Earthquake physical damage and serviceability of lifeline networks

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ABSTRACT: The connectivity of lifeline systems is studied in this paper in terms of two ratios: 'damage ratio', which is the degree of physical damage to the system, and is defined as the expected number of failures per link; and 'service ratio', which is a measurement of system operation. First, a method is developed for determining the relationship between the two ratios in simple systems immediately following an earthquake. Density function, expected value and covariance of service ratio are calculated as functions of the damage ratio. Even in a simple system, the density function of the service ratio has two peaks. Then, the variation in reliability of the network is examined with respect to differences in scale and configuration of the network, which may accompany growth of the community.

1 INTRODUCTION

In the early phases of study of lifeline systems, several analysis techniques were developed to determine system reliability, e.g. super series systems in parallel (SSSP) (Taleb-Agha 1975; Shinozuka, Takada and Kawakami 1977) Boolean algebra (Pratta and Montanari 1973), and the Monte Carlo technique (Shinozuka, Takada and Ishikawa 1979; Tamura and Kawakami 1978). Thus, it is possible to evaluate connectivity or serviceability even for cases of relatively complicated networks. However, obtained numerical results for connectivity have neither been summarized nor explained in a coherent and unified manner even in the case of simple networks.

The connectivity of lifeline systems is studied in this paper in terms of two ratios: 'damage ratio', which is the degree of physical damage to the system; and 'service ratio', which is a measurement of system operation.

The purpose of this paper is to develop a method for determining a relationship between the two ratios described above immediately following an earthquake, and also to examine the variation in reliability of the network with respect to difference in scale and configuration, which may accompany the growth of the community.

The physical and functional restoration processes for lifeline systems are studied based on survey reports (Katayama 1979) of the Miyagi-ken-oki earthquake of 1978 (magnitude 7.4). The restoration of physical structures and of system serviceability are shown in Figure 1 for the gas system in Sendai City. The solid curve shows the ratio of repaired pipe failures to the total number of failures. The dashed line shows the ratio of normally-supplied houses to the total number in the system. These two ratios naturally increase monotonically with time. However, they do not coincide with each other. The rate of increase in the serviceability in relation to the restoration of physical structures is not constant during restoration. This indicates the necessity to draw a distinction between the physical damage (damage ratio) and the functional damage (service ratio).
2 THEORY OF RELATIONSHIP BETWEEN THE TWO RATIOS

The following assumptions are made:

1. The system is a network composed of links and one supply station. The supply station does not fail, but failures occur along links;
2. The failure probability per unit length is the same for all locations in the network, and locations of failures are mutually independent;
3. The damage ratio is defined as the expected number of failures per link;
4. The portion connected to the supply station, by use of one or more safe paths, is called the 'connected portion'. For the network shown in Figure 2, with failures indicated by cross symbols, connected portions are represented by solid lines. Service ratio is then defined as the ratio of total length of the connected portions to that of all links. In the case of Figure 2(b), the service ratio is 100 percent, even though two failures occurred.

2.1 One-link system with supply station at one end

The simplest network, a system composed only of one link, having a length \( L \), and a supply station at one end, is considered in Figure 3. When \( N \) failures occur in this system, the probability that the \( n \)-th nearest failure to the supply station is located between \( x \) and \( x+dx \), \( p(E) \), is

\[
p(E) = \frac{N!}{(n-1)! (N-n)!} f_X(x) f_X(x) dx [1-F_X(x)]^{N-n} \tag{1}
\]

in which \( f_X(x) \) and \( F_X(x) \) are the probability density and distribution functions of the failure location \( X \), respectively.

As the failure probability per unit length is assumed to be the same for all locations in the network, as described in assumption 2,

\[
f_X(x) = \frac{1}{L} \tag{2}
\]

and

\[
F_X(x) = \frac{x}{L} \tag{3}
\]

Then, the density function of the \( n \)-th failure location, \( f_{X_n}(x) \), is

\[
f_{X_n}(x) = \frac{N!}{(n-1)! (N-n)!} x^{n-1} (L-x)^{N-n} \quad (0 \leq x \leq L) \quad \tag{4}
\]

When the link length is equal to unity, i.e. \( L=1 \), the density function of the service ratio \( Z \), \( g_2(z) \), equals the density function of the first damaged point, \( f_X(z) \), because the portion between the supply station and

\[
\text{Supply Station}
\]

\[
\text{Figure 2. Definition of service ratio.}
\]

\[
(x: \text{damaged point})
\]

\[
\text{Figure 3. One-link system with a supply station at only one end.}
\]

\[
g_2(z) = f_{X_1}(z) = (N-1-z)^{N-1} \tag{5}
\]

The expected value and covariance of the service ratio are given as

\[
z = \int_0^1 z g_2(z) dz = \frac{1}{N+1} \tag{6}
\]

and

\[
\sigma^2 = \int_0^1 (z-z)^2 g_2(z) dz = \frac{N}{(N+1)^2 (N+2)} \tag{7}
\]

2.2 Loop-type system

The loop-type system shown in Figure 4(a) can be transformed to a one-link system by cutting the supply station into two halves, and each half serving as a terminus of the system, as shown in Figure 4(b). This system differs from that in Figure 3 because this one has supply stations at both ends of the link, while there was only one supply in Figure 3.

Then, the density function of service ratio \( g_2(z) \) is (Kawakami 1990)
\[ g_2(z) = N(N-1)(1-z)^{N-2} z \]  \hspace{1cm} (8)

and the expected value and covariance are

\[ z = 2/(N+1) \]  \hspace{1cm} (9)

and

\[ \sigma^2 = 2(N-1)/\{(N+1)^2(N+2)\} \]  \hspace{1cm} (10)

2.3 Network system

As shown in the preceding sections, an exact density function of service length can be computed for much more complicated networks.

First, determine the pattern of failure location. All the links can be classified into one of the following groups:

a) A link having one or more failures, and both ends connected with the supply station through one or more safe paths;

b) A link having one or more failures, and only one end connected with the supply station;

c) A link having no failures, and being connected with the supply station;

d) A link disconnected from the supply station.

Considering that group a corresponds to the loop-type system in Figure 4, and that group b is the one-link system in Figure 3, the expected service ratio and covariance can be formulated.

Figure 4. Loop-type system.

Figure 5. Damage ratio vs. service ratio for a tree-type system (1000 simulations).

Figure 6. Damage ratio vs. service ratio for a checkerboard-type network (1000 simulations).
3 NUMERICAL EXAMPLES

For a much more complicated system, locations of failures are simulated using the Monte Carlo technique. By repeating this simulation a number of times, approximate density function, expected value and covariance of the service ratio are computed. As examples, density functions of the service ratio for the tree-type and the checkerboard-type system, related to damage ratio, are shown in Figures 5 and 6. Expected values with each damage ratio are also shown by the solid lines.

From Figures 5 and 6, it is seen that as the damage ratio increases, the service ratio decreases. However, even for such a simple network, the density function does not show a simple, gradual change as the damage ratio increases. Two peaks in the density function exist. As the damage ratio increases, the service ratio decreases but the range of values of the density function is very large for moderate damage ratios. The extreme of nearly a 0 percent service ratio is produced by failures of links which are directly connected with the supply station. This result indicates that stronger pipes near supply stations increase network robustness.

4 EFFECTS OF SYSTEM COMPLEXITY ON CONNECTIVITY

Variation of the relationship between the two ratios, with growth of the system, is detailed in Figure 7 for the tree-type systems, and in Figure 8 for the checkerboard-type networks.

For the tree-type systems in Figure 7, the service ratio decreases sharply as the damage ratio increases.

However, for the checkerboard-type networks in Figure 8 it should be noted that similar values are sometimes obtained for different networks: for a damage ratio less than 0.3/link, the service ratio is more than 80 percent in all networks; for a damage ratio of about 1/link, the service ratio is less than 50 percent in all networks. However, details of the relationship vary with the complication of the network, and the service ratio for complex networks is as large as, or larger, than that for simple networks at small damage ratios (less than 0.3/link). This pattern is, however, reversed for large damage ratios, where for a damage ratio of 0.3/link or more, the service ratio for complex networks is smaller than that for simple networks. These results indicate that a complicated network is safer against small earthquakes, but in large earthquakes its reliability is drastically reduced. If such an indication is not considered sufficiently, complicated lifeline networks, especially in urban areas, may completely malfunction in future large earthquakes.

Figure 9 shows the variation of the relationship between the expected value and the standard deviation of the service ratio for the networks shown in the right portion of the figure. The standard deviation of the service ratio reaches a maximum when its expected value is 60–70 percent. It is interesting that this value is greater than 50 percent. When the expected value of the service ratio is small, its variance is small, and the probability that the service ratio is approximately 100 percent is
negligible. Meanwhile, when the expected value is large, the variance is large, and the probability that the service ratio is almost zero is not negligible.

The curves in Figure 9 are similar in shape. However, the standard deviation is larger when the supply station is connected directly to only two neighboring links (solid lines) than when it is connected to three or more (broken lines).
5 CONCLUSIONS

The main results of this paper may be summarized as follows:

1. The connectivity of lifeline systems can be studied in terms of two ratios: the 'damage ratio', which pertains to the degree of physical damage to the system and is defined as the expected number of failures per link; and the 'service ratio', which is a measurement of system operation and system connectivity.

2. Density function, expected value and covariance of service ratio can be calculated as functions of the damage ratio in simple systems.

3. Even for a simple system, the density function of the service ratio has two peaks. It should be noted that a peak at a service ratio around zero is apparent. This results from the possibility of malfunction of the total system caused by even small physical damage.

4. For tree-type systems, the service ratio decreases sharply as the damage ratio increases.

5. For checkerboard-type networks, when the damage ratio is small, the service ratio of a complex network is as large, or larger, than that of a simple network. This pattern is, however, reversed for large damage ratios.

6. The standard deviation of the service ratio reaches a maximum when its expected value is 60-70 percent.

REFERENCES


