section (θ_{pl}) has been assumed as a plastic deformation parameter. The previous results obtained for the beam-element are now used for the plastic rotation calculation. The θ_{pl}, parameter has been defined as:

$$\theta_{pl} = \frac{L}{Mu} \int_{My}^{Mu} [\rho(M) - \rho_y] dM$$

and evaluated according to the bending moment-mean curvature relationship, approximated with a piecewise linear relation defined by the cracking, first yielding and ultimate bending moments, as given in Fig. 7.



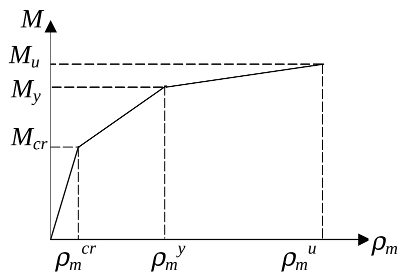


Fig. 6 – Beams and column models

Fig. 7 – Simplified moment –mean curvature relation

Numerical examples and parametric studies have been developed for a simply supported beam with a length of 6.00 m and a rectangular 30x60 cm cross section. The steel hardening ratio, and the bars percentage are the parameters, while the ultimate steel strength is constant. The concrete behaviour is given Fig. 1. Some of the obtained results are given in Fig .8 and Fig. 9, where the plastic rotation versus the steel percentage is plotted, for three hardening ratio and for two values of reinforcement in compression.

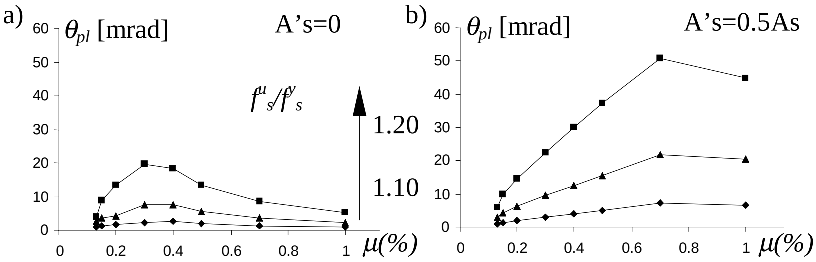


Fig. 8 – Plastic rotation versus steel percentage

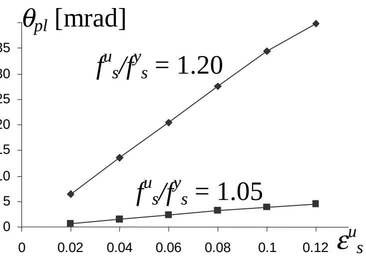
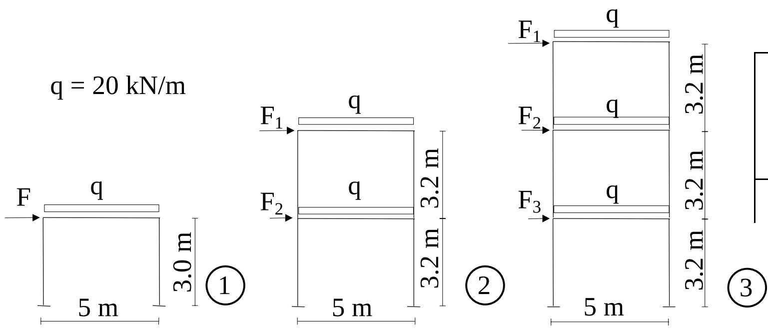


Fig. 9 – Plastic rotation versus ultimate steel strain

The obtained patterns are similar to those already discussed for the single beam-element. Once again in the absence of compression steel bars, the ranges related to the failure of the two materials are well evident. The influence of the steel properties is relevant when the collapse is due to the steel, and the plastic rotation can be reduced of a factor ten when the ratio f^u_s/f^y_s varies from 1.20 to 1.05. The rotation capacity reduction is not relevant only for steel percentage higher than 1%. When the compression reinforcement is considered the ductility is mainly related to the hardening parameters and quite independent from the tensile steel percentage. The influence of the ultimate steel strain on the plastic rotation, in the case of the steel failure, has been finally analysed, with reference to the hardening ratio f^u_s/f^y_s equal to 1.05 and 1.20.

In Fig. 9 the results related to μ = 0.4% are plotted. When f^u_s/f^y_s = 1.05 the plastic rotation is reduced of a factor three when the ultimate strain varies from 12% to 2%. For the same range of ε^u_s variation, the ductility parameter is reduced of a factor five when the hardening ratio is equal to 1.20.

Finally some simple framed schemes have been analysed. The behaviour of these structures is more complex and governed by the plastic hinges number and location, and by the rotation capacity they are able to exploit. The present design criteria aim at obtaining structures able to grant a ductile ultimate behaviour, and to dissipate the energy in a plastic field. The adoption of steel with low values of hardening ratio and ultimate strain could lead to a brittle collapse due to an untimely failure of a single section. In order to study this aspect a first parametric analysis has been performed for the schemes in Fig. 10. The considered steel (named Feb44 e B500 Tempcore) properties are shown in the same figure. The frames are subjected to a constant uniform vertical load and to horizontal forces, with a linear distribution, increasing up to the collapse of the structure. The model correspond to the case of framed structures under seismic static forces. The schemes have been analysed with a numerical analysis according to a finite element discretization of the beams and the columns. The previous given beam model has been applied to the single element, assumed to be loaded by constant bending moment and axial force.



	Feb44	B500 Tempcore
f^y_s [Mpa]	440	525

Fig. 10 – Frame models

The results obtained for the frames characterised by equal beams and columns sections (30 cm x 50 cm) symmetrically reinforced with As = A’s = 6 cm^2 are summarised in Fig. 11. For each scheme and for the two analysed steel properties the base shear (V) is plotted versus the top displacement δ.

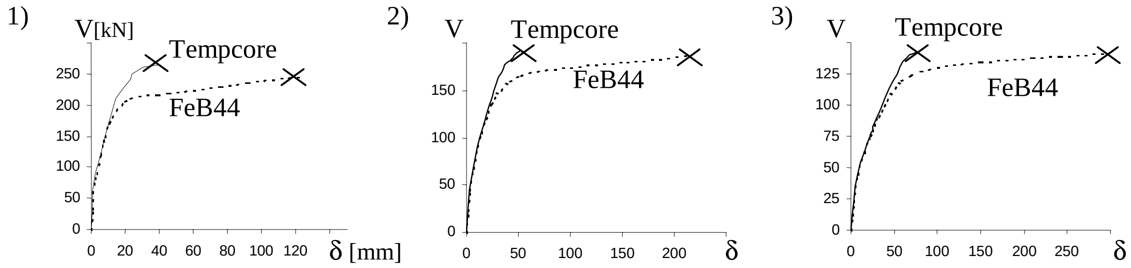


Fig. 11 – Base shear force versus top displacement

The single-storey frame (Fig. 10) reinforced with Tempcore bars, compared with to the same scheme with FeB44 steel, shows a large ultimate displacement reduction, of about 1/3, but a similar ultimate strength. When the number of storeys increases, the ultimate load for the two frames is almost equal, but the ultimate displacement is reduced of about ¼ for the B500 reinforced schemes. In these cases the untimely collapse of one section (at the base of the first column), does not allow the formation of an adequate number of yielded zones with sufficient rotational capacity, and therefore a ultimate failure with a global collapse mechanism is not attained.

The obtained results have been interpreted according to a traditional limit analysis, by introducing plastic hinges in which the yield effects are concentrated. Since the sixties the problem of the plastic hinge length has been studied leading to simplified formulations [Baker, Amarakone 1964, Sawyer 1964, Corley 1966, Mattock 1967, Riva, Cohn 1991, Cosenza et al. 1993, Manfredi, Pecce 1997]. The proposed model allows the evaluation of an equivalent plastic hinge length (l_p), based on the material properties, and on the bending moment distribution. In particular, when the moment variation is linear along the beam the length (l_p) can be expressed as:

$$\frac{l_p}{z} = \frac{\phi}{4\tau l}\left(1 - \frac{M_y}{M_u}\right)\left(\sigma_u^s - f_s^y\right)$$

where z is the distance between the sections with zero and maximum bending moment, f^y_s the yield steel stress and σ^u_s the steel stress related to the ultimate curvature (equal to f^u_s in the case of steel failure). The τl/∅ parameter, related to the ultimate tensile stress in the concrete, to the section geometry and to the amount of axial forces, can be easily evaluated by means of simple expressions [Grimaldi et al. 1997, Rinaldi, 1998]. The obtained results have been compared with the formulation developed by [Riva e Cohn, 1994], showing a good agreement. [Como et al., 1999]

The ductility differences in the examined frame schemes, can be explained on the base of the plastic yield length. In the case of frames reinforced with more ductile steels the failure mechanism is characterised by plastic hinge length comparable with the section depth. When using rebars with low values of hardening ratio, a brittle collapse occurs due to the failure of one section. In this case the plastic hinge lengths are smaller, with values equal to 1/4 - 1/5 than the one corresponding to more ductile steels (Feb 44).

CONCLUSIONS

The developed analysis and the numerical studies have pointed out and confirmed the relation between the ultimate behaviour of r.c. structural elements and the steel mechanical properties.

The basic model is a beam element loaded by axial force and constant bending moment, with length corresponding to the distance between stabilised cracks.

The model allows to point out that at yielding a relevant strain increment in the steel bar can occur, near the crack, and this strain concentration is high dependent on the hardening steel modulus. In particular, in the limit case of elastic-perfectly plastic behaviour of the steel, at yielding, strain localization occurs causing rebar crisis and a global brittle collapse.

The steel properties strongly affect the element behaviour, also increasing the applied deformation up to the final failure of the structure.

The influence of the hardening ratio and the ultimate steel strain on the structural ductility has been emphasised by numerical studies on single elements.

Similar analyses have been developed for simple beams and framed models. The obtained results show mainly the influence of the hardening ratio on the rotation capacity or on the global ductility of the structures, and therefore the necessity, particularly in the case of seismic actions, of requiring suitable minimum values of the steel ductility parameters.

REFERENCES

1. Baker A.L.L, Amarakone A. M. L. (1964), "Inelastic Hyperstatic Frames Analysis", *Proc. of the International Symposium on the Flexural Mechanics of Reinforced Concrete*. ASCE-ACI, Miami, pp.85-142.
2. Bazant Z. P., Oh B. H. (1983) "Spacing of cracks in Reinforced Concrete" *Journal of Structural Engineering*, ASCE, Vol.109, No. 9.
3. Beeby A.W. (1997), "Ductility in reinforced concrete: why is it needed and how is it achieved?" *The Structural Engineering* Vol. 75/No 18, pp.311-318.
4. CEB Bulletin d'Information n° 242 (1998), "Ductility of Reinforced Concrete Structures".
5. CEB FIP (1991), "Model Code 1990", Bulletin d'Infornation n° 203
6. Commission of the European Community (1993) "Eurocode 2. Common unified rules for concrete structures".
7. Commission of the European Community (1994) "Eurocode 8. Design provision for earthquake resistance of structures". CEN/TC250/SC8, ENV 1998-1-1/2/3, Lisbon.
8. Como M., Grimaldi A., Rinaldi Z. (1999), "Duttilità e resistenza ultima di telai in c.a.: influenza delle caratteristiche degli acciai.", *Proc. VIII Convegno Nazionale ANIDIS "L'Ingegneria Sismica in Italia*", Torino, Italy.
9. Corley W.G. (1966), "Rotational Capacity of Reinforced Concrete Beams", *Journal of Structural Division, ASCE*, Vol. 92 ST5, pp.121-146.
10. Eligehausen R., Langer P. (1987), "Rotation capacity of plastic hinges and allowable moment redistribution", CEB Bulletin d'Information No.175.
11. ENV 10080 (1995) "Steel for reinforced of concrete-Weldable ribbed reinforcing steel B500 – Technical delivery conditions for bars coils and welded fabric". UNI.
12. Franchi A. Riva P. Ronca P. (1997), "Meccanismi di rottura di armature al piede di pilastri in c.a. soggetti a carichi ciclici", *Atti delle Giornate AICAP 1997*, Roma
13. Gambarova P.G., Iori I., Vallini P. (1998) "Correlazione tra curvatura media e curvatura locale in elementi monodimensionali in conglomerato armato". *Relazione conclusiva gruppo di studio CNR "Rapporto tra curvatura media e locale in elementi in c.a."*.
14. Giuriani E., Ronca P. (1979), "Il metodo di moirè per trasparenza per lo studio di travi inflesse in cemento armato", *Proc. VII Convegno Nazionale A.I.A.S.* pp. 6.55-6.68.
15. Grimaldi A., Rinaldi Z. (1997), "Influenza delle caratteristiche degli acciai sulla duttilità di elementi inflessi in c.a.", *Proc. VII Convegno Nazionale ANIDIS "L'Ingegneria Sismica in Italia*", Taormina,.
16. Macchi G., Pinto P.E., Sanpaolesi L. (1996), "Ductility requirementes for reinforcement under Eurocodes" *IABSE* No.4.
17. Manfredi G, Pecce M. (1997), "Influenza delle proprietà dell'acciaio sulla duttilità di colonne in c.a." *Proc. VII Convegno Nazionale ANIDIS "L'Ingegneria Sismica in Italia*", Taormina, Italy.
18. Marti P., Alvarez M., Kaufmann W., Sigrist V. (1998), "Tension chord model for structural concrete" *Structural Engineering International* 4/98; pp. 287-198 IABSE.
19. Mattock A.H. (1967), Discussion of "Rotational Capacity of Reinforced Concrete Beams" , by W.G. Corley. *Journal of Structural Division, ASCE*, Vol. 93, ST2, April 1967, pp.399-492.
20. Ngo D., Scordelis A. C. (1967), "Finite element analysis of reinforced concrete beams", *ACI Journal*. Vol. 64(3). Pp. 152-163.
21. Plauk G., Hees G. (1981), "Finite element analysis of reinforced concrete beams with special regard to bond behaviour", *IABSE Colloquium on Advanced Mechanics of Reinforced Concrete*. Delft.
22. Rinaldi Z. (1998), "Duttilità e resistenza di elementi in c.a.: influenza della localizzazione delle deformazione nell'acciaio" PhD Thesis, University of Rome "Tor Vergata".
23. Riva P., Cohn M.Z, (1990), "Engineering Approach to Nonlinear Analysis of Concrete Structures", *Journal of Structural Engineering*, Vol.116, N. 8.
24. Riva P., Cohn M.Z. (1991), "Plastic Rotation Capacity of structural concrete member" *Università degli studi di Brescia – Dipartimento di Ingegneria Civile, Rapporti tecnici.*

26. Sawyer H.A. (1964), "Design of Concrete Frames for Two Failure States", *Proc. of the International Symposium on the Flexural Mechanics of Reinforced Concrete*. ASCE-ACI, Miami, pp.405-431.