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Seismic reliability assessment of power transmission networks by simulation technique

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ABSTRACT: The study is concerned with the seismic risk assessment of power transmission networks, with reference to medium voltage networks having a regional dimension. Two kind of procedures are compared: the first one is valid if the serviceability criterion is expressed in terms of a connectivity requirement between the source nodes and the demand nodes; the second, based on a Monte Carlo simulation improved with an importance sampling technique, can be used for the reliability assessment of complex systems, and in order to take into account the real capacitative nature of the network. The practical feasibility of the two approaches is demonstrated through an application to a network in Central Italy, located within reach of four seismogenetic regions, and surrounding the town of Pescara.

1 INTRODUCTION

The study, that represents an extension of a previous one (Ciampoli *et Al.*, 1991), is concerned with the seismic risk assessment of power transmission networks.

As for any lifeline system, functional and structural redundancy, spatial extension, and the presence of a vast number of vulnerable elements are the primary features of a power distribution system, this being in fact a multiply-connected network, having at least a regional extension, and comprising a large number of components.

Attention is mainly focussed on medium voltage networks, for which the assumption is made that vulnerability lies essentially in the (power generating, branching, connecting and transforming) stations, the transmission lines being seismically much safer.

For simplicity, the further assumption is made that the stations, which are by themselves complex redundant systems, made up of electrical, mechanical and structural elements, can only be either fully functional or failed. Consequently, the fragilities of the stations are defined as the probability of complete loss of capacity.

Actually however, each station may link many transmission lines; therefore, the alternatives corresponding to having cut off only some of the connections between these lines must be considered. The resulting multistate behaviour of the station can be still analyzed in terms of a binary state function, if the station is represented by an appropriate assemblage of the binary elements corresponding to all the functional connections between input and output lines.

In (Ciampoli *et Al.*, 1991) the serviceability criterion has been stated in terms of simple connectivity among the "supply" or "sink" nodes, located along the contour of the examined portion of the network, and a number of selected "demand" nodes (the "objective" stations), disregarding the capacitative nature of the network.

Instead, a more realistic formulation of the serviceability criterion should correspond to satisfy a quantitative, multiple-target type of requirement, expressed by the ability to carry an amount x of electricity to a demand node X, *and* an amount y to a node Y, and so on.

Since the power which is available at the sources, and which can be transported throughout a line is limited by the electrical characteristics of the network, the simple connectivity requirement is not sufficient to guarantee this quantitative criterion. Furthermore, the state function of the system cannot be expressed as a simple function of the state of the stations, in the latter case: in fact, given the collapse of a certain number of stations, and after checking the connectivity between the source node(s) and the demand node(s), a global circuit analysis of the damaged network must be performed to determine the power available at each objective site, and then to evaluate the survival of the system.

Finally, it must be considered that the behaviour of electricity-carrying networks is distinguished by the peculiar characteristic that the flow in the various branches can be varied accordingly to the global amount and spatial distribution of the demand, so that flow inversion may well occur in some branches and through some nodes.

Therefore, it seems very difficult to get an analytical solution of the more general problem, by describing the system behaviour in terms of an evolutive state function; perhaps then, the only useful approach is the one that utilizes a Monte Carlo procedure, involving the simulation both of the seismic activity of the sources affecting the objective sites, and of the response of the power network vulnerable components.

By means of this procedure, each resulting network configuration can be readily verified with respect to any selected serviceability criterion. In fact the simulation approach can be used in conjunction with both the connectivity and the quantitative criterion, the only difference being in the greater computational demand of this latter.

To verify its practical feasibility, a Monte Carlo simulation procedure, whose efficiency has been optimized through the use of an importance sampling technique

(Harbitz, 1983), has been developed.

In the following the analytical procedure for the seismic reliability assessment of a power transmission network with regard to the simple connectivity requirement is firstly summarized, this step being always necessary to examine the possibility of carrying some electricity flow to the objective(s) site(s); then, the Monte Carlo simulation procedure is illustrated, along with the results of a comparative numerical test regarding a portion of the actual network located in Central Italy. To verify the efficiency of the Monte Carlo simulation, a connectivity serviceability criterion has been selected, since exact results are available for this case.

2 FORMULATION OF THE CONNECTIVE MODEL

2.1 Functional logic of the network

If the serviceability criterion is formulated in terms of simple connectivity, the functional logic of the network can be described in terms of minimal path sets (MPS). In fact, as seriality prevails on redundancy, as it is the case of medium voltage networks, the number of MPS's is less than that of minimal cut sets, and hence the MPS approach is computationally more economical.

The survival of the system can then be defined as the state for which one or more selected objectives are connected with the supply nodes, as connectivity automatically allows any required amount of flow to be carried.

Therefore, the problem of finding the MPS's for the system is equivalent to that of finding all the minimal physical paths connecting the source with the objective(s): the survival of all the stations along a minimal physical path guarantees the survival of the system, and then defines a MPS.

The algorithm for finding all the MPS's of a given network has been set up in (Ciampoli *et Al.*, 1991), and it has proven to be efficient for single and multiple objectives and also in the case of complex topologies and doubly oriented links among the stations.

As a special feature in comparison with other algorithms derived by graph theory, the proposed algorithm finds the MPS's related to n objectives through the reduction of the paths found for the single objective.

In fact, the MPS's for each objective are found first; then the sum of any combination of n MPS's (one per objective) is a path set for the multiple objectives case. The elimination of the redundant path sets yields all and only the MPS's for this case.

The adopted formulation of the serviceability criteria and the binary state assumption for the vulnerable elements lead to a state function S of the whole system also of the binary type. Denoting by S_{ij} the state function of the i-th element of the j-th subsystem, the minimal path sets representation of the system state is:

$$S = 1 - \prod_{i=1}^{n_p} \left[1 - \prod_{j=1}^{n_{pi}} S_{ij} \right] \tag{1}$$

The survival probability of the system is equal to the expected value of S. If, given epicentral location and intensity, the failures of the vulnerable elements can be assumed as independent events (this assumption appears logical, because the elements are usually several kilometers apart and structurally different, although some of the basic electrical components are standardized), the expected value of S is expressed by the relation:

$$P_S = \sum_{i=1}^{n_p} \prod_{j=1}^{n_{pi}} E[S_{ij}] - \sum_{i=1}^{n_p - 1} \sum_{j=i+1}^{n_p} E[S_i S_j] + \sum_{i=1}^{n_p - 2} \sum_{j=i+1}^{n_p - 1} \sum_{k=j+1}^{n_p} E[S_i S_j S_k] - \dots\dots \tag{2}$$

where:

$$S_i = \prod_{h=1}^{n_{pi}} S_{ih} \tag{3}$$

is the state function of the i-th path.

The value of P_s can be evaluated by expression (2) only if the total number of MPS's is not greater than 7+8: otherwise, the computational effort would be excessive. Alternatively expression (2) can be bounded by the second-order bounds proposed by (Ditlevsen, 1979), according to the inequalities:

$$P_S \geq P_{S1} + \sum_{i=2}^{n_p} \max \left\{ 0,\ P_{Si} - \sum_{j=1}^{i-1} E[S_i S_j] \right\}$$

$$P_S \leq \sum_{i=1}^{n_p} P_{Si} - \sum_{j=2}^{n_p} \max_{k<j} \{ E[S_i S_k] \} \tag{4}$$

where:

$$P_{Si} = E[S_i] = E\left[\prod_{j=1}^{n_{pi}} S_{ij} \right] = \prod_{j=1}^{n_{pi}} E[S_{ij}] = \prod_{j=1}^{n_{pi}} P_{Sij} \tag{5}$$

$$E[S_i S_j] = E\left[\prod_{h=1}^{n_{pi}} S_{ih} \prod_{k=1}^{n_{pi}} S_{jk} \right] = \prod E[S_{ir}] = \prod P_{Sir} \tag{6}$$

with the last product extending to all different elements of the paths i and j.

2.2 Seismic reliability assessment

The seismic hazard model is the classical Cornell model (1968), which considers diffused seismic sources. The activity of each seismogenetic area is described by the doubly-truncated Gutemberg-Richter (G.R.) expression, whose parameters are evaluated on the basis of the historical seismicity. In fact catalogue data are first filtered to eliminate fore and after-shocks; then, having determined the equivalent completeness period for each intensity, a maximum likelihood procedure (Weichert, 1980) is applied to evaluate these parameters.

The site intensity I_s is related to the source intensity by site-specific non circular attenuation laws, that include also a random term ε describing the scatter around the mean. The attenuation laws are of the form (Ortolani and Giannini, 1988):

$$\Delta I_s(r,\theta) = a + b\,(r+r_0) + c(\theta) \log (r+r_0) + \varepsilon \tag{7}$$

where:

$$c(\theta) = c_0 + \sum_{k=1}^{n} (c_{2k-1} \cos k\theta + c_{2k} \sin k\theta) \tag{8}$$

with a, b, r_0, c_0, c_i constant parameters, and r, θ the geographical polar coordinates of the site measured with reference to the epicenter.

The random variable ε is assumed as $N(0, \sigma)$, and two different sets of constants are introduced in the attenuation law, depending on the epicentral intensity.

Therefore, given an event of intensity I_Q, occurring within a small area Q of a generic seismogenetic area, the local intensity at all nodes of the system is evaluated through expression (7); using the fragilities of all the vulnerable elements, the conditional probabilities of failure $P_{fkj}(Q, x_Q)$ of the j-th element of the k-th path are evaluated by convolution with the corresponding density function of the intensity, and then the conditional probability of failure of the system $P_f(Q, x_Q)$ according to expressions (2) or (4).

Finally, the total probability is obtained by sequential convolutions with the density functions of the intensity and position of the epicenter.

3 THE MONTE CARLO SIMULATION

The alternative approach followed in this study is based on the reliability assessment by simulation technique. The Monte Carlo procedure, that has been implemented for the evaluation of the yearly failure probability of the network, can be summarized in the following steps:

1. For any seismogenetic area repeat steps 2 to 8.
2. For any subarea, derived from a close-meshed discretization of the current area, repeat steps 3 to 8.
3. Generate - from a uniform p.d.f. - n values of the intensity of an event with source in the current subarea, and for each value repeat steps 4 to 8.
4. Select a station to be verified.
5. Having generated a value of the error associated to the attenuation law (from the corresponding normal p.d.f.) evaluate the resulting intensity of the seismic event at the site of the selected station.
6. Generate a value of the strength of the station, this being expressed by the value of the maximum seismic intensity which the station can withstand.
7. Verify the status of the station (failed or survived).
8. Based on the status of the stations which have been already verified, determine the status of the network (failed, survived or undefined) according to the selected serviceability criterion. If the status is undefined, go to 4.
9. After completing the loops defined in steps 1, 2 and 3, compute the failure probability of the network from the number of system failures, weighting the contribution of each sample. The weight, given by: $h(i) = G(i)/U(i)$, reflects the use of a rectangular, $U(i)$, instead of the Gutemberg-Richter (i.e. truncated exponential) p.d.f., $G(i)$, for the simulation of each seismic event.

The best estimator of the failure probability of the system is then equal to:

$$E[P_f] = \sum_{r=1}^{n_a} \frac{N_{a0}(r)}{A(r)} \sum_{j=1}^{n_s} \Delta A_j \sum_{k=1}^{n} D_{i_k}\, h(i_k) / n \qquad (9)$$

where:

- $D_{i_k} = 1$ if the network is failed, and $D_{i_k} = 0$ otherwise;
- i_k is the k-th value of the seismic intensity at the j-th subarea, (with area ΔA_j) of the r-th seismogenetic area, of area $A(r)$;
- $N_{a0}(r)$ is the yearly mean number of seismic event occurring within each seismogenetic area.

The standard deviation of this estimator, relative to each epicentral location, is expressed by the relation:

$$\sigma[P_f] = \sqrt{\frac{(\Sigma\, h(i_k))^2 - \Sigma\, h(i_k)^2/n}{n(n-1)}} \qquad (10)$$

If the safety check is related only to the connectivity requirement, it is possible to substantially improve the computational efficiency of the algorithm if the stations are verified in a specific order. The objective is to determine the status of the network with the verification of the status of the least number of stations.

A rational choice would be that of selecting a station which belongs either to the path set or to the cut set which has the highest probability of failing or surviving respectively. But this requires the preliminary knowledge of the failure probability of the stations which could be estimated a priori and then updated as the simulation proceeds. To limit the computing time, for a cut set representation of the functional logic of the network, only the cut sets of lower order should be determined.

Alternatively, a simpler euristic approach, which has been used in this implementation, is that of giving priority to the path sets with fewer stations, and, within each of them, of choosing the stations which belong to more path sets. This method does not require the evaluation of the cut sets, and it has proved to be particularly efficient in the comparative cases illustrated in the following.

4 COMPARATIVE EXAMPLE

In Fig. 1, the area of interest for the pilot application of the procedures for the seismic reliability assessment of a power transmission network with respect to both serviceability criteria is shown.

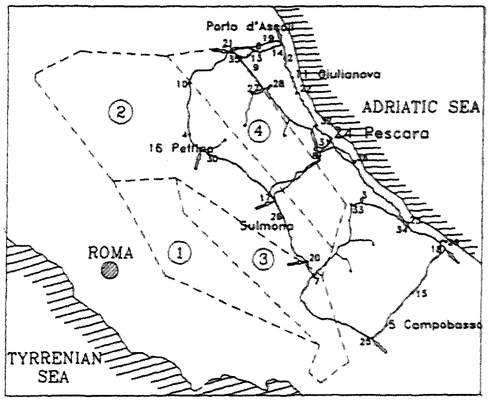

Figure 1. Medium voltage power distribution network in Central Italy, with the four seismogenetic areas.

It includes a portion of the Italian medium voltage network, centered around a number of sites near the town of Pescara, and comprising 37 stations.

The extension of the examined portion of the power transmission network has been delimited such as to satisfy two requirements: to ensure that the supply of electricity in the sites of interest (the objectives) is practically independent from the state of the rest of the network; to guarantee that the excluded portion is sufficiently away from the seismic sources affecting the objectives, such that its failure for an event occurring within these sources has negligible probability.

As concerns the seismic risk model, it has been evaluated (Giannini *et Al.*, 1984) that only four seismogenetic area sources (roughly depicted in Fig. 1) can affect the objectives.

The activity of these sources has been modeled in (Giannini *et AL.*, 1984): the parameters of the G.R. law, in terms of I_{MM} intensity, are summarized in Table 1, where I_0 ed I_m represents the lower and the upper truncation values for each of the four seismogenetic areas.

The separation value in the attenuation law has been set at: I = 8; while, with reference to expression (8), for each area, eight coefficients c_i have been evaluated, and r_0 has been taken equal to 3 km. The coefficient of variation of ε has been set up equal to 0.20.

Table 1. Parameters of the G.R. law for the four areas

AREA	α	β	I_0	I_m
1	0.77	1.214	5	10
2	0.60	1.035	5	12
3	1.14	0.857	5	11
4	0.51	1.197	5	10

Three different cases have been considered: two single objectives (stations 24 and 32), and one multiple objective including both.

In Fig. 2, a schematic representation of the network is given; moreover the orientation of the links in the network, describing a typical service condition for the system, and the MPS's corresponding to a single objective (station 24) are reported.

In order to examine also the effect of the system capability of adapting itself to variations in power demand and to eventual failures of the stations, and consequently to take into account also that the directions of flow in the various branches are not univocally fixed, the assumption that the electric power can flow in either directions among any two connected nodes is made.

Obviously, the reliability of the system characterized by this behaviour (to which corresponds the case denominated of "reflexive" network) provides an upper bound of the reliability of an optimally adaptative electric power network.

Therefore, the previous three cases have been repeated with the assumption of reflexive network.

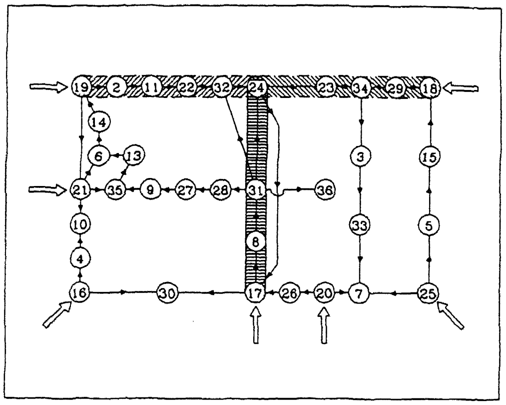

Figure 2. Schematic representation of the network; pointed out the MPS's relative to objective 24 (for the not reflexive network case).

The fragilities of the stations, whose exact evaluation is the subject of separate experimental and analytical studies, has been given the following expression:

$$P_{fs}(x) = \alpha \left(e^{\gamma I} - 1\right) \qquad (11)$$

that is a simple exponential distribution defined by the two parameters: $\alpha = 0.275 \cdot 10^{-4}$; $\gamma = 0.875$.

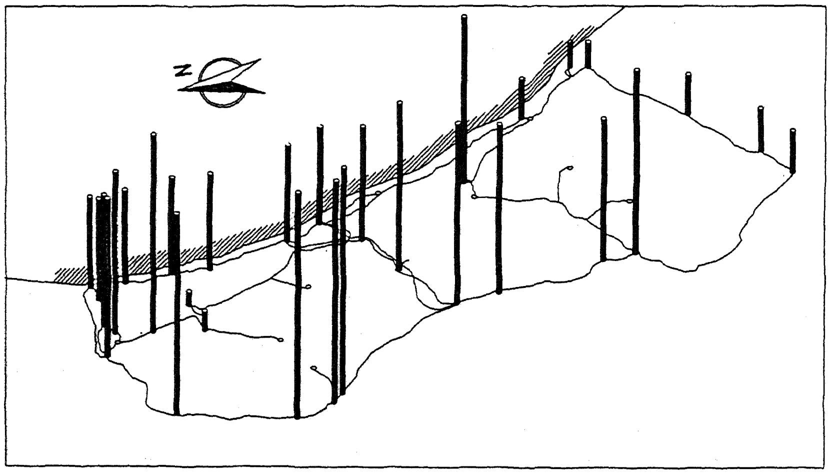

Figure 3. Probabilities of failures of each station.

Table 2 - Results of the analysis.

Objectives	*Doubly oriented links*	*No. of MPS*	*No. of sites*	P_f *(rel. 2)*	P_f *(rel. 3)*	P_f *(MC sim.)*
24	NOT	3	13	$.118 \cdot 10^{-2}$	$.22 \cdot 10^{-3} \div .12 \cdot 10^{-2}$	$.106 \cdot 10^{-2}$
32	NOT	2	8	$.134 \cdot 10^{-2}$	$.13 \cdot 10^{-2}$	$.107 \cdot 10^{-2}$
24 - 32	NOT	2	9	$.251 \cdot 10^{-2}$	$.25 \cdot 10^{-2}$	$.198 \cdot 10^{-2}$
24	YES	6	24	$.118 \cdot 10^{-2}$	$.12 \cdot 10^{-3} \div .12 \cdot 10^{-2}$	$.102 \cdot 10^{-2}$
32	YES	7	24	$.126 \cdot 10^{-2}$	$\leq .39 \cdot 10^{-2}$	$.103 \cdot 10^{-2}$
24 - 32	YES	6	23	$.243 \cdot 10^{-2}$	$.60 \cdot 10^{-3} \div .24 \cdot 10^{-2}$	$.186 \cdot 10^{-2}$

The corresponding marginal probabilities of failure of each station, evaluated according to the previously defined seismic risk model, and to the data reported in Table 1, are plotted in Fig. 3.

The results of the analyses carried out are reported in Table 2. In the Monte Carlo simulation, for each epicentral location 500 values of seismic intensity have been considered.

Looking at the results reported in Table 2, it appears evident that the reliabilities that are evaluated by applying the inclusion-exclusion principle, result approximately equal to the corresponding values of the upper bound proposed by Ditlevsen (1979). In the second and third case, relative to the single objective (station 32) and to the double objective (stations 24 and 32), the number of MPS's is equal to two, and therefore the only value resulting from expressions (4) coincide with the exact one.

It can be observed that the probability of failure evaluated considering the reflexive network case and a single objective (station 24) is equal to the value derived by considering the non reflexive case, and assuming the orientation of the flow in the branches that corresponds to a standard service condition.

This suggests that the redundancy introduced to consider also the adaptative capacity of the network has no significant effects on the system reliability. If the three additional paths that corresponds to the reflexive case are examined, it results that they include a larger number of vulnerable elements than the first three just examined in the non reflexive case: this explain their negligible contribution to the global system reliability.

Similar conclusions can be derived also for the cases of the single objective represented by the station 32, and of the double objective, for which the possibility of varying the directions of flow has negligible influence on the system reliability.

As concerns the simulation procedure, it gives results that are well in agreement with the exact ones.

The computational effort is also comparable: in fact the computing time is about $1.5 \div 2$ times greater than the computing time required by the application of expression (2) (that varies between 5 and 10 hours on a 386-33 Mhz personal computer), and about 3 times greater than the time required by the application of expression (4).

5. CONCLUSIONS

As just noted, the proposed procedures for the seismic reliability assessment of power transmission networks can be applied with a reasonable computational effort to a system having a regional dimension and moderate redundancy: the first is able to handle the problem when the serviceability criterion is expressed in terms of a connectivity requirement; the second needs to be applied when the actual capacitative nature of the network has to be examined.

As concerns the former procedure, the minimal path sets approach has proven to be efficient for single and multiple objectives, and also in the case of complex topologies.

Furthermore, for the case of reflexive networks, the ease of identification of the MPS's makes it possible to estimate an upper bound of the system reliability, taking into account the adaptability of the network to local failures.

As concerns the Monte Carlo simulation technique, that is evidently more onerous, the following advantages can be pointed out:

- without any significant effort, it can be applied to the reliability assessment of networks anyhow complex, i.e. that are characterized by a multistate behaviour, and therefore, cannot be interpreted according to the minimal path sets or minimal cut sets representations;
- the difference between the computational effort required by the two approaches tends to vanish as the redundancy of the examined network increases.

In fact the number of terms that must be evaluated in applying expression (2) goes up exponentially; therefore, the inclusion-exclusion principle becomes rapidly inefficient to evaluate the probability of the intersection between several events.

On the other hand, the bounds expressed by relation (4), tends to become too large in same cases, as pointed out in Table 2.

Even if it is possible to adopt some tricks to improve the accuracy of these bounds, it must be observed that the computational effort required by the Monte Carlo simulation increases only linearly as a function of the number of the stations; thus, this approach can result competitive for the reliability assessment of actual systems, also to verify the simple connectivity requirement.

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