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Analysis of piping systems for non-stationary support excitation

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ABSTRACT: A response analysis of piping systems to random support excitation is presented. The piping system is supported by primary structures or block foundations. The dynamic resistance offered by the supports are specified in terms of stiffness and damping. The excitations at the supports are non-stationary filtered white-noise process. The filters denote the primary structures and the soil mass with evolutionary white-noise representing bed rock motion. A space state transition matrix formulation is used to obtain the response in time domain. Using the proposed method of analysis, few example problems for a typical piping system are solved. Also, the responses for a particular example obtained by spectral method of analysis are compared with those obtained by the proposed method in order to investigate the relative efficiencies of both methods.

1 INTRODUCTION

The piping systems are generally supported at certain points by independent structres (which are classified as primary structures) including block foundations. If the piping system is attached to more than one type of supporting structres, excitations at different points of supports produced due to earthquake will be different. In addition, the supports of the piping system may be flexible and may also provide different kinds of damping to the system, depending upon the nature of the support. Thus, the problem of seismic response of piping system is a complex one including multi-support excitation, non-classical damping and support flexibility.

Seismic analysis of piping systems subjected to multiple support excitations commonly uses either time history (Chiba, Koyanagi et.al.,(1989), Lee & Penzien(1983)) or the response spectrum method (Gupta, Jhaveri et.al.,(1984) & Sujuki, Sone(1989)). Unfortunately, these methods cannot take into account cross correlation effect between the multiple support excitations. Therefore, more rigorous stochastic seismic analysis technique should be employed for piping systems. For stochastic analysis of piping sytems under random ground motion, spectral analysis in frequency domain can be developed considering flexibility of supports and their damping. However, when the input ground motion is treated as non-stationary random process, the determination of evolutionary rms response of the system with the help of frequency domain analysis may become computationally prohibitive because of the repeated application of spectral analysis at each time step in the evolutionary process.

In the present paper, an alternative time domain solution for the problem is presented. The input excitations at the supports are provided in terms of the time history of rms ground accleration, and a cross correlation function between the multiple support excitations. Using the proposed method of analysis, a piping system is analysed for various types of supports and excitation conditions. Also, the efficiency of the proposed method of analysis is compared with that of the frequency domain method of analysis.

2 SUPPORT EXCITATION

Excitations to the piping system are those produced by the dynamic movements of the supporting points. They are defined by the evolutionary rms acclerations which are the outputs of filters excited by the evolutionary white noise which is considered as the seismic excitation at the bed rock level. For the excitation points attached to the ground, the filters represent the soil media ; whereas for those attached to the supporting structres, the filters represent the combined soil and supporting systems. The flexibility and damping of supports are represented by spring-dash pot systems. Seismic excitations which are represented by evolutionary white noise are assumed to act in the principal directions (u,v,w) of earthquake (that is the componenets of ground motions are uncorrelated) with shear wave velocity incident at an angle w.r.t. a set of global axes. The evolutionary rms acceleration of each component of excitation is specified. The spatial correlation between the excitations at two points is given by a frequency independent correlation function (Loh (1985)).

$$\rho_{mn} = \exp(-a|L|) \cos 2\pi K_o L \qquad (1)$$

where a and K_o are parameters which depend on the direction of wave propagation as well as wave type, earthquake location and magnitude. Values of 2.756 and 4.769 have been given for a and K_o respectively (Loh (1985)). L is the distance between stations m and n measured in the direction of wave propagation.

For time domain analysis, the filters are augmented to each translational degree of freedom associated with the support points and represented by

$$\ddot{x}_{fi} + 2\xi_{fi}\, w_{fi}\, \dot{x}_{fi} + w^2_{fi}\, x_{fi} = S_i + W_i(t) \qquad (2a)$$

$$\ddot{S}_i + 2\xi_{si}\, w_{si}\, \dot{S}_i + w^2_{si}\, S_i = -W_i(t) \qquad (2b)$$

where $i = 1,2,\ldots 3n_1$; $3n_1$ is the number of translational degrees of freedom of the supporting points for the piping system, each support point having 3 d.o.f. The output of each one of these filters is the input excitation to the piping system along the specified translational DOF. w_{fi}, & w_{si} and ξ_{fi} & ξ_{si} are respectively the frequencies and damping ratios characterizing the i^{th} filter. x_{fi} is the response of the i^{th} filter which is the input to the piping system. S_i is an intermediate response of the filter. $W_i(t)$ is an evolutionary input white noise of the form $\Psi_i(t).\bar{W}_i$ which is different for each supporting structure ($\bar{W}_i$ is a stationary white noise and $\Psi_i(t)$ is modulating time function). The vector of evolutionary white noise $\{W(t)\}$, is having a covariance matrix,

$$\Sigma_{ww}(t,t+\tau) = [Q(t)]\ \delta(\tau) \qquad (3)$$

where $\delta(\tau)$ is the Dirac delta function, $[Q(t)]$ is the matrix of white noise intensities or strengths of components.

By integrating Eq.(3), it can be seen that $[Q(t)]$ is the integral of the covariance matrix of $\{W\}$. Typical elements of the covariance matrix of the evolutionary vector valued white noise, (Eq.(3)) are given by (considering $W_i(t)$ as zero mean process)

$$\mathrm{Cov}[w_i(t),w_j(t)] = \rho_{ij}[q_{ii}(t)q_{jj}(t)]^{\frac{1}{2}}\ \delta(\tau) \qquad (4)$$

where ρ_{ij} is the correlation coefficient between support excitations at supports i & j, and $q_{ii}(t)$ is the evolutionary intensity of i^{th} evolutionary white noise.

The response of each filter x_{fi} is characterized by an evolutionary one sided power spectral density function, or time history of rms response.

$$S_{xfi}(w,t) = \Psi^2_i(t)\ S_{xfi}(w) \qquad (5)$$

$$\sigma_{xfi}(t) = \Psi_i(t).\sigma_{xfi} \qquad (6)$$

in which $S_{xfi}(w)$ is the power spectral density function which depends on the spectral value of white noise W_i and the filter coefficients.

3 ANALYSIS OF PIPING SYSTEM

Fig.(1) shows a model of a piping system supported at different levels. The supporting structures have different dynamic characteristics. The interaction between the piping system and supporting points is represented by linear springdashpot system with stiffness and damping values denoted by K_s and C_s respectively. Seismic excitations get modified and are transferred to the piping system through the supporting points. The dynamic d.o.f are considerd as 3 translations (in X,Y,Z directions) at each node.

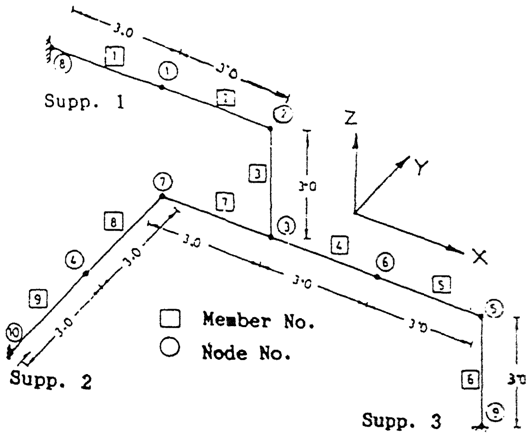

Figure 1. Typical piping system

3.1 Equations of motion

The equations of motions for the piping system in terms of absolute displacements may be written in the following form :

$$\begin{bmatrix} m_{11} & \\ & m_{22}\end{bmatrix}\begin{Bmatrix}\ddot{X}_p\\ \ddot{X}_s\end{Bmatrix} + \begin{bmatrix} C_{p11} & C_{p12}\\ C_{p21} & C_{p22}+C_s\end{bmatrix}\begin{Bmatrix}\dot{X}_p\\ \dot{X}_s\end{Bmatrix} +$$

$$\begin{bmatrix}K_{p11} & K_{p12}\\ K_{p21} & K_{p22}+K_s\end{bmatrix}\begin{Bmatrix}X_p\\ X_s\end{Bmatrix} =$$

$$\begin{bmatrix}0\\ K_s\end{bmatrix}\{X_f\} + \begin{bmatrix}0\\ C_s\end{bmatrix}\{\dot{X}_f\} = \begin{Bmatrix}0\\ P(t)\end{Bmatrix} \qquad (7)$$

If the number of non support nodes and support nodes are n_1 and n_2 respectively, then the size of $\{X_p\}$ and $\{X_s\}$ are $3n_1$ and $3n_2$ respectively.

In the above equations $[m_{11}]$, $[C_{p11}]$ and $[K_{p11}]$ are matrices of pipe mass (diagonal), damping and stiffness corresponding to d.o.f. at nonsupport nodes with size of $3(n_1 \times n_1)$. $[C_{p12}]$ and $[K_{p12}]$ are matrices of pipe damping and stiffness representing the coupling between non-support and support degrees of freedom; $[m_{22}]$, $[C_{p22}]$ & $[K_{p22}]$ are matrices of pipe mass, damping and stiffness corresponding to support d.o.f. of size of $3(n_2 \times n_2)$. $[C_s]$ and $[K_s]$ are matrices of support damping and stiffness corresponding to support d.o.f. $\{X_s\}$ is the total displacements at support d.o.f. i.e., $\{X_s\} = \{u\} + \{X_f\}$, where $\{u\}$ is the vector of relative displacement at the support d.o.f. and $\{X_f\}$ is the vector of prescribed displacements in the directions of the d.o.f. at the supports.

C_{p11} is assumed to be proportional to m_{11} and K_{p11} and is determined by assuming 5% modal damping in first two modes(Clough & Penzien(1982)). Coupling terms in damping matrices ie., C_{p11} and C_{p21} are assumed to be zero. Further, C_{p22} is assumed to be much smaller in comparison to C_s and is neglected.

3.2 Solution of eq. of motion in time domain

For time domain formulation, the filter degrees of freedom $\{X_f\}$ and $\{S\}$ are added to the responce vector. Then, combining Eq. 2 and 7, a modified equation of motion can be written in the following form

$$[M]\{\ddot{Y}\} + [C]\{\dot{Y}\} + [K]\{Y\} = [F]\{W(t)\} \qquad (8)$$

$$\{Y\} = \begin{Bmatrix} X_p\\ X_s\\ X_f\\ S\end{Bmatrix}_{\bar{m} \times 1} \qquad (9)$$

The size of $[M]$, $[C]$ and $[K]$ is of size $\bar{m} \times \bar{m}$ where $\bar{m}=3(n_1+3n_2)$; $\{W(t)\}$ is a vector of evolutionary white noise which is the input to the filters; $[F]$ is a rectangular matrix with size $(\bar{m} \times 3n_2)$, it contains the load coefficients of the white noise vector.

The matrix $[F]$ contains zero values as load coefficients corresponding to x_p and x_s, and it contains diagonal blocks of 3x3 matrix (matrix b), which contains the component coefficients of the evolutionary white noise in the directions of global axes, arranged in the lower $3n_2 \times 3n_2$ segment of the matrix.

The filters are directed towards d.o.f. in the global directions at the supports so that

$$[b] = \begin{bmatrix} -\cos\alpha & \sin\alpha & 0\\ -\sin\alpha & -\cos\alpha & 0\\ 0 & 0 & 1\end{bmatrix} \qquad (10)$$

where {W} is the vector of input evolutionary white noise to the filters in principal directions u, v and w; u being at an angle of α with the x-axis.

3.3 Space state formulation of dyn. system

For the space state formulation, the state variables are considered as the displacements and their derivatives. Defining a space state vector Z, Eq.(8) can be written in terms of Z as

$$\{\dot{Z}\} + [A]\{Z\} = [B]\{W(t)\} \tag{11}$$

Equation (11) describes a system of 2m first order differential equations. The solution of Eq. (11) in the time domain is given by Mortensen (1987).

$$\underline{Z}(t)=[\phi(t,t_o)]\underline{Z}(t_o)+\int_{t_o}^{t}[\phi(t,t_o)][B]\{W(\tau)\}d\tau \tag{12}$$

$$\phi(t,t_o) = \exp\{-[A](t-t_o)\} \tag{13}$$

where $\phi(t,t_o)$ is the state transition matrix.

3.4 Evolutionary cov. matrix of state vector

If the excitation {W(t)} is evolutionary Gaussian white noise, the response Z is an evolutionary Gauss-Markov random process (Bryson & Ho (1969), Gasparini (1979)). The covariance matrix of the state vector is obtained in the following form, assuming that the response vector at time (t_o) i.e., {Z(t_o)} is independent of the excitation {W(t)} (Bryson & Ho (1969), Gasparini (1979)).

$$\underline{\Sigma}_{zz}(t) = \phi(t,t_o)\ \underline{\Sigma}_{zz}(t_o)\ \underline{\phi}^T(t,t_o)$$

$$+ \int_{t_o}^{t} \underline{\phi}(t,\tau)\ \underline{B}\ \underline{Q}(\tau)\ \underline{B}^T\underline{\phi}^T(t,\tau)\ d\tau \tag{14}$$

If the mean of the exciting vector is assumed to be zero (i.e. $\underline{\mu}_w = 0$), then Eq.(14) fully describes the state of output vector {Z(t)}. Thus, the covariance matrix of response can be calculated at any time t provided that the covariance matrix at any previous time t_o, and the matrix of strengths of the excitation (i.e. intensity matrix [Q(t)] of {W(t)}) are known. Covariance matrix for member end forces is determined from the covariance matrices of the member end displacements and rotations. The latter is obtained from the covariance matrix of displacements (for condensed d.o.f.) using standard procedure (Nigam (1983)).

3.5 Calculation of Intensity Matrix Q(t)

Given the time histories of rms responses of filters (ie, the input excitation at the support d.o.f.), the intensity matrix Q(t) for the evolutionary white noise, which is the input to the filters (or it represents the bed-rock motion) is calculated. The fictitious piece-wise linear strength envelope (intensity function of white noise) needed to match any prescribed time history of rms motion can be directly obtained by analyzing the filters before augmenting them to the supports of the piping system. The procedure is outlined in Ref.(Bryson & Ho (1969), Hany (1991)).

4 NUMERICAL STUDY

With the help of the proposed method of analysis, the piping system shown in Fig. 1 is analysed for numerical study. The piping system is subjected to same type of non-stationary excitation at each support, i.e., same filter (A) is used for all support points. The filter characteristics are w_g=25.0, w_f=10.0, ξ_g=0.5 and ξ_f=0.5. The rms accelerations are assumed to be same for all three components. The problem is solved for two support conditions (flexibility and damping) which are shown in Table 1.

The time histories of rms accelerations of the motion at the supports are specified for principal directions of ground motions which are assumed to coincide with x, y, and z directions of the piping system and assumed to be same for all supports. Also, rms excitations in three principal directions are assumed to be same. The time history of rms acceleration to major direction of the motion is shown in Fig. 2.

Table 1. Support stiffness & damping values

Case No.	Sup. No.	K_{sx}	K_{sy}	K_{sz}	C_{sx}	C_{sy}	C_{sz}
1	1	116.1	109.4	133.1	1.56	1.56	2.7
	2	116.1	109.4	133.1	1.56	1.56	2.7
	3	116.1	109.4	133.1	1.56	1.56	2.7
2	1	1290.3	1215.9	1479.0	5.2	5.2	9.0
	2	1290.3	1215.9	1479.0	5.2	5.2	9.0
	3	1290.3	1215.9	1479.0	5.2	5.2	9.0

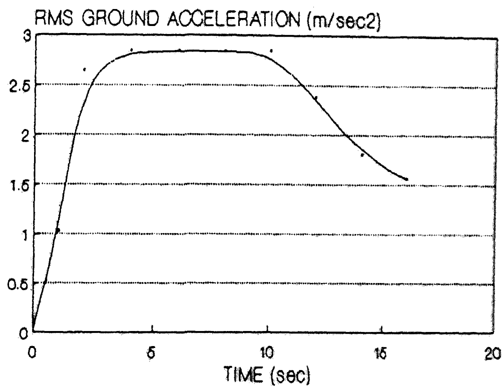

Figure 2. Evolutionary rms acceleration

The first five undamped frequencies of the piping system for the support conditions shown in Table 1 are given in Table 2.

Table 2. Undamped natural frequencies (rad/sec)

Mode No.	Case 1	Case 2
1	7.105	7.680
2	11.789	13.211
3	12.668	14.440
4	19.390	31.750
5	21.360	41.300

The results for the displacement response at a selected node for two cases are shown in Fig. 3. The responses for the first case are greater than those for the second. This is due to the fact that for the first case, 3rd & 4th natural frequencies of the system happen to fall within the range of frequencies where excitation energy is significantly high.

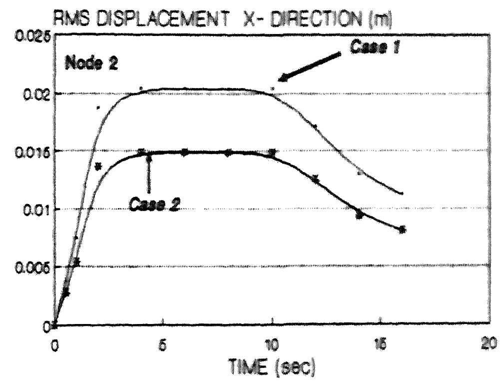

Figure 3. Evolutionary rms displacement in x - direction

In order to evaluate the efficiency of the proposed method of analysis, the responses obtained by both frequency and time domain analyses are compared for the example problem (case 2) shown in Table 1. In the frequency domain analysis, spectral approach is used in which the input to the supports are the evolutionary power spectral density functions of the outputs from the filters attached to the supports. The spectral analysis is performed at each time step (Δt taken same as that for the time domain analysis) in the evolutionary process and the time history of rms response are determined. Note that for both time and frequency domain analyses, same frequency independent spatial correlation function as given by Eq.1 is used. The details of the spectral method of analysis for piping system with support flexibility and damping for non-stationary excitation are given in Ref. (Hany (1991))

Some typical results of the two analyses are presented in Fig.4. It is seen that the frequency domain analysis provides higher response. However, the difference is not very significant (is of the order of 8%). The reason for this difference is essentially due to the difference in the solution technique employed for the two methods. The frequency domain solution for non-stationary excitation is in fact based on quasi-stationary approach which directly provides the rms response at any instant of time in the evolutionary process. The time domain solution provides the response at any time station treating the response as Gauss-Marcov process.The rms response at any instant of time t depends on the response at previous time station and hence, the initial conditions influence the response

The computational time required by time domain method is 1150 sec as against 1615 sec required by spectral analysis (in ICL 3980 system).

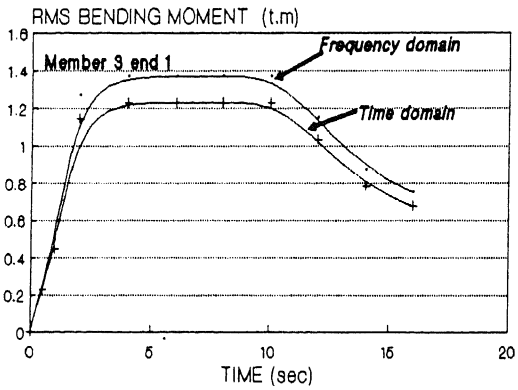

Figure 4. Evolutionary rms bending moment about y-y axis

5 CONCLUSIONS

A time domain analysis for piping system with flexible supports and with different types of damping is presented for multi-point non-stationary random excitations. The method of analysis is based on space state formulation with state transition matrix used to determine covariance of response at different time steps in the evolutionary process of excitation. The piping system represents typical secondary system which is suported by primary structure at different points including block foundations directly attached to the ground. With the help of the proposed method, some typical problems of piping system are solved to illustrate its applicability for different cases.

The computational time required by the proposed method is much less compared to that required by the frequency domain spectral analysis. Therefore, the method is proved to be more efficient for piping systems subjected to non-stationary multiple support random excitations.

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