IDENTIFICATION OF ANTI-RESONANCE FREQUENCY IN BUILDINGS BASED ON VIBRATION MEASUREMENTS

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SUMMARY

A method for locating the anti-resonance frequencies (ARF) of existing buildings from modal analysis was developed in this study . The application of ARF to protect nonstructural elements in earthquake environments was proposed and found effective for shear type buildings in other studies. Most of the seismic protection methods for nonstructural elements were designed after the structures were built. It is necessary to know the ARF of building floors for seismic protection design. By using effective modal analysis and mode shape regression, the first few ARF modes in existing buildings can be found based on vibration measurements. A forced vibration test was designed to verify the sensitivity of the proposed method. The results show that a floor's ARF can be identified only by partial vibration measurements without much error.

INTRODUCTION

When earthquakes attack a building, each floor vibrates according to its dynamic property and the characteristics of the ground motion. Non-structural elements attached to a floor are therefore base-excited by the floors' vibration, which can be viewed as a filtered ground motion. The filter, in general, is a multi-DOF system and its properties are described by Frequency Response Functions (FRF). In an FRF curve, the relationship between the responses at a certain DOF when excited at other DOF is established. A typical FRF curve, as shown in Figure 1, has peaks and valleys. The peaks correspond to the natural frequencies, while the valleys correspond to the anti-resonance frequencies [Ewin, 1986]. If the excitation frequency is near an ARF, the response diminishes to zero when the inherent damping approaches zero.

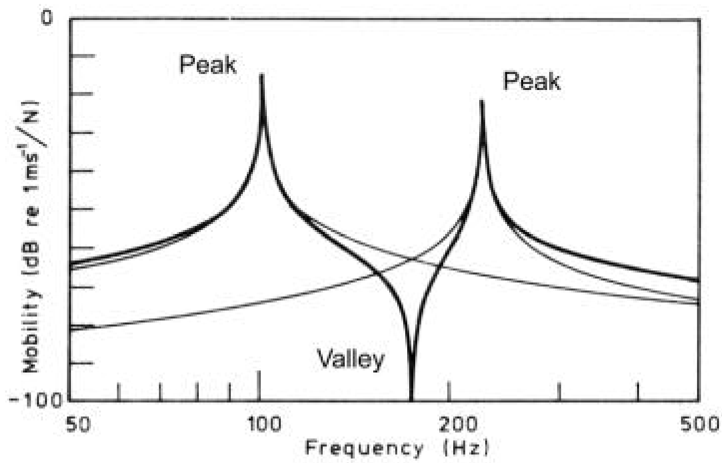

Figure 1: FRF for a 2-DOF system

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ARF engineering applications to reduce vibrations at a certain locations in a system are versatile. Most of the successful applications have been in the mechanical [Kajiwara, Agamatsu, and Seto, 1989] and aerospace industries [Shepard, 1985]. Yao and Lien [Lien and Yao, 1997] proposed an idea to apply ARF theory in the protection of nonstructural elements against earthquakes in a museum. The basic concept is to tune a flexible nonstructural element's natural frequency to the ARF of the supporting floor. When a building vibrates in an earthquake, the frequency component at the ARF of the supporting floor is minimal and so is the vibration of the nonstructural elements. The sensitivity of the ARF protection method to variations in mass ratio [Yao and Lien, 1998] and column yielding [Lien and Yao, 1998] was small, as already discussed in several papers. It was found that within a reasonable range, the ARF protection method could drastically reduce vibrations in nonstructural elements in an earthquake.

Usually, the seismic protection of nonstructural elements is designed after buildings were built. Actual structures are very complicated and have large amounts of DOF. It is very difficult to identify all of the dynamic properties of a system. In this study , the authors attempted to derive the ARF from the response measurements in existing buildings.

ANTI-RESONANCE FREQUENCY IN EARTHQUAKE ENVIRONMENT

For an N-story shear type building, assuming a lumped mass stick model with a fixed base, as shown in Figure 2, the equation of motion for this building in vibration can be expressed as:

$$\mathbf{M}\ddot{\mathbf{x}}(t)+\mathbf{C}\dot{\mathbf{x}}(t)+\mathbf{K}\mathbf{x}(t)=\mathbf{f}(t) \tag{1}$$

where $\mathbf{x}=\{x_j\}$ is the displacement vector, and each element represents the horizontal translation at each floor for a shear type building. $\mathbf{M}$ $\mathbf{K}$ is the mass and stiffness matrixes, and $\mathbf{f}=\{f_j\}$ is the forcing vector. In the frequency domain this equation is expressed as:

$$(\mathbf{K}+\mathrm{i}\Omega\mathbf{C}-\Omega^2\mathbf{M})\mathbf{X}(\Omega)=\mathbf{F}(\Omega) \tag{2}$$

where $\mathbf{X}$ and $\mathbf{F}$ are the Fourier transform of **x** and **f,** and Ω is the excitation frequency.

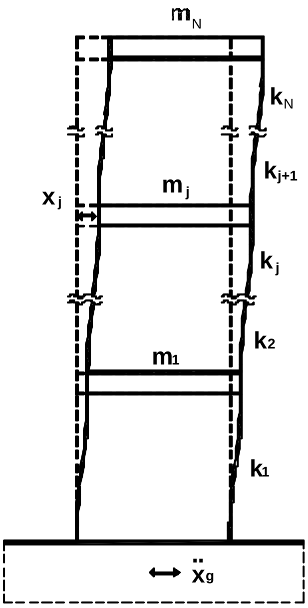


Figure 2: An N-story building model

Assuming a proportional damping system, the frequency domain response can be expressed as:

$$\mathbf{X}(\Omega)=\mathbf{\Phi}\mathbf{Z}(\Omega) \tag{3}$$

where $\mathbf{Z}=\{Z_j\}$ is the modal displacement vector, and $\mathbf{\Phi}=[\Phi_j]$ is the mode shape matrix. Substituting equation (3) into equation (2) and applying the orthogonal properties of the mass and stiffness matrix:

$$\mathbf{Z}(\Omega) = \mathrm{diag}[\frac{1}{w_j^{\,2} - \Omega^2 + \mathrm{i}2\Omega x_j w_j}]\mathbf{F}^{\mathrm{T}}\mathbf{F}(\Omega) \tag{4}$$

where $\mathbf{F} = [\ddot{O}_j]$ is the matrix consisting of normal mode shapes, $\ddot{O}_j$, w_j is the natural frequency of the *j*th mode and x_j is the percentage of critical damping of the *j*th mode, and $\mathrm{diag}[\cdot]$ denotes a diagonal matrix. Defining the modal structure transfer function as:

$$H_j(\Omega) = \frac{1}{w_j^2 - \Omega^2 + \mathrm{i}2\Omega x_j w_j} \tag{5}$$

Therefore the structural displacement can be solved from equation (3), (4) and equation (5) as

$$\begin{aligned} \mathbf{X}(\Omega) &= \mathbf{F}\mathbf{H}(\Omega)\mathbf{F}^{\mathrm{T}}\mathbf{F}(\Omega) \\ &= \mathbf{B}(\Omega)\mathbf{F}(\Omega) \end{aligned} \tag{6}$$

where $\mathbf{H}(\Omega)$ is the diagonal matrix with corresponding $H_j(\Omega)$ for $j = 1 - N$. $\mathbf{B}(\Omega)$ is the structural transfer function matrix.

If the *r*th floor is to have a zero displacement, the *r*th element in equation (6) must be equal to zero.

$$X_r = \sum_{k=1}^{N} (B_{rk}(\Omega) F_k) = 0 \tag{7}$$

B_{rk} can be expressed as the ratio of two polynomials [3]:

$$B_{rk}(\Omega) = \frac{b_0 + b_1\Omega^2 + b_2\Omega^4 + \cdots + b_{N-1}\Omega^{2(N-1)}}{d_0 + d_1\Omega^2 + d_2\Omega^4 + \cdots + d_N\Omega^{2N}} \tag{8}$$

The numerator has a solution of 2(N-1) roots, but only the positive values comprise the meaningful frequency value and therefore there are (N-1) ARF modes in an N-DOF system at every floor.
When ground movements excite a building, the equation of motion is expressed as :

$$\mathbf{M}\ddot{\mathbf{x}}^*(t) + \mathbf{K}\mathbf{x}^*(t) = -\mathbf{M}\mathbf{1}\,\ddot{\mathbf{x}}_g(t) \tag{9}$$

where x^* is the relative displacement and **1** is a displacement vector of unity in a shear type building due to unit ground translation .

Substituting the right hand side of equation (9) into equation (2), the ARF of the *r*th floor can be found from the solution of the following equation.

$$\sum_{k=1}^{N}((\sum_{j=1}^{N}\frac{({}_r\Phi_j)({}_k\Phi_j)}{w_j^2 - \Omega^2}) \times (-m_k)) = 0 \tag{10}$$

where ${}_r\Phi_j$ is the mode shape value of the *r*th DOF in the *j*th mode.

For a five-story building, whose basic properties are shown in Table 1, the FRF of the top and bottom floors with respect to the excitation at the ground floor, are plotted in Figure 3. The fine solid lines indicate the calculated locations of the ARFs' from equation (10),and the dotted lines depict the natural frequencies [5]. The successful match with the ARF location shows that equation (10) is valid.

Table 1. System parameters for the 5-story shear building

Element	Section (mm)	Type	Young's Modulus (KN/m^2)
Column	H308×250×10×24	A3*	2.1×10^{10}
Beam	H250×118×10×13	A3*	2.1×10^{10}

* PRC product

Mode	Natural Frequency				
	1st	2nd	3rd	4th	5th
Hz	1.85	5.37	8.38	10.63	11.96

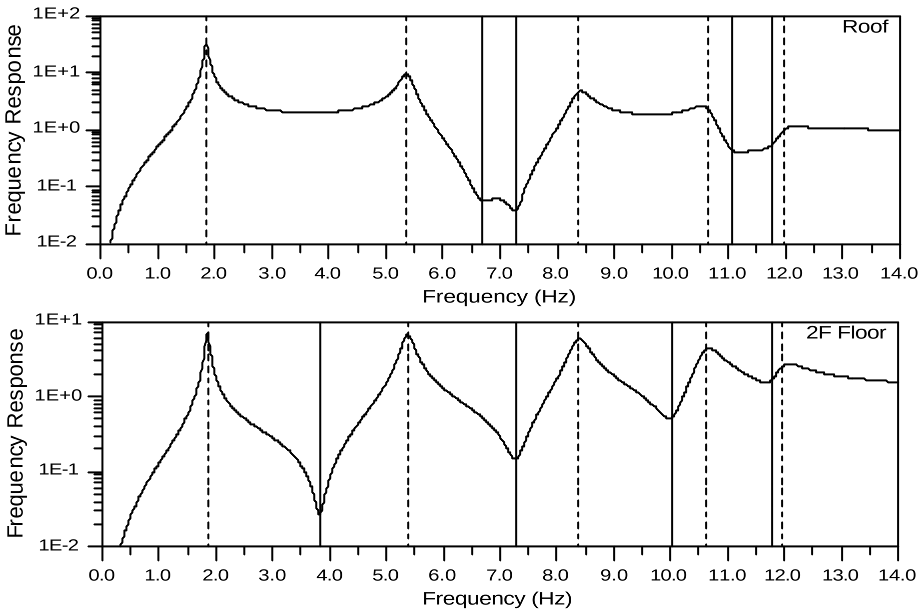


Figure 3: Frequency response functions for top and bottom floors

ANTI-RESONANCE FREQUENCY IN EXISTING BUILDINGS

Usually, the seismic protection of nonstructural elements is designed after buildings have been built. A real structure is very complicated and has a lot of DOFs'. It is very difficult to identify all the dynamic properties of such a system. In this section, derivation of ARF for building structures is demonstrated, and it can be modified by modal superposition method to reduce computation effort.

In addition to uncoupling the equations of motion to solve the problem with SDOF oscillators at each mode, a more attractive feature of the modal superposition method is that, in general, the response of a linear structure to a seismic loading is dominated by first few lower modes of vibration. Even for complicated systems, analysis via the superposition of only a few vibration modes usually produces satisfactory results, provided that the governing system of equations can be decoupled. To do so, the structural frequency response is approximated by

$$\mathbf{X}(\Omega) \approx \mathbf{F}_n \mathbf{Z}_n(\Omega) \tag{11}$$

where $\mathbf{F}_n$ is the $N\times n$ matrix containing first n mode shapes of a structure and $\mathbf{Z}_n$ is the $n\times 1$ column vector containing modal displacements of the first n modes. So the structural displacements can be approximated using

$$\mathbf{X}(\Omega) \approx \mathbf{F}_n \mathbf{H}_n(\Omega) \mathbf{F}_n^{\mathrm{T}} \mathbf{M1}\,\ddot{X}_g(\Omega) \tag{12}$$

Respectively, $\mathbf{H}_n(\Omega)$ is $n\times n$ diagonal matrix containing the modal structure transfer functions of the first n modes.

The first *n-1* ARFs of the *r*th floor can be found from the following governing equation.

$$\sum_{k=1}^{N}((\sum_{j=1}^{n}\frac{({}_{r}\Phi_{j})({}_{k}\Phi_{j})}{w_{j}^{2}-\Omega^{2}+\mathrm{i}2\Omega x_{j}w_{j}})\times(-m_{k}))=0 \tag{13}$$

To demonstrate the proposed methodology, a modal superposition algorithm was used to find the first few floor ARF modes . When all five vibrational modes of a structure are considered, the ARFs obtained by equation (13) are the same as the theoretical solutions using equation (10). To evaluate the quality of the approximation using only a few lower modes of vibration, the first 2 to the first 4 normal modes were plotted in Figure 4 for comparison. It is observed that the ARF with limited modes included has excellent agreement with the theoretical values in each mode. The errors and error ratios from Figure 4 are shown in Table 2. Most of the errors are under 0.1Hz. Even using the first 2 modes, the maximal error is 0.35Hz and the error ratio is only 6.03% . Therefore, using an efficient modal analysis method to calculate the ARFs is accurate enough when the structural information is not complete.

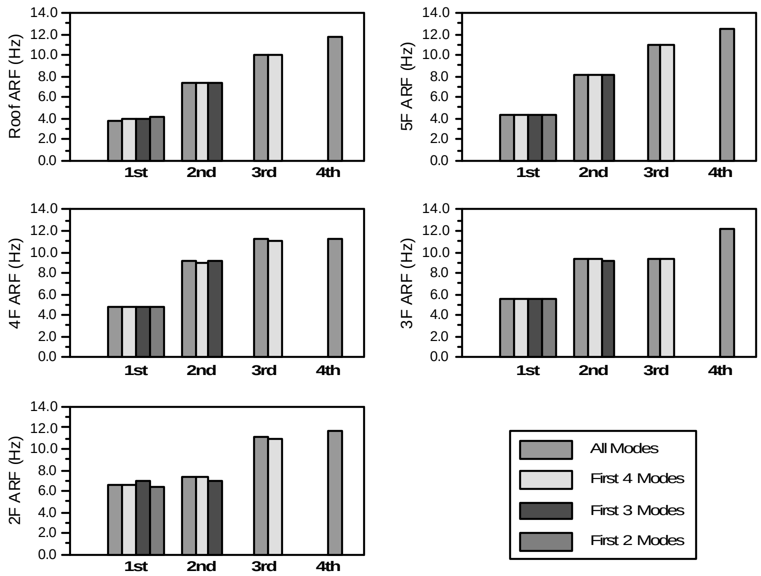


Figure 4. The ARFs of each floors with efficient modal analysis

Table 2. the sensitivity of ARFs with efficient modal analysis

Modes Used	Floor	1st ARF		2nd ARF		3rd ARF	
		Hz Diff.	(%)	Hz Diff.	(%)	Hz Diff.	(%)
First 4 Modes	RF	0.02	0.44	0.03	0.45	0.05	0.49
	5F	-0.01	0.26	-0.01	0.15	0.08	0.75
	4F	0.01	0.12	-0.05	0.58	-0.01	0.89
	3F	0.00	0.05	-0.06	0.69	-0.06	0.69
	2F	-0.03	0.43	0.04	0.49	-0.05	0.46
First 3 Modes	RF	0.08	2.00	0.16	2.22		
	5F	-0.03	0.67	-0.04	0.46		
	4F	0.00	0.02	0.04	0.48		
	3F	-0.01	0.12	-0.34	3.60		
	2F	0.28	4.25	-0.33	4.51		
First 2 Modes	RF	0.23	**6.03**				
	5F	0.01	0.24				
	4F	-0.04	0.86				
	3F	0.01	0.23				
	2F	**-0.35**	5.21				

An efficient modal analysis method can locate the ARFs using only the first few structural modes from equation (13). A completed mode shape vector for each of the lower modes is still required. In order to obtain a complete mode shape vector, massive instrumentation is required, which is expensive and rarely done. However, there were examples [Lin, Gau and Wang, 1995] [Loh and Yang, 1997] of the interpolation of complete mode shape vectors from limited instrumentation. This study attempted to perform a similar interpolation to locate the ARFs.

One of the interpolation methods involves X^n polynomial curve fitting. For each mode of a shear type building, the mode shape equation can be expressed as

$$y = f^{\mathrm{T}} A \tag{14}$$

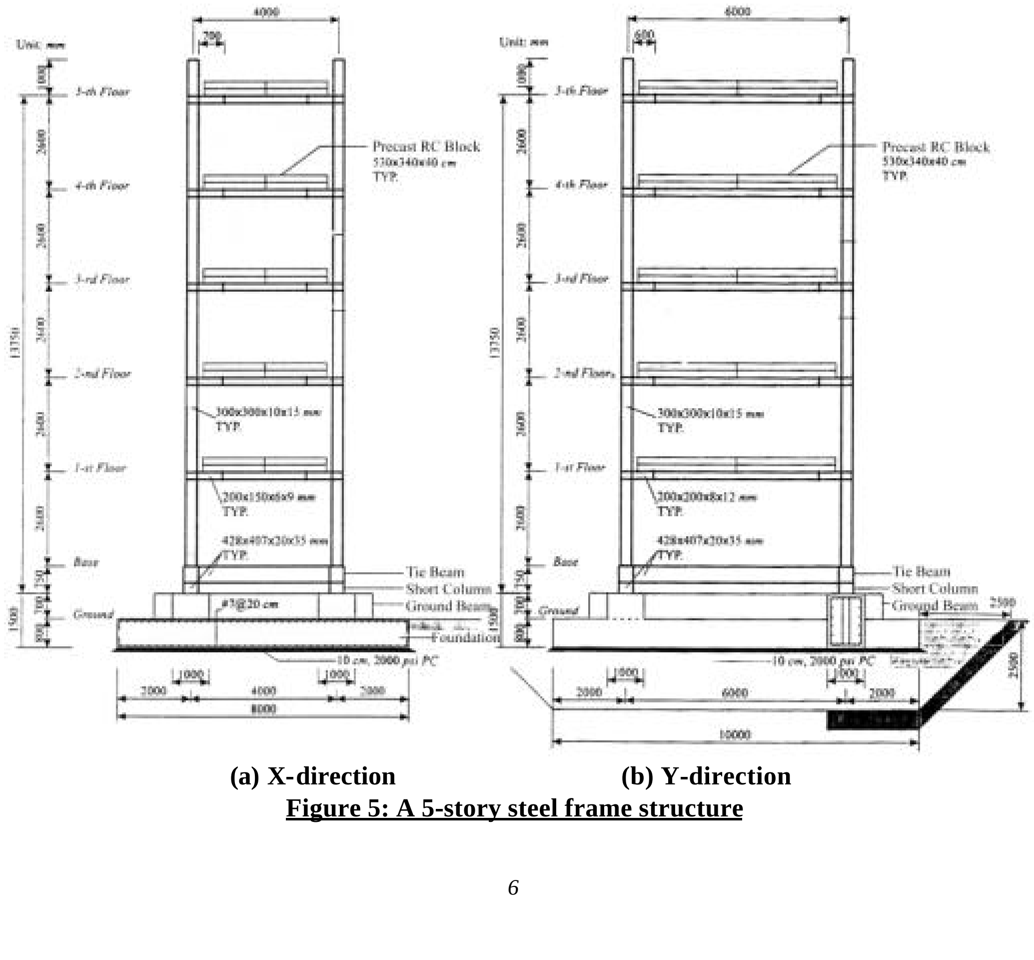
where $f = [h, h^2, \cdots, h^n]^{\mathrm{T}}$, h is the foundation to story height, $A = [a_1, a_2, \cdots, a_n]^{\mathrm{T}}$ is the coefficient vector of regression, y is the mode shape value with respect to h, and n is the power of the regression, which must be less than the amount of data. If there are m sensors installed in the structure, the error of the mode shape equation can be determined using the least mean square method. The coefficient vector estimate is then determined using

$$A = \left[\sum_{i=1}^{m} f_i f_i^{\mathrm{T}} \right]^{-1} \left[\sum_{i=1}^{m} f_i y_i \right] \tag{15}$$

When the coefficient vector is determined, the first n mode shapes can be interpolated by introducing the height of each floor. The mode shapes should be satisfied by the orthogonality conditions and must be modified by $\mathbf{F}_n^{\mathrm{T}} \mathbf{M} \mathbf{F}_n = \mathrm{diag}[1]$. The efficiency of this method is demonstrated below.

FORCED VIBRATION TESTING

A 5-story steel frame structure was constructed on the campus of the National I-LAN Institute of Technology by National Center for Research on Earthquake Engineering (NCREE), as shown in Figure 5. The structure consisted of frames parallel in two directions [Yeh and etc. 1996]. The SAP90 program was used to identify the modal dynamic properties including natural frequencies and mode shapes. The anti-resonance frequencies of each floor could be determined from equation (10). All of these characteristics are shown in Table 3.



(a) X-direction **(b) Y-direction**

Figure 5: A 5-story steel frame structure

Table 3: Dynamic characteristics of testing model in Y direction

Mode	Natural Frequency (Hz)				
	1st	2nd	3rd	4th	5th
(Hz)	0.82	2.71	5.14	7.64	9.37
Floor	Mode Shape				
	1st	2nd	3rd	4th	5th
Roof	0.0938	-0.0833	0.0610	-0.0370	-0.0167
5F	0.0817	-0.0056	-0.0667	0.0845	0.0523
4F	0.0629	0.0673	-0.0586	-0.0526	-0.0796
3F	0.0389	0.0869	0.0596	-0.0367	0.0839
2F	0.0143	0.0452	0.0766	0.0913	-0.0673

Floor	Anti-resonance Frequency (Hz)			
	1st	2nd	3rd	4th
Roof	3.40	4.50	7.97	9.25
5F	2.74	6.02	6.62	9.62
4F	2.34	5.53	8.50	----
3F	2.04	4.79	7.85	9.87
2F	1.81	4.19	6.89	9.13

Forced vibration testing was designed to verify the sensitivity of the proposed method on anti-resonance frequency in an existing building. A shaker was fastened to the top floor of the structure so that applied harmonic exciting forces can be generated to excite the structure. The sensors are located on the top, the forth and the second floor to measure the translation response of each floor. The frequency response curves of each measured floor could be created in sine sweep tests. Because of the capacity limit of the shaker, the excitation frequency must be less than 9 Hz. This is too low to identify the higher modes of the structure. Only the first 3 modes could be excited.

According to the measured frequency response curves in the NCREE report[Yeh and etc., 1996], the mode shape values were corrected as displacements related to the foundation. The measured vibration characteristics of the structure, via system identification, are shown in Table 4. The complete mode shapes of the first 3 modes can be determined using a polynomial and its orthogonality condition, as shown in Table 5 [Lien, 1999]. From equation (13), combined with the mode shape and natural frequency verified by system identification, the first 2 anti-resonance frequencies of each floor in an earthquake environment could be determined.

Table 4. Dynamic characteristics of testing model by vibration measurements

Mode		1st	2nd	3rd
Natural Frequency (Hz)		0.82	2.68	5.01
Mode Shape	Roof	0.998	1.007	0.774
	4F	0.657	-0.936	-0.677
	2F	0.146	-0.571	0.987

Table 5. Mode shapes of First 3 modes with mode shape regression

Floor	Mode Shape		
	1st	2nd	3rd
Roof	0.998	1.007	0.774
5F	0.883	-0.348	-0.938
4F	0.657	-0.936	-0.677
3F	0.388	-0.947	0.363
2F	0.146	-0.571	0.987

The error, with respect to the numerical analysis value and error ratio of the ARF to the partial measurements, is shown in Table 6. The error ratio is defined as the percentage of the error divided by the baseline value. It is shown in Table 6 that the maximal error is 0.72 Hz, and the maximal error ratio is 12.0%. Most ARF errors in each floor are under 0.3Hz. The error ratios are less than 10.0%.

Table 6. Variation sensitivity of ARF for each floor

Mode		1st ARF		2nd ARF	
Modes included	Floor	Error (Hz)	(%)	Error (Hz)	(%)
First 3 modes	RF	0.28	8.33	-0.09	1.91
	5F	-0.24	8.69	**-0.72**	**12.00**
	4F	-0.16	6.78	-0.33	6.06
	3F	-0.09	4.25	0.10	2.19
	2F	-0.09	4.75	0.32	7.70

CONCLUSIONS

The theoretical results illustrate that a Single Degree-of-Freedom system would experience minimal vibrations if its fundamental frequency coincides with the supporting floor's ARF. The ARFs of each floor in an existing building could be determined based on vibration measurements. According to the vibration measurements, the structural dynamic characteristics, including natural frequency and mode shape, could be determined by system identification. Also, the ARFs can be identified using an efficient modal analysis and mode shape regression. A forced vibration test verified that the sensitivity of the ARF obtained from the proposed method is good. Even only partial response measurements could produce acceptable results. The proposed seismic protection method for nonstructural elements could be applied easily when the ARFs of the floors are identified.

ACKNOWLEDGEMENTS

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