TORSIONAL RESPONSE OF RC BUILDINGS RETROFITTED WITH STEEL FRAMED BRACES

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SUMMARY

In a retrofit design, well-balanced placement of retrofit elements in a building is most essential to ensure sound seismic performance during earthquakes. For this purpose the Japanese Guidelines for Seismic Evaluation and Retrofit regulate that indices representing unbalanced distribution of laterally resisting members in plan and elevation of a structure be smaller than certain criteria. However, in the case of retrofitting an RC building with steel framed braces, their unbalanced distribution is often considered a minor problem due to their stiffness lower than RC walls.

To investigate the effects of unbalanced distribution of high-strength-but-low-stiffness members, torsional response analyses of RC building structures retrofitted with steel framed braces are carried out using simplified model structures. The results show that responses are highly dependent on the unbalanced distribution of lateral resistance of retrofit elements rather than that of their elastic stiffness. The authors also discuss on the relationship between the torsional responses of the model structure and indices representing the unbalanced distribution of laterally resisting members, and conclude that an index proposed in this paper can be a candidate to estimate the maximum torsional angle during seismic excitations.

INTRODUCTION

In retrofitting an existing RC building, the scheme to infill new RC walls into existing bare frames had been most conventionally applied in Japan since numerous practical experiences as well as experimental and analytical researches were extensively made on this technique. Although it has been one of the most reliable strategies to retrofit a seismically vulnerable RC building, *infilling* often causes less flexibility in architectural and environmental design and/or the increase in building weight sometimes leads to costly redesign of foundation. Steel framed braces, therefore, have been more widely applied recently in Japan, particularly after the 1995 Kobe Earthquake, to overcome shortcomings resulting from the conventional RC walls stated above.

In the retrofit design, well-balanced placement of retrofit elements in a building is most essential to ensure sound seismic performance during earthquakes. For this purpose, the Guidelines [JBDPA, 1990a and b] regulate that indices representing unbalanced distribution of laterally resisting members in plan and elevation of a structure be smaller than certain criteria. However, in the case of retrofitting an RC building with steel framed braces, their unbalanced distribution is often considered a minor problem mainly because (1) the indices representing unbalance distribution of laterally resisting members are calculated based on their elastic stiffness rather than their lateral resistance, (2) the elastic stiffness of a steel framed brace is much lower than an RC wall even if they

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are designed to have same lateral resistance, and (3) the indices based on the elastic stiffness of a steel framed brace are therefore often smaller than criteria in the Guidelines and the unbalanced distribution is neglected in the retrofit design. However, the unbalanced distribution of lateral resistance may cause unfavorable torsional response of a building retrofitted using high-strength-but-low-stiffness elements such as steel framed braces when it is subjected to a major earthquake and responds beyond the elastic range.

To investigate the effects of unbalanced distribution of high-strength-but-low-stiffness members, torsional response analyses of simplified model structures retrofitted with steel framed braces are carried out. In this paper, the relationship between torsional responses and unbalanced distribution of lateral resistance in plan will be mainly discussed.

BASIC ASSUMPTIONS

Model Structures

In the numerical investigation herein, an idealized single-story building model which represents a low-rise RC building is employed as an original bare frame structure. The bare frame structure is assumed to have 3 bays in X-direction and 2 bays in Y-direction, each span length of which is 4.5 m and 6.0 m, respectively. The model consists of a rigid rectangular floor slab supported on 12 lateral load resisting columns having a cross section of 60 x 60 cm. The mass is assumed to be uniformly distributed across the slab. Three sets of yield strength V_{yo} of the bare frame, i. e., 0.3W (30 % of the total building weight W), 0.4W and 0.5W are considered to simulate a typical RC building designed in accordance with dated seismic codes in Japan.

To investigate the effects of unbalanced distribution of stiffness and strength on the torsional response of retrofitted structures, which may be dependent on the location and the amount of retrofit elements, the following parameters as shown in **Table 1** are considered.

Table 1: Parameters for numerical analyses

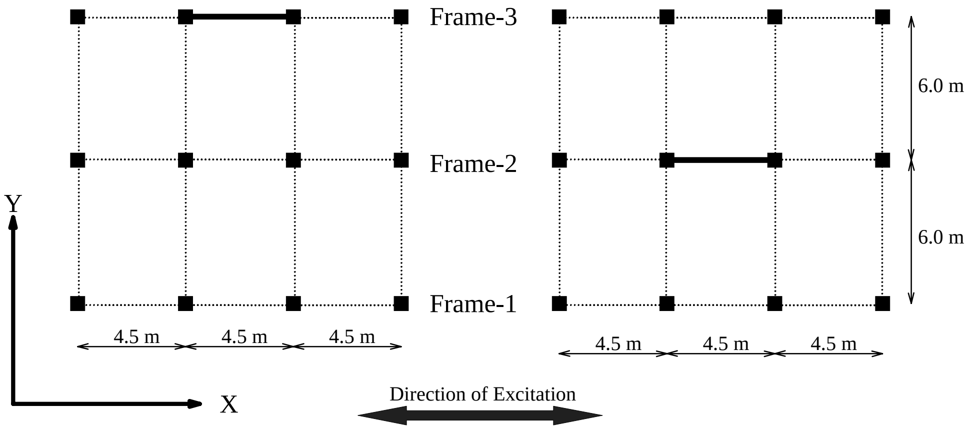
	V_{yo}											
	0.3W				0.4W				0.5W			
ΔVy	0.1W	0.2W	0.3W	0.4W	0.1W	0.2W	0.3W	0.4W	0.1W	0.2W	0.3W	0.4W
$\Delta Ke/Ke$ **	0.15 0.45	0.30 0.90	0.45 1.35	0.60 1.80	0.15 0.45	0.30 0.90	0.45 1.35	0.60 1.80	0.15 0.45	0.30 0.90	0.45 1.35	0.60 1.80
f_{eK}**	0.04 0.10	0.08 0.16	0.10 0.19	0.12 0.21	0.04 0.10	0.08 0.16	0.10 0.19	0.12 0.21	0.04 0.10	0.08 0.16	0.10 0.19	0.12 0.21
T_1**	0.46 0.44	0.45 0.43	0.44 0.43	0.43 0.42	0.46 0.44	0.45 0.43	0.44 0.43	0.43 0.42	0.46 0.44	0.45 0.43	0.44 0.43	0.43 0.42
T_2**	0.34 0.30	0.32 0.26	0.30 0.23	0.28 0.20	0.34 0.30	0.32 0.26	0.30 0.23	0.28 0.20	0.34 0.30	0.32 0.26	0.30 0.23	0.28 0.20

Note ΔVy : yield strength increment due to retrofit
$\Delta Ke/Ke$: (elastic stiffness increment due to retrofit) / (overall elastic stiffness of an original bare frame)
f_{eK} : $= e_K / \sqrt{B^2 + L^2}$, stiffness unbalance index defined in the Guideline [JBDPA, 1990a]
where e_K : eccentricity, i.e., distance between the center of mass and the center of stiffness
B, L : width and length of a building (see also **Eqs. (9)** and **(10)** defined later)
T_1, T_2 : natural period (sec.) for the first and second mode, respectively
** upper row : SFB lower row: RCW

(1) Retrofit schemes:
Even when a frame retrofitted with steel framed braces (referred to as SFB) is designed to have the lateral resistance equal to a frame retrofitted with post-installed RC walls (referred to as RCW), the stiffness of SFB is generally much lower than that of RCW. To investigate the effects of fundamental properties of retrofit elements, RCW which has high strength and *high stiffness* and SFB which has high strength but *low stiffness* are considered as retrofit schemes investigated herein.
(2) Location of retrofit element: To simulate the torsional response of a retrofitted structure, a monosymmetric and hence torsionally unbalanced (referred to as TU) building model, whose distribution of stiffness and strength

is assumed to be symmetric about the transverse Y-axis but asymmetric about the longitudinal X-axis as shown in **Figure 1(a)**, is employed. In addition, a fully symmetric and hence torsionally balanced (referred to as TB) model structure as shown in **Figure 1(b)** is investigated to compare with the performance of TU structural model.
(3) Strength increment due to retrofit: The lateral strength increment ΔVy due to retrofit is assumed to vary from $0.1W$ through $0.4W$ at an increment of $0.1W$, where W signifies the total building weight.

Considering Japanese retrofit design practices [JBDPA, 1900b] and assuming that the increase in elastic stiffness of RCW is 3 times of SFB, the elastic stiffness increment due to retrofit ΔKe is determined in the following manner. When the yield strength increment ΔVy due to retrofit is $0.1W$, ΔKe is 45% of overall stiffness of the bare frame structure for RCW, while 15% for SFB. For both retrofit elements, the stiffness increment ΔKe is assumed to be proportional to their strength increment ΔVy.



(a) Retrofitted in the exterior frame (Torsionally unbalanced (TU) structure) **(b) Retrofitted in the central frame (Torsionally balanced (TB) structure)**

Figure 1: Model structures

Numerical Solution for Torsional Response Analyses

Assuming an idealized single story structure and rigid floor system in both bare and retrofitted model structures described above, the fundamental equation of motion for numerical integration considering both translational and torsional responses can be expressed in **Eqs. (1)** through **(3)**. To simulate inelastic behaviors of model structures, the Takeda hysteretic model shown in **Figure 2** is employed for both columns and retrofit elements. The yield displacement is determined from the drift angle at yielding as shown in **Figure 2** and the equivalent building height, assuming that (1) the model structure represents a 4 storied building, (2) each story is 3.5 m high, and (3) the equivalent building height is 3/4 of overall building height.

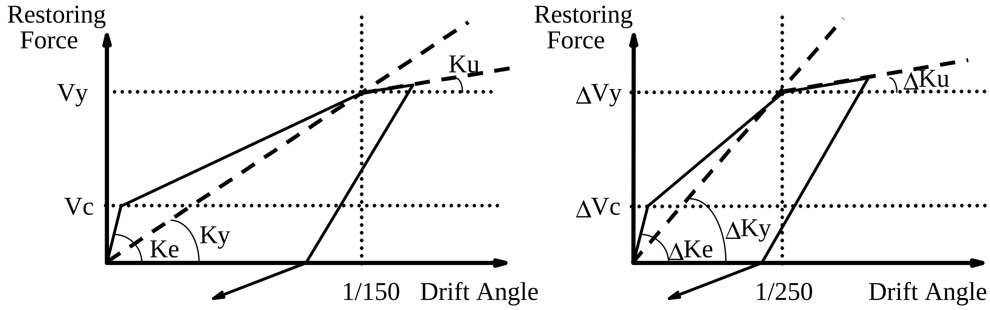
To simplify the subsequent discussions, a unidirectional earthquake ground motion is considered in the computation as shown in **Figure 1**, and the Hachinohe EW component recorded during 1968 Tokachi-oki Earthquake is used for $\ddot{x}_0$, scaling the peak ground acceleration to 0.4 g, while $\ddot{y}_0$ and $\ddot{\theta}_0$ is assumed 0.

$$m(\ddot{x}+\ddot{x}_0)+\sum_i {}_iC_x(\dot{x}+{}_il_y\dot{\theta})+\sum_i {}_iK_x(x+{}_il_y\theta)=0 \tag{1}$$

$$m(\ddot{y}+\ddot{y}_0)+\sum_i {}_iC_y(\dot{y}-{}_il_x\dot{\theta})+\sum_i {}_iK_y(y-{}_il_x\theta)=0 \tag{2}$$

$$I(\ddot{\theta}+\ddot{\theta}_0)+\sum_i {}_iC_x(\dot{x}+{}_il_y\dot{\theta})\cdot{}_il_y-\sum_i {}_iC_y(y-{}_il_x\theta)\cdot{}_il_x+\sum_i {}_iK_x(x+{}_il_y\theta)\cdot{}_il_y-\sum_i {}_iK_y(y-{}_il_x\theta)\cdot{}_il_x=0 \tag{3}$$

Where, m, I : mass and moment of inertia of model structure, respectively
x, y : response displacements at the center of mass (CM) in X- and Y-direction, respectively
θ : torsional response angle
iCx, iCy : damping coefficient
iKx, iKy : instantaneous stiffness of member i
ilx, ily : distance between member i and CM
xi, yi : response displacement of member i ($xi = x + ilx\,\theta$, $yi = y - ily\,\theta$)



(a) Column **(b) Retrofit element (SFB and RCW)**

Note: $V_y = 3\,V_c$ $\Delta V_y = 3\,\Delta V_c$

$K_e = 4\,K_y$ ΔK_e can be defined from the assumptions shown in **Table 1**

$K_u = K_e / 1000$ $\Delta K_u = \Delta K_e / 1000$

Drift angle at yielding is assumed 1/150 for columns and 1/250 for retrofit elements.

Figure 2: Hysteresis models employed in the numerical analyses

PERFORMANCE OF RETROFITTED STRUCTURES

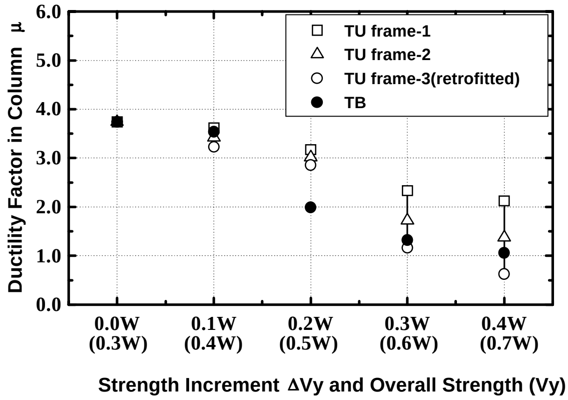
Effects of unbalanced distribution of stiffness and lateral resistance on torsional responses

Figures 3(a) and **(b)** show the relationship between column ductility factors μ and strength increment ΔV_y of structures having original lateral strength V_{yo} equal to $0.3W$. In the figures, μ is defined as the ratio of response displacement in each frame to yield displacement when the frame-1 reaches the maximum displacement. As can be seen from the figures, the ductility factors μ of both retrofit types of RCW (**Figure 3 (a)**) and SFB (**Figure 3 (b)**) having torsional unbalance generally decrease with increase in the lateral strength increment ΔV_y. The ductility factor of TU (torsionally unbalanced) structure is larger than that of TB (torsionally balanced) structure in the non-retrofitted frame-1 while generally smaller in the retrofitted frame-3. However, the torsional response increases and hence the discrepancy of ductility factors μ between frames-1 and -3 becomes more significant with increase in ΔV_y. It should be also noted that the discrepancy of ductility factors between frames-1 and -3, which corresponds to the torsional response, is approximately same for both retrofit types with identical ΔV_y.

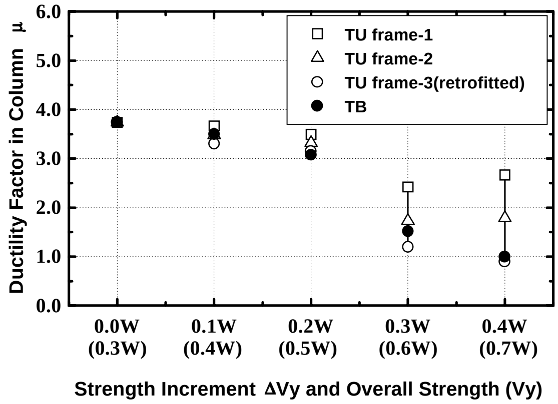
Figure 3(c) summarizes the relationship between the strength increment ΔV_y and maximum torsional angle θ_{max}. Although RCW is assumed to have the stiffness increment ΔK_e 3 times as much as SFB, the maximum torsional angle θ_{max} is almost identical for both retrofit types when they have identical ΔV_y. This figure clearly indicates that the strength increment rather than the elastic stiffness increment provided in the exterior frame-3 governs the torsional response of retrofitted buildings. This result demonstrates that the structural design should be more carefully done considering the unbalanced distribution of *lateral resistance* since the torsional response may not be neglected in the presence of unbalanced distribution of lateral resistance, even when a building is retrofitted with SFB and hence the unbalanced distribution of *stiffness* is insignificant. This is also suggesting that indices representing structural unbalance including inelastic range and their criteria to ensure sound performance during a major earthquake need to be developed considering lateral resistance unbalance.

Effects of yield strength of overall structure on torsional responses

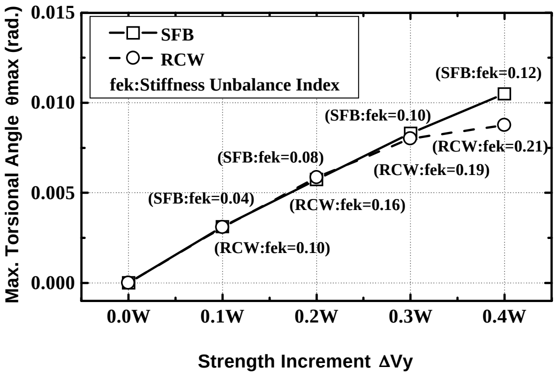
To investigate the effects of yield strength of overall structure after retrofit, torsional responses of structures having different strength are compared. **Figure 3(d)** shows the relationship between column ductility factors μ and strength increment ΔV_y of an SFB structure having $V_{yo} = 0.5W$. As can be found from **Figures 3(b)** and **(d)**, column ductility factors μ for a structure with $V_{yo} = 0.5W$ are generally smaller than those for a structure with $V_{yo} = 0.3W$. It should be noted, however, that the discrepancy of ductility factors between frames-1 and -3 is similar in both structures when they have same ΔV_y. This result implies that the torsional response is dependent on ΔV_y more significantly than V_y. **Figure 4** summarizes the relationship among the yield strength of overall structure V_y after retrofit, strength increment ΔV_y, and maximum torsional angle θ_{max} of SFB-TU structure. This figure also shows that θ_{max} is dependent on ΔV_y more significantly than V_y provided that all frames including retrofitted frame-3 respond beyond yielding to the input excitation.

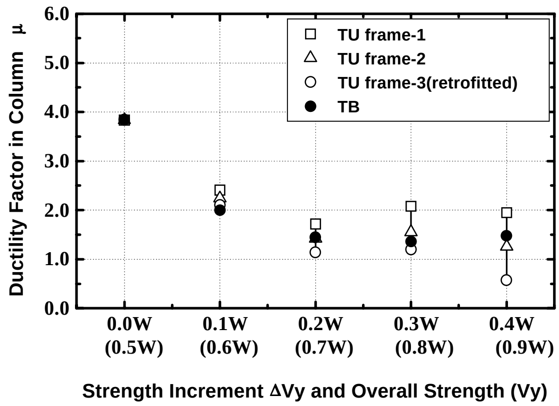


(a) RCW structures with V_{yo} = 0.3 W



(b) SFB structures with V_{yo} = 0.3 W





(c) ΔV_y vs. θ_{max} for structures with V_{yo} = 0.3 W **(d) SFB structures with V_{yo} = 0.5 W**

Figure 3: Relationship among strength increment ΔV_y, column ductility factor μ and maximum torsional angle θ_{max}

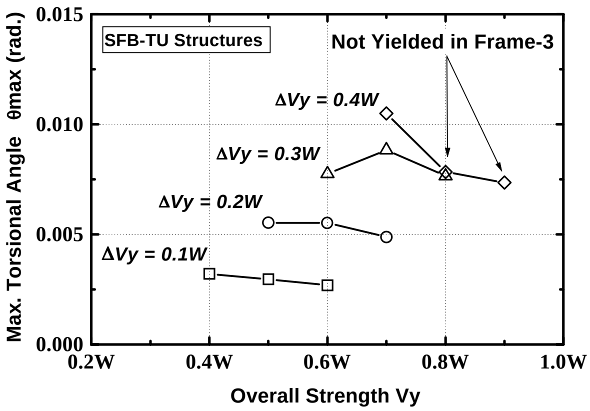


Figure 4: Relationship among V_y, ΔV_y, and θ_{max} of SFB-TU structures

Relationship between torsional response and torsional moment acting on the structure

To understand what affects torsional responses of the retrofitted structures most significantly, the relationship between torsional moment and torsional response angle θ is investigated subsequently. Neglecting damping forces (i.e., $C_x = C_y = 0$) and torsional component of input motion (i.e., $\ddot{\theta}_0 = 0$) to simplify the subsequent discussions, **Eq. (3)** can be rewritten as **Eq. (4)**. Considering the response shear forces in each frame and $y = 0$ for a monosymmetric structure subjected to unidirectional input motions in X-direction as shown in **Eqs. (5)** and **(6)**, **Eq. (3)** leads to **Eq. (7)**. **Eq. (7)** implies that the torsional response may be highly depending on the torsional moment ($\Sigma\, {}_iV_x\, {}_il_y$) acting on the structure.

$$I\ddot{\theta}+\sum_i {}_iK_x(x+{}_il_y\cdot\theta)\cdot{}_il_y-\sum_i {}_iK_y(y-{}_il_x\cdot\theta)\cdot{}_il_x=0 \tag{4}$$

$$\sum_i {}_iK_x(x+{}_il_y\cdot\theta)\cdot{}_il_y=\sum_i {}_iV_x\cdot{}_il_y \tag{5}$$

$$\sum_i {}_iK_y(y-{}_il_x\cdot\theta)\cdot{}_il_x=-\sum_i {}_iK_y\cdot{}_il_x^2\cdot\theta=-K_{\theta y}\cdot\theta \tag{6}$$

$$I\ddot{\theta}+K_{\theta y}\cdot\theta=-\sum_i {}_iV_x\cdot{}_il_y \tag{7}$$

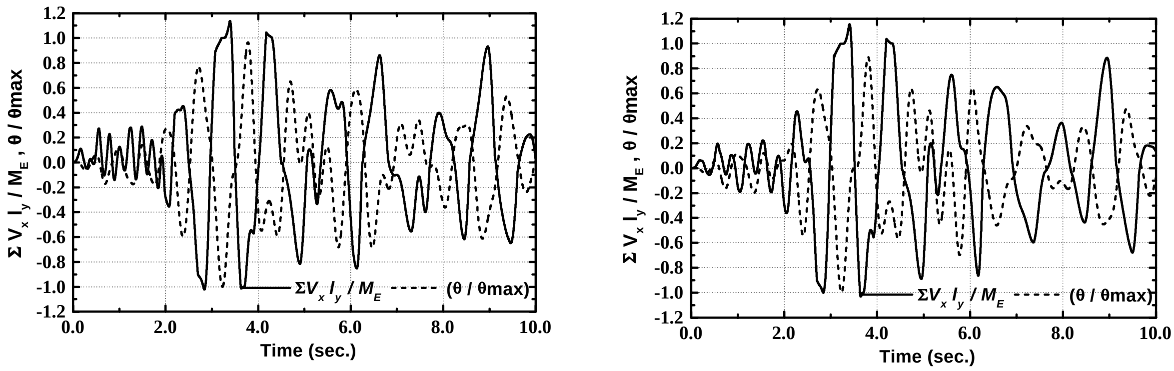
where ${}_iV_x$: response shear force of member i

Figure 5 shows the time history of the torsional moment ($\Sigma\ {}_iV_x\ {}_il_y$) and torsional response angle θ normalized by M_E and θ_{max}, respectively, for RCW-TU and SFB-TU structures having V_{yo} = 0.3W and ΔV_y = 0.3W. In the figure, M_E is defined as **Eq. (8)** assuming that each frame reaches the yielding strength during the excitations.

$$M_E=\sum_i {}_iV_{yx}\cdot{}_il_y \tag{8}$$

where ${}_iV_{yx}$: yield strength of member i

This figure shows that the ($\Sigma\ {}_iV_x\ {}_il_y$ / M_E) and (θ / θ_{max}) mutually correlated over the response duration for both RCW-TU and SFB-TU structures. The maximum values of ($\Sigma\ {}_iV_x\ {}_il_y$ / M_E) are approximately 1.0 for both structures because they reaches the yielding strength in each frame at the same time. This result implies that the maximum value of torsional moment $(\Sigma\ {}_iV_x\ {}_il_y)_{max}$ can be approximated by M_E defined in **Eq. (8)**, providing that each frame of a structure reaches the yielding strength simultaneously.



(a) RCW-TU structure **(b) SFB-TU structure**

Figure 5: Time history of ($\Sigma\ {}_iV_x\ {}_il_y$ / M_E) and (θ / θ_{max}) for TU structures with V_{yo}=0.3W and ΔV_y=0.3W

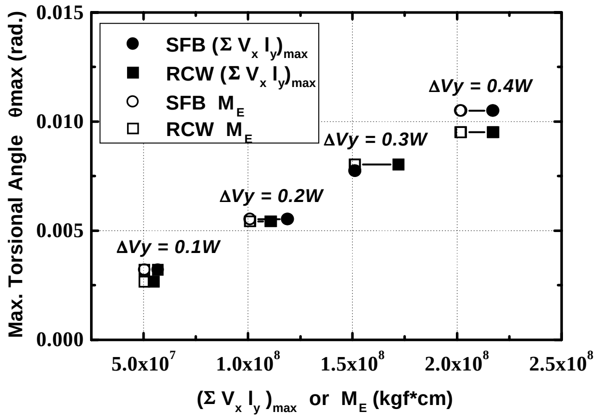


Figure 6: Relationship among $(\Sigma\ {}_iV_x\ {}_il_y)_{max}$, M_E, and θ_{max} for TU structures with V_{yo} = 0.3W

Figure 6 summarizes the relationship among the maximum value of torsional moment acting on the structure (Σ ${}_iV_x\,{}_il_y)_{max}$, M_E, and maximum torsional angle θ_{max} for structures having V_{yo} = 0.3W. As can be seen from the figure, θ_{max} is roughly proportional to (Σ ${}_iV_x\,{}_il_y)_{max}$ and (Σ ${}_iV_x\,{}_il_y)_{max}$ can be approximated by M_E.

ESTIMATION OF TORSIONAL RESPONSET BY ECCENTRICITY INDICES

To obtain a better index to estimate the torsional responses of TU structures, the correlation of maximum torsional angle θ_{max} and the following three different indices, f_{eK}, f_{eV}', and f_{eV} are investigated.

As stated earlier, an index to represent the structural unbalance of laterally resisting members is generally based on their elastic stiffness in the conventional structural design procedures. **Eq. (9)** shows an example index f_{eK} based on the elastic stiffness [JBDPA, 1990a]. **Figure 7(a)** shows the relationship between f_{eK} and θ_{max} for structures investigated in this study. As can be easily understood from the previous discussions, f_{eK} does not correlate well with θ_{max}.

f_{eV}' in **Eq. (11)** is an index to incorporate the effects of unbalanced distribution of lateral resistance, where e_K in **Eq. (9)** is simply replaced by e_V in **Eq. (12)** to define an index with analogous form to **Eq. (9)**. **Figure 7(b)** shows the relationship between f_{eV}' and θ_{max}. Although the correlation is better than the results in **Figure 7(a)**, different f_{eV}' indices give similar θ_{max} and f_{eV}' is still unsatisfactory index to estimate θ_{max}. Bearing in mind that θ_{max} is dependent on M_E as shown in **Figure 6** but independent of V_y as shown in **Figure 4**, and that e_V in **Eq. (12)** can be rewritten as (M_E / V_y), one might easily understand that f_{eV}', which is a function of M_E and V_y, may not be the best index to estimate θ_{max}.

Considering the above and results obtained from the numerical simulations as discussed in section 3.3, i.e., "*(a) θ_{max} is roughly proportional to (Σ ${}_iV_x\,{}_il_y)_{max}$,* and *(b) (Σ ${}_iV_x\,{}_il_y)_{max}$ can be approximated by M_E defined in Eq. (8), if the structure responds beyond yielding in all frames*," a new index f_{eV} is proposed as shown in **Eqs. (13)** and **(14)**. Based on the result (a) described above, f_{eV} is assumed to be a linear function of (Σ ${}_iV_x\,{}_il_y)_{max}$. Considering the second result (b) and Σ ${}_iV_{yx}$ = C_B W, f_{eV} can be expressed by **Eq. (13)**. Setting α in **Eq. (13)** equal to $1/(\sqrt{a^2+b^2}\;W)$ to obtain an analogous form with **Eq. (9)**, f_{eV} can be rewritten as **Eq. (14)**.

$$f_{eK} = e_K \Big/ \sqrt{a^2+b^2} \tag{9}$$

$$e_K = \sum_i {}_iK_x \cdot {}_il_y \Big/ \sum_i {}_iK_x \tag{10}$$

$$f_{eV}' = e_V \Big/ \sqrt{a^2+b^2} \tag{11}$$

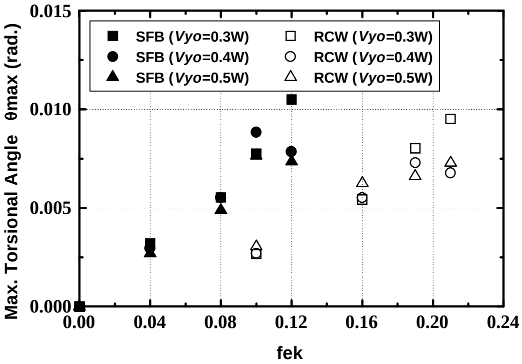
$$e_V = \sum_i {}_iV_{yx} \cdot {}_il_y \Big/ \sum_i {}_iV_{yx} \left(= M_E / V_y\right) \tag{12}$$

$$\begin{aligned} f_{eV} &= \alpha \cdot (\sum_i {}_iV_x \cdot {}_il_y)_{\max} \approx \alpha \cdot \sum_i {}_iV_{yx} \cdot {}_il_y \\ &= \alpha \cdot \left(\sum_i {}_iV_{yx} \cdot {}_il_y \Big/ \sum_i {}_iV_{yx} \right) \cdot \sum_i {}_iV_{yx} = \alpha \cdot W \cdot e_V \cdot \sum_i {}_iC_{yx} \\ &= \alpha \cdot e_V \cdot C_B \cdot W \end{aligned} \tag{13}$$

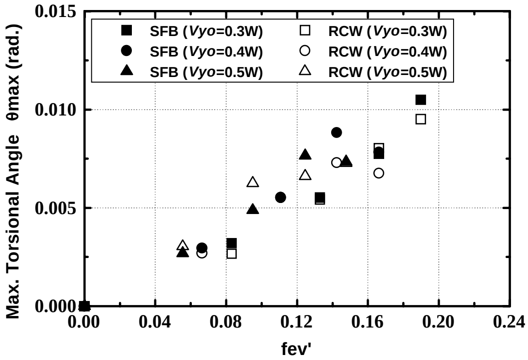
$$f_{eV} = \left(e_V \Big/ \sqrt{a^2+b^2} \right) \cdot C_B \tag{14}$$

where e_K, e_V : eccentricity based on the ***stiffness*** and ***strength***, respectively
a, b : building length and width
${}_iV_{yx}$, V_y : yield strength of member i and overall structure (= Σ ${}_iV_{yx}$), respectively
${}_iC_{yx}$, C_B: shear capacity coefficient of member i (= ${}_iV_{yx}$ / W) and base shear coefficient (= Σ ${}_iC_{yx}$ = V_y / W)

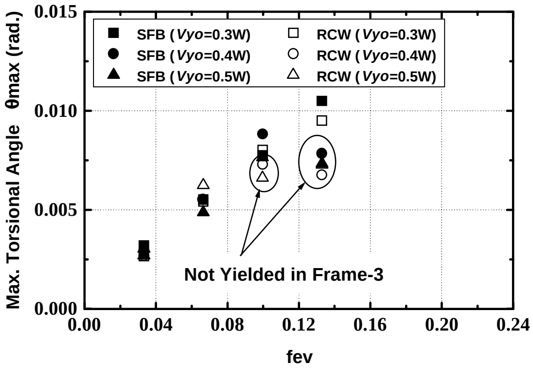
Figure 7(c) shows the relationship between f_{eV} and θ_{max}. As can be found in the figure, f_{eV} correlates well with θ_{max} except for several cases where the retrofitted frame-3 does not yield. The reason for the above exceptions is due that these cases do not meet the second result (b) described above and hence M_E overestimates (Σ ${}_iV_x$ ${}_il_y)_{max}$, resulting in the overestimation of f_{eV}. It can be concluded, however, that the proposed index f_{eV} can be a candidate to estimate θ_{max}, provided that all frames in a structure respond beyond elastic range due to torsional responses under seismic excitations.



(a) f_{eK} - θ_{max} relationship



(b) f_{eV}' - θ_{max} relationship



(c) f_{eV} - θ_{max} relationship

Figure 7: Relationship among different indices representing structural unbalance and θ_{max}

CONCLUDING REMARKS

To investigate the effects of unbalanced distribution of high-strength-but-low-stiffness members, torsional response analyses of RC building structures retrofitted with steel framed braces were carried out using simplified model structures, and their responses were compared with those retrofitted with RC walls. Although the investigated cases are limited, major findings obtained in this study can be summarized as follows.

(1) With increase in the strength increment ΔV_y of retrofit elements provided in frame-3, maximum response displacements of TU structures generally decreased. Because of torsional responses, however, the discrepancy of ductility factors between frames-1 and -3 became more significant.

(2) The major factor which affected torsional responses of TU structures was unbalanced distribution of lateral resistance rather than that of elastic stiffness. This result suggested that indices representing structural unbalance including inelastic range and their criteria to ensure sound performance during a major earthquake needed to be developed considering lateral resistance unbalance.

(3) Structural unbalance index f_{eV} based on lateral resistance proposed in this paper could be a candidate to give a satisfactory estimation of maximum torsional angle θ_{max} during a major earthquake, provided that all frames yielded during excitation.

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