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SEISMIC RESPONSE OF YIELDING SYSTEMS UNDER THREE COMPONENT EARTHQUAKES

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SUMMARY

This paper is aimed at analysing the interaction phenomena between vertical and horizontal forces (tri-axial interaction), which affect to a large degree strength and ductility demands in columns of framed structures, with a simple elastic-plastic model, including vertical earthquake component among the actions and taking into account *P-Δ* effects. It has been found that, in the presence of gravity loads, including the vertical component in the input ground motion does not lead to significant variations in ductility demands with respect to those evaluated considering an horizontal ground motion. Furthermore, the influence of *P-Δ* effects on inelastic response is generally not substantially amplified by the earthquake vertical component.

INTRODUCTION

It is well known that interaction phenomena between axial and lateral forces influence strength and ductility demand of a structural system. As an example, for columns of framed buildings, the variation in axial force results in modifying yielding conditions, which may lead to smaller bending capacity and, then, to a greater ductility demand. Moreover, for framed structures, interaction phenomena may work against the activation of the intended strong column - weak beam dissipation mechanism, since the reduction in column strength can modify the resistance hierarchy among structural members.

In this respect, it should be noticed that the evaluation of effects due to interaction phenomena require a proper definition of the actions to be applied, considering all three earthquake components; namely, response to earthquakes with a strong vertical component should be assessed since significant variations in axial forces in columns [Elnashai 1997, Elnashai and Papazoglou 1996] arise, which interact with bending components. In addition, a significant vertical earthquake component may influences *P-Δ* effects, which have been shown to lead to further increase in ductility demands and, eventually, to onset of dynamic instability [De Stefano and Rutenberg 1999]. However, most studies dealing with response of systems under vertical ground motion do not make allowance for *P-Δ* effects.

Until early '90s, following the well known studies by Newmark et al. (1973), the issue of effects of vertical ground acceleration was practically dismissed since peak vertical acceleration was considered definitely lower than its horizontal counterpart. The analysis of records from recent major earthquakes [Bozorgnia et al. 1995], instead, has shown that the peak vertical ground acceleration can be even higher than the horizontal ones at sites near to the epicentre (near field earthquakes). Therefore, in recent years, a growing interest in the evaluation of effects of earthquake vertical component has developed in the scientific community [Bozorgnia et al. 1998, Elnashai and Papazoglou 1997].

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This paper is aimed at analysing the interaction phenomena between vertical and horizontal forces (tri-axial interaction) for a simple elastic-plastic model, including vertical earthquake component among the actions and taking into account *P-Δ* effects. As shown elsewhere (Como et al.1999), despite the magnitude of peak vertical ground accelerations, plastic demands are slightly influenced by the vertical seismic component; however, the interaction between axial forces due to gravity loads only and lateral forces due to seismic excitation govern inelastic system response and cannot be neglected.
The study shows that the major effect of vertical excitation in the inelastic range of behaviour regards plastic residuals in the horizontal direction at the end of excitation which present significant amplifications. It is also demonstrated that *P-Δ* destabilizing forces are largely modified by seismic vertical accelerations; however, for structures designed to sustain moderate inelastic actions, variations in ductility demands and plastic residuals due to the seismic vertical component appear not substantial.

MODEL CHARACTERISTICS

The examined model is a simple three-degree-of-freedom oscillator characterised by fundamental periods T_x, T_y and T_z in the three principal directions *x*, *y* and *z* and by a damping coefficient ν=5% for the three vibration modes. This model is representative of a single ductile column and may be extended, under simplifying assumptions, to one-storey regular buildings. In order to take into account interaction among bending moments and axial force in the plastic field, an ellipsoidal elastic-plastic domain is introduced (Figure 1) to govern yielding behaviour. The two horizontal d.o.f.s are coupled with the vertical one even in the elastic range because of the geometrical stiffness matrix (Eqns.5).

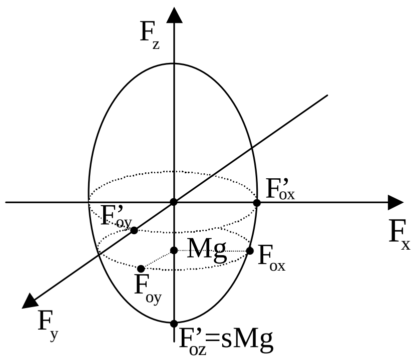


Figure 1: Plastic domain.

This plastic condition is expressed by:

$$\left(\frac{F_x}{F'_{ox}}\right)^2+\left(\frac{F_y}{F'_{oy}}\right)^2+\left(\frac{F_z}{F'_{oz}}\right)^2=1 \tag{1}$$

where F'_{ox}, F'_{oy} and F'_{oz} (Fig.1) represent ultimate strengths under each one of the horizontal and vertical forces F_x, F_y and F_z respectively. F'_{oz} is proportional to gravity loads by means of a safety coefficient *s*:

$$F'_{oz} = s\,M\,g \tag{2}$$

The design strength in the *x* horizontal direction has been obtained from the constant ductility inelastic spectrum S_μ of the seismic component acting in the same direction:

$$F_{ox} = M\,S_\mu \tag{3}$$

This is the lateral design strength corresponding to application of gravity load *Mg*; as a consequence, the x-direction system strength in absence of gravity loads ($F_z = 0$), F'_{ox}, can be deduced from the condition of laying contemporaneously on the plastic domain and on the *x* -direction axis:

$$F'_{ox}=\frac{F_{ox}}{\sqrt{1-1/s^2}} \tag{4}$$

In this manner, system capacity is defined according to the provisions of current major seismic codes: design strengths in the horizontal directions are obtained by reducing the elastic ones thanks to system ductility, while the system vertical strength is derived by amplifying gravity loads.
In the same way ultimate strength in the *y* direction, F'_{0y}, has been obtained.

EQUATIONS OF MOTION

The system equations of motions can be expressed as follows:

$$\begin{cases} M\ddot{x} + 2\nu\omega_x M\, \dot{x} + K_x\left(1-\theta_0\left(1+\dfrac{\ddot{z}}{g}\right)\right)x = -M\ddot{x}_{gr.} \\ M\ddot{y} + 2\nu\omega_y M\, \dot{y} + K_y\left(1-\theta_0\left(1+\dfrac{\ddot{z}}{g}\right)\right)y = -M\ddot{y}_{gr.} \\ M\ddot{z} + 2\nu\omega_z M\, \dot{z} + K_z z = Mg - M\ddot{z}_{gr.} \end{cases} \tag{5}$$

where x_{gr}, y_{gr} and z_{gr} are the ground motion acceleration components along the x, y and z directions, K_x, K_y and K_z are the system stiffnesses, ω_x, ω_y and ω_z are the corresponding system frequencies. The equations are expressed by means of the so-called stability coefficient θ_0 which, in the x-direction, is given by:

$$\theta_0 = \frac{Mg}{K_x H} \tag{6}$$

where H is the system height.
During excitation, the vertical acceleration $\ddot{z}$ modifies the acting axial force; therefore, it can be defined an 'equivalent' stability coefficient $\theta(t)$ given by:

$$\theta(t) = \frac{M(g+\ddot{z})}{K_x H} \tag{7}$$

Values of coefficient θ_0 must be carefully selected to represent actual sensitivity of building structures to P-Δ effects. A well-established approach is the one proposed by Bernal (1987) which expresses θ_0 as the ratio of limiting interstory drift Ψ to elastic seismic coefficient C:

$$\theta = \frac{\Psi}{C} \tag{8}$$

Following Eurocode 8 provisions, it has been assumed for Ψ its limit-value at the ultimate limit state supposing that secondary elements does not interfere with the structure, and for the seismic coefficient C values obtained from the elastic spectrum for soils type B with a peak ground acceleration $a_{gr} = 0.35\, g$.
The system equations of motion have been numerically integrated defining $\theta(t)$ at the time step i with the value of $\ddot{z}$ obtained at the time step i-1.

PARAMETRIC FIELD, DAMAGE INDICES AND SEISMIC INPUT

Analysis have been conducted by varying parameters characterizing elastic and inelastic behaviour of the system within ranges that are believed to cover most actual situations. Elastic periods in the two horizontal principal directions - T_x and T_y - have been assumed to be equal, in order to not introduce effects arising from different flexibility of the system in the horizontal directions, and to vary from 0.1 *sec* to 1.5 *sec*, to represent structures with medium and medium-low lateral stiffness. The vertical period T_z has been obtained with a simple formulation which relates it to the horizontal one; this relationship again takes into account the limitation imposed by Eurocode 8 to the inter-storey drift Ψ as shown elsewhere (Como et al. 1999). For steel structures, the proposed formulation satisfactorily correlate values reported by Elnashai e Papazoglou (1996) for real structures (Fig.2).

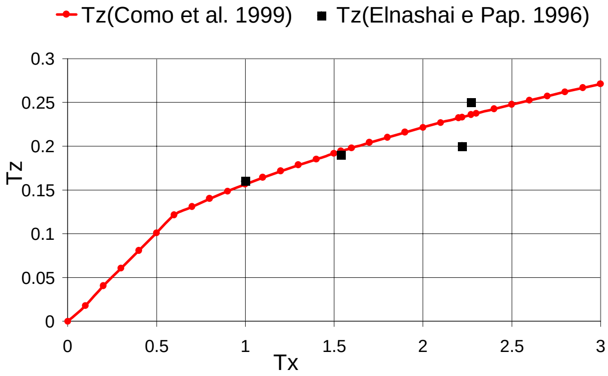


Figure 2: Comparison between the proposed relationship (Como et al. 1999) between T_x and T_z and values reported by Elnashai and Papazoglou (1996) for steel structures.

Inelastic system parameters define the interaction domain. Particularly, the horizontal system strengths are dependent on the design ductility μ used to evaluate the inelastic spectra S_μ of the horizontal earthquake components; the vertical system strength is defined by the safety coefficient against axial forces *s*. For the ductility μ values of 1, 2, 3 and 4 have been chosen in order to investigate structures designed to experience low and medium plastic excursions. For the safety coefficient *s*, it has been adopted *s=2*; this value is thought to be relevant to not very slender steel structures as the ones having lateral periods included between 0.1 *sec* and 1.5 sec. For more slender structures the onset of the instability phenomenon would impose to adopt higher values of safety coefficient *s*.

For what concerns the assessment of the plastic demand experienced by the system, it has to be observed that the available studies are far from exhausting the topic of plastic demand evaluation under multi-directional excitations (De Stefano and Faella 1996). It has been chosen to use kinematic parameters instead of hysteretic ones, since this study deals with ductile structures whose collapse is essentially caused by the attainment of a limit displacement (Cosenza et al. 1991). Particularly, the kinematic ductility demands μ_x, μ_y and μ_z along the three principal directions have been evaluated:

$$\mu_x = \frac{u_{x,\max}}{u_{xo}} \qquad \mu_y = \frac{u_{y,\max}}{u_{yo}} \qquad \mu_z = \frac{u_{z,\max}}{u_{zo}} \tag{9}$$

where $u_{x,max}$, $u_{y,max}$ and $u_{z,max}$ are the maximum displacements during excitation and u_{xo}, u_{yo} and u_{zo} are the yielding displacements along the three directions.

In a previous study (Como et al. 1999), in order to represent plastic demand caused by multi-directional excitations, it has been considered the radial ductility μ_{rad} that is a direct three-dimensional extension of the well-known concept of kinematic ductility for uni-directional action. Namely, μ_{rad} is the ratio of the maximum displacement modulus to the yielding displacement modulus in the same direction:

$$\mu_{rad} = \max\left|\frac{u_x(t)^2}{u_{xo}^2} + \frac{u_y(t)^2}{u_{yo}^2} + \frac{u_z(t)^2}{u_{zo}^2}\right|^{\frac{1}{2}} \tag{10}$$

It can be also defined a parameter of horizontal kinematic ductility μ_h obtained as the ratio of the maximum displacement horizontal projection modulus to the corresponding horizontal yielding displacement in the same direction:

$$\mu_h = \max\left|\frac{u_x(t)^2}{u_{xo}^2} + \frac{u_y(t)^2}{u_{yo}^2}\right|^{\frac{1}{2}} \tag{11}$$

This last parameter μ_h is directly comparable to the widely used kinematic ductility to evaluate plastic demand under uni-directional horizontal action.

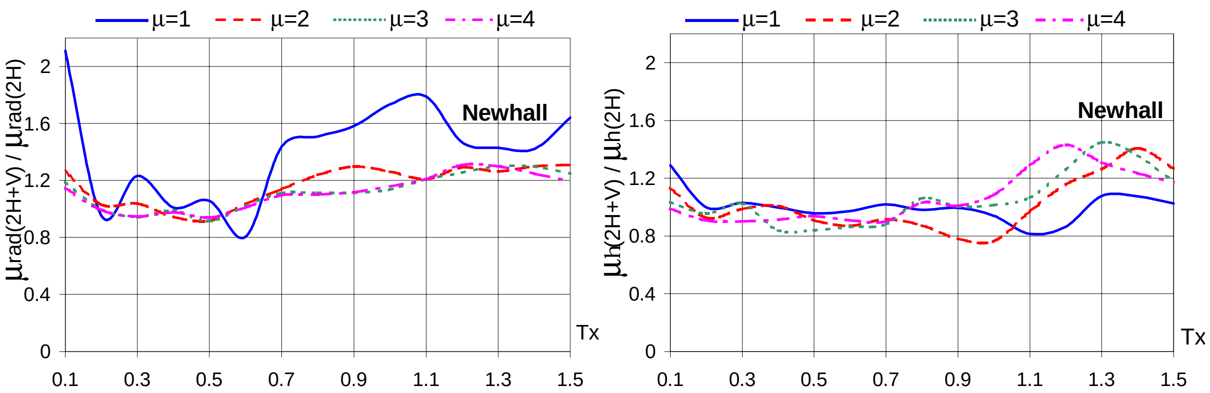
In this study records from the Northridge earthquake (17-1-1994) and from the Kobe earthquake (17-1-1995) have been used as input ground motion. In this way, the investigation covers system response under seismic excitations characterized by significant vertical accelerations, as shown in Table 1 where peak ground accelerations of horizontal (A_H) and vertical (A_V) components are reported.

Table 1: Seismic input.

Earthquake	Station	Components	PGA m/s^2	A_V /A_{Hmax}
Hyogoken-Nanbu (Kobe) 1995	JMA	NS EW UP	8.17 6.17 3.32	0.406
Northridge 1994	Newhall, L.A. County Fire Station	Ch.1 Ch.3 UP	5.78 5.72 5.37	0.930

ANALYSES

The analyses aim primarily to understand if the presence of the vertical seismic component in addition to the horizontal ones and to gravity loads has important effects on the structural response. In a previous paper (Como et al. 1999) it has been shown, with reference to a simple model not allowing for *P-Δ* effects, that the interaction phenomena induced by gravity loads and seismic horizontal components are dominant and are slightly modified by the vertical component. As an example, curves of Figures 3 and 4 report increase in the radial ductility demand and in the horizontal ductility demand due to the vertical component of the Newhall record obtained by comparing responses to three-component (*2H+V* loading condition) and two-component (*2H* loading condition) excitation, for different values of the design ductility μ.



Figures 3 and 4: Radial and horizontal ductility increments due to the vertical earthquake component

It can be seen from Figure 3 that only for structures designed to resist seismic actions in the elastic field (μ=1), vertical component leads to a significant increment in the plastic radial demands. Figure 4 shows increments in plastic demand along the horizontal direction which appear not very important, independently of the design ductility μ.

In Figure 5 the values of radial, horizontal and vertical ductility of systems designed imposing μ =4 and *s*=2 and subjected to bi-directional and three-directional seismic action are reported.

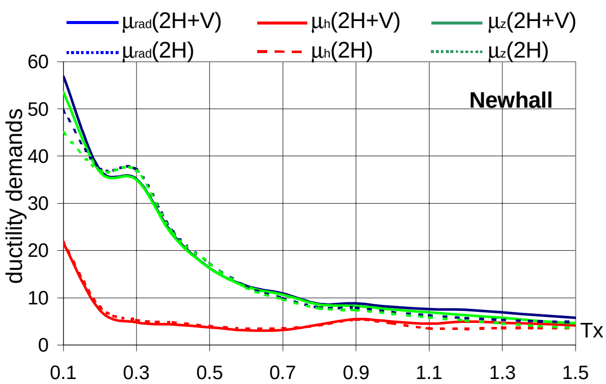
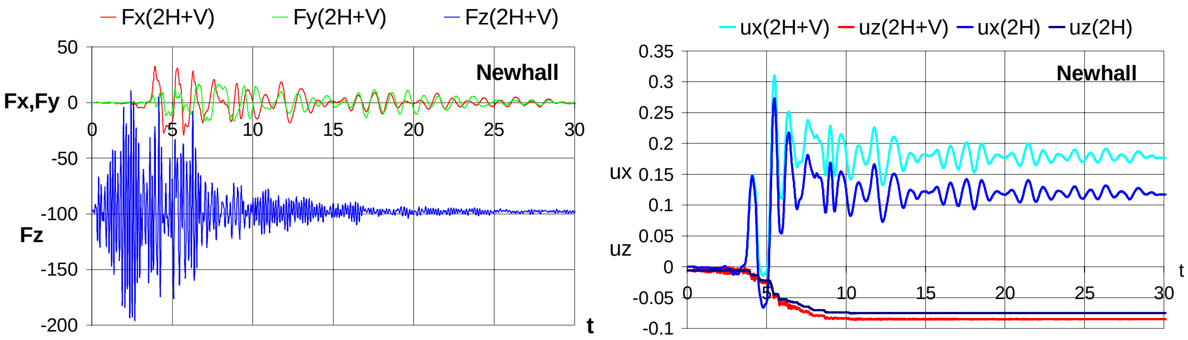


Figure 5: Ductility demand for systems under bi-directional and tri-directional excitations

It emerges that the largest plastic excursions develop along the vertical direction, even when the vertical component is not considered. As previously underlined, this shows that interaction phenomena between axial force due to gravity loads and lateral earthquake forces are quite remarkable since they govern peak inelastic excursions as it is demonstrated by the small differences in values of ductility demands (μ_{rad}, μ_h, μ_z) computed with and without the vertical component (*2H+V* vs *2H*). For a better understanding of previous results, in Figures 6 and 7 the force and displacement time-histories of a system presenting T_x=T_y= 1 *sec*, T_z = 0.16 *sec* and designed imposing μ =4 and *s*=2 are reported.



Figures 6 and 7: Displacement and force time-histories for systems under bi-directional and three-directional excitations

Figure 6 shows that frequency of the force vertical component F_z is quite higher than the horizontal ones due to the different stiffnesses of the system along *x*, *y* and *z* directions. By comparing F_z time-history to the horizontal displacement (u_x) time-histories reported in Figure 7, it can be seen that, during a single horizontal oscillation, several cycles of the force vertical component occur. It appears, then, that the effect of the variation in force vertical component on horizontal displacement tend to vanish primarily for what concern peak horizontal displacements (Figure 7), thus leading to almost coincident μ_h. However, after a large plastic excursion at about 5 sec, lateral displacement oscillate around values which are influenced by the *z* direction seismic excitation. Therefore, in the horizontal direction, plastic residuals at the end of the excitation are expected to amplify when vertical component is acting. Regarding vertical displacements, which induce the largest plastic excursions, they monotonically accumulate during the excitation, both under bi-directional and under three-directional excitation.
Previous results are obtained from response analysis of a simplified system not including geometrical non-linearity. When considering *P-Δ* effects, an important parameter is the above-defined stability coefficient $\theta(t)$, which varies during excitation both under bi-directional ground motion, due to vertical motion induced by triaxial interaction, and, of course, under three-directional ground motion. For a system having T_x=T_y=1 *sec* and T_z =0.16 *sec*, designed with *s*=2 and μ =4 and subjected to the Newhall record, the ratio $\Delta\theta = \theta(t)/\theta_0$ is reported in Figure 8 in the two cases of bi-directional and three-directional ground motion.

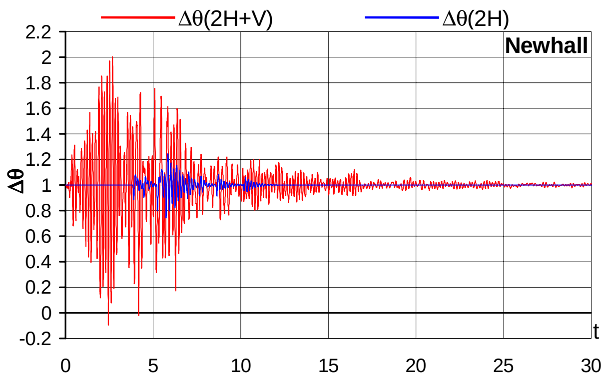
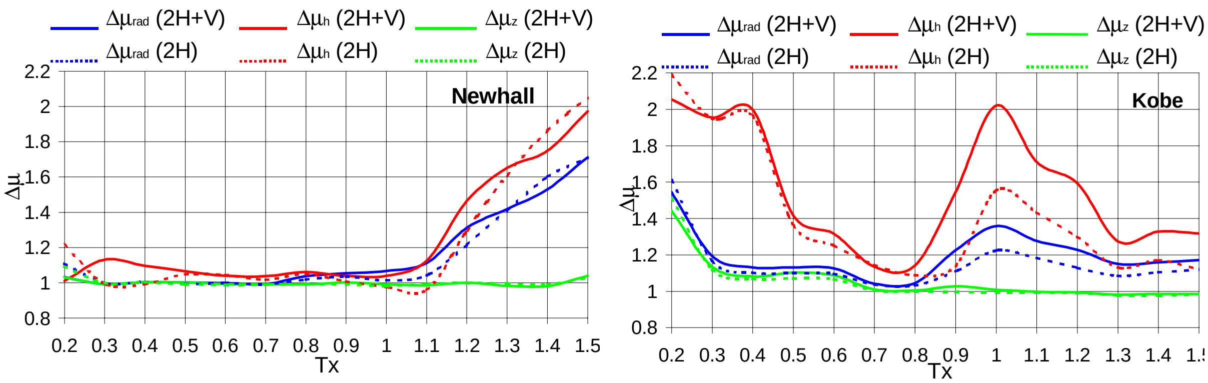


Figure 8. Increments in the stability coefficient θ for the two loading conditions.

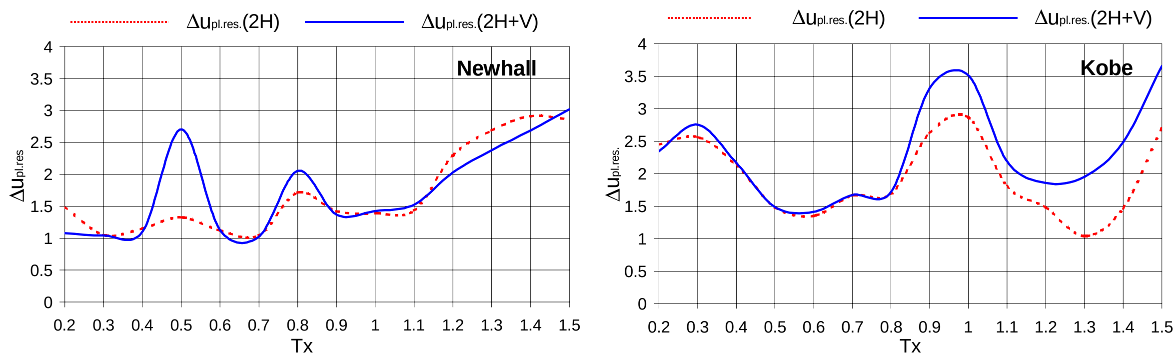
It can be seen that, in the presence of vertical component, large excursions in $\theta(t)$ with respect to θ_0 occur; namely, $\theta(t)$ amplifies and reduces up to 100%. Very small variations in $\theta(t)$ are induced by tri-axial interaction when bi-directional excitation is considered.
In order to isolate *P-Δ* effects under bi- and tri-directional excitations, the ratio $\Delta\mu$ of ductility demands evaluated including geometrical non-linearity to ductility demands evaluated without *P-Δ* effects has been

computed for all three ductility parameters μ_{rad}, μ_h and μ_z for systems subjected both to the Newhall and to the Kobe records (Figures 9 and 10). Curves show similar values and trends independently of the loading condition (*2H+V*, *2H*), demonstrating that the vertical excitation does not affect significantly amplifications in ductility demands due to *P-Δ* effects. This may be due to the reversals in sign of the quantity ($\theta(t)$- θ_0) which does not lead to significant variations in the average destabilizing lateral forces and in peak displacements.
The maximum amplifications in ductility demands due to *P-Δ* effects occur at different periods depending on the input ground motion: for the Newhall record, noticeable increments are obtained for periods larger than 1.1 *sec*; for the Kobe record the highest values of $\Delta\mu$ are found within period ranges from 0.2 *sec* to 0.6 *sec* and from 0.9 *sec* to 1.2 *sec*.



Figures 9 and 10. Ductility increments due to P-Δ effects for the Newhall and Kobe records

Figures 11 and 12 show the ratio $\Delta u_{pl.res.}$ of horizontal plastic residuals $u_{pl.res.}$ (moduli of the horizontal displacement vector having u_x and u_y components at the end of excitation) computed for systems with and without *P-Δ* under two and three component Newhall and Kobe excitations. It can be observed that, over a wide range of periods for the Kobe record, amplifications of plastic residuals due to *P-Δ* effects are affected substantially by the earthquake vertical component. However, this is not the case of the Newhall record for which no significant differences in plastic residuals due to vertical component have been found. Analysis of response under different earthquakes (not reported for brevity) has further shown that in most cases both ductility demands and plastic residuals due to P-Δ effects are not significantly affected by the earthquake vertical component.



Figures 11 and 12. Plastic residuals due to P-Δ effects for the Newhall and Kobe records

CONCLUSIONS

In this paper, the effects of earthquake vertical component have been dealt with considering a simplified three-degree-of-freedom system accounting both for mechanical and for geometrical non-linearity. It has been found that the inclusion of the vertical component in the input ground motion does not lead to significant variations in the kinematic ductility demands with respect to those evaluated considering the horizontal ground motion, because of the dominant effect of interaction phenomena between axial forces due to gravity loads only and lateral forces due to horizontal seismic components. However, plastic residuals at the end of excitation are seen to be amplified in presence of vertical excitation. For what concerns *P-Δ* effects, which, as it is well known,

affect to a large degree dynamic response of yielding systems, it has been found that generally their influence on ductility demands and plastic residuals is not substantially amplified by the earthquake vertical component.
Further research is needed in order to investigate on the hysteretic aspects of seismic response under multi-directional ground motions, which in turn involves definition of suitable damage parameters and analysis of degrading systems.

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