Notes on subsystems sizing criteria in active aseismic protection of strategic buildings

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ABSTRACT: The last 20 years have seen major theoretical advances in active structural control (ASC) for civil engineering applications. Over the last decade tests on reduced scale physical models have also undergone notable development.
The active alternative offers a number of distinct advantages for protection against brief and intense disturbances occurring at random during the life of a construction.
The state space environment is used to discuss a criteria for sizing and management of the control law of a general electrohydraulic subsystem for active structural control in civil applications. A preliminary physical feasibility assessment of the proposed approach is developed.

INTRODUCTION

The possibility of using active structural control (ASC) technology for protecting civil structures under extraordinary horizontal (seismic or wind) loads has been discussed in the literature for at least twenty years.

Traditional antiseismic protection usually involves elastic design in compliance with serviceability limit states in the expectation of moderate earthquakes (ground acceleration 4-6 times weaker than the maximum foreseeable). Under severe earthquakes, energy is dissipated by plastic deformation with inevitable, possibly serious, structural damage. The active system can be viewed as an alternative approach to building safety for cases where continued functioning is all-important, for example in "1st class" buildings, which must remain usable in a post-earthquake emergency.

In the event of a relatively brief but intense horizontal disturbance, an on-line servodrive which intervenes, adapting its control action to the level of the disturbance, could constitute an economically worthwhile, high performance mechanical alternative to traditional protection. Costs will be addressed specifically later.

Technical contributions and simulation studies in this field are numerous, and certain sophisticated active solutions are already finding application in high technology areas such as aeronautics and space. The building industry is much less receptive to new ideas and even slower in initiating pilot projects; however there has undoubtedly been a widespread increase in interest in the active approach over the past five years. We feel that full-scale ASC projects could be imminent, but schemes must be based on imaginative engineering solutions which combine innovative ideas and traditional building techniques; in that way they will attract the attention of building investors and public administrators concerned with civil protection.

An attempt must be made, in particular, to summarise standard criteria for pre-sizing components and subsystems, stating simple relations between known global structural parameters and the characteristic variables of an active plant: this would provide the design engineer with qualitative and quantitative guidelines for evaluating the effectiveness of the active alternative when approaching protection in structural engineering, so he is a able to make a rational choice between various active and passive solutions.

MORPHOLOGY. THE NEED FOR DESIGN STANDARDS IN BUILDINGS WITH ACTIVE INSTALLATIONS. CONTROL STRATEGIES.

Active (and passive) structural control systems may be schematically divided into:
(i) those which artificially adjust the stiffness of the structure;

(ii) those which artificially control the damping.

In the first case, with reference to a simplified one degree of freedom structure, the stiffness is artificially controlled by the application of a corrective force whose time-history is instantaneously proportional to building lateral displacement. The displacement signal comes from transducers, is amplified and manipulated in the controller and sent to drive the actuators. If the latter are placed at the base of the building, the corrective action is transmitted to the upper floors by auxiliary structures (e.g. pre-stressed cable systems). In the second case the control time-history is implemented by means of an action which is instantaneously proportional and opposite to building sway velocity, thus imparting artificial damping to the structure. For the protectin of r.c. multistorey residential buildings, the active plant must be compact, have high MTBF and be attachable to the r.c. frame without extensive retrofitting.

For r.c. rectangular-plan, low or medium rise buildings (up to 9 stories) of type in common use in southern European seismic zones, a practical solution is to attach active plant in each end wall - which thus become "active" walls, AW. Such AWs consist of two conceptually distinct parts: the power subsystem located in the base and consisting of power unit, pump, wich command the auxiliary tendons, and the auxiliary bracing system itself (ABS), which receives the actions of the actuators at the base and transmits them to the building superstructure. When these actuators are clamped or when the lower ends of the ABS are fixed to the building foundations, the system reduces to a passive wall. The following conclusion can be stated : practical design specifications for the active system and ABS are (a) all the hydraulic and mechanical equipment located on the ground floor (or below) of the building; (b) active control structures to be distributed as widely as possible throughout the entire building; (c) seismic energy dissipated by plastic deformation of elements of the bracing system, should the control system not be activated or whenever the energy imparted by the earthquake exceeds that the system was designed to cope with (>>30% g), possible configurations for an ABS are: (i) a "double fan" cable structure with or without pretension, (ii) r.c. walls fitted with active tendons acting vertically, (iii) as in (ii) but with tendons acting curvilinearly. Our preliminary feasibility investigations have indicated that case (i) plus pre-stressed tendons, offers a number of practical advantages in terms of cost, easy installation, easy inspection and maintenance: more detailed investigations were therefore carried out on it. In such a structure the two "fans" of cables are simultaneously and symmetrically pre-stressed so that they are never subjected to compressive forces during sustained seismic disturbance. The stressing operation is performed once only during installation (there could be a check 2-3 years after installation to take up viscosity and relaxation slack) and has the consequence that high compressive stresses are applied to the corner columns and to the beams bounding the floors. For the case of a typical r.c. 9 storey residential building, 30x12 m plan, greatest pre-stressing, at around 80 tonnes, would be applied to the longest cable; this would reach 160 tonnes under a disturbance of 30% g, with the symmetrically opposite tendon becoming tensionless.

Pre-tensioning produces a compressive stress of about 600×10^3 Kgf at the base of the corner columns, it therefore allows the total force exerted by the active control system to be transferred from the four points at the base of the four corner pillars to a total of 36 points (4 per floor) thus distributing the total force on the structure over a wide area. Pre-tensioning the cable fans also permits conformity to the minimum stiffness condition for each fan and this in turn stabilises the active control system without the need to oversize the cables (otherwise by a factor of 2) and attenuates the overall delays in control transmission. Two design variables may be varied to optimise the response of the structure: (i) the axial stiffness of each cable may be varied without changing the overall rigidity of the fan; (ii) the angle of inclination of the actuating tendons to the horizontal.

Both these parameters significantly influence structure response.

Cable size is determined more by axial rigidity than strength and it is not necessary to use cable-quality steel ($\sigma = 180$ Kgf/mm^2); steels normally employed for steel construction are sufficient, with the stays in bar- or pressed-steel (with one or two intermediate joints). The high ductility of these steels confers considerable passive ductile strength to the structure.

Numerical simulations permit the following conclusions:

(a) The passive effect provided by a pre-tensioned ABS installed at each end of a building is sufficient to protect the serviceability limit state (building response always linear elastic) under a peak disturbance of 0.1 g.

(b) These ABS systems are also able to

protect, via steel plasticisation, the ultimate limit state of the structure (though it sustains considerable deformation and damage) under a severe earthquake of say, 0.33 g at peak. It also provides emergency protection in the event that the active control system fails to operate.

(c) As part of a closed loop active protection system the ABS system constrains the building response within the elastic range (serviceability limit state) under an earthquake with peak at 0.33 g.

Two factors which might encourage the interest of construction companies and design professionals in ASC, and hence promote the start to ASC field trials are, in our opinion: (i) the establishment of integrated design standards for active-protected buildings; (ii) the establishment of standardised criteria for active control onset and management in a wide range of situations, from which a specific field strategy may be chosen.

An outline of a possible standard procedure for the design of active-protected buildings is: (a) The designer begins by considering a basic ideal building designed to respond elastically to a standard wind excitation of pressure ≈ 100 Kgf/m^2 or to a weak earthquake ($<0.05g$). (b) The building is modified by installation of an ABS in each of the end walls. (c) The active servodrive is math-modelled and sized using the strategy that the uncontrolled peak displacement of the building due to wind or weak earthquake is assumed as the admissible displacement, Y_{adm}, under a 0.33g-peak earthquake and active control. (d) The 4th step consists of re-sizing certain structural elements and subsystems of the r.c. frame, adapting them to receive the control action without sustaining damage. In the case of the standard 30 x 12m plan, 9 storey, r.c. building made up of 3 longitudinal frames and 8 transverse frames held together by a rigid horizontal structure at each floor level, two end AWs would be installed. Assuming a peak ground acceleration of 0.33 g no modification of floor systems would be required. Within the floor systems, the energy is dissipated as follows: the inertial force of each floor is balanced by the resultant shear forces of all the columns and by the active and passive forces transmitted by the tendons converging to the 4 corners of that floor. Even if the entire inertial force of a floor were supported by the 4 corners, there would be no need to change floor system specifications from those employed in non-seismic areas. Modifications to other structural subsystems would be required:

(1) The end pillars would have to be considerably reinforced since they have to bear a load of approximately, 1100 tonnes, 12-14 times the static load.

(2) The boundary beams edging each floor would be subjected to an additional compressive force induced by the tension in the stressed tendons; this force is of the order of 50-60 tonnes per floor but acts eccentrically with respect to the beam axis. It would therefore be necessary, in order to re-centre the axes of the edging beams (to within the plane of the active wall), to elongate them, projecting them to the exterior and providing there holes to allow the tendons to pass through. In this way earthquake forces distribution from the tendons to the floor system takes place gradually utilising the whole length of a boundary beam.

(3) The foundations of the 4 corner pillars must be able to resist very high forces during a seismic disturbance in condition of equilibrium with the total building inertia force. These compression and traction forces are, in the case under examination, equal to about 1200 tonnes inclined an angle of 70° to the horizontal, and may be allowed for by employing inclined foundation piles resistant to traction and compression.

MATH-MODELLING

The attention is now focused on the lateral response of a wide structural and disturbance classes, hereinafter respectively modelled as vertical cantilever-type beam fixed at the base, and as a pulsating resonating load-density uniformely distributed along the height. We contribute criteria for designing and managing the control law implemented by an electrohydraulic subsystem which controls the 1st dominant mode of the structure: in this approach the natural dynamics (L) of that mode (pulsation ω and damping ξ) is progressively forced to track a given reference dynamics (L*), of parameters $\omega*$ and $\xi*$.

The following abbreviations are also adopted: motion (D) for the disturbed motion, without active control; motion (LC) for the motion (L) plus active control; motion (DC) for the motion (D) plus active control.

With a vertical x-axis upward oriented, let H be the height of the construction, $m(x)$ the mass density (unitary height), $P(x)$ the total weight upon the section of abscissa x, and $I(x)$ the law of the cross sectional moment of inertia.

Let the disturbance be modelled as a space and time square-wave, the latter being:

$$F_r(t) = F_r^* \{\sin \frac{2\pi}{T} \cdot t + \frac{1}{3} \sin \frac{6\pi}{T} \cdot t +$$

$$+ \frac{1}{5} \sin \frac{10\pi}{T} \cdot t\}$$

Let $M_a(t)$ be the time-history of the active control moment applied at height h ($h<H$), and $[t_o, t_f]$ the control time interval.

With the dominant mode $\psi(x) = 1 - \cos\alpha x$, ($\alpha = \pi/2H$) the stationary and in-phase dynamics $y(x,t) = \psi(x) \cdot Y(t)$ is driven by:

$$E \frac{\partial^2}{\partial x^2} (I(x) \frac{\partial^2 y}{\partial x^2}) + c_v \frac{\partial^2}{\partial x^2} (I(x) \frac{\partial^3 y}{\partial x^2 \partial t}) +$$

$$+ P(x) \frac{\partial^2 y}{\partial x^2} + P'(x) \frac{\partial y}{\partial x} + m(x) \frac{\partial^2 y}{\partial t^2} = \qquad (1)$$

$$= F_r(t) + M_a(t) \cdot \delta'(x-h)$$

in which $\delta'(dx)$ indicates the derivative of the Dirac spatial distribution $\delta(x)$, and c_v the internal damping.

After an integral transformation of (1), the modal lagrangian dynamics with disturbance and control results in:

$$\ddot{Y}(t) + 2\omega\xi \dot{Y}(t) + (\omega^2 + C)Y(t) = \frac{4}{\pi m} F_r(t) + B u(t);$$

$$Y(t_o) = Y_o, \dot{Y}(t_o) = V_o \qquad (2)$$

in which:

$$\xi = \frac{c}{c_{cr}} = \frac{c}{2m\omega} \quad ; \quad B = \frac{\pi \bar{K} b}{mH^2} \sin\alpha h \ ;$$

$$C = \frac{\pi^2 \bar{K} b^2}{2m \ H^3} \sin^2\alpha h$$

and $u(t)$, $\bar{K}$ and b are, respectively, the control force in output from the actuator, the structural stiffness possibly interposed between the actuator (e.g. the piston head of an hydraulic cylinder) and the construction, and the arm associated with the control moment $M_a(t)$.

If the following instantaneous state vector of the vibrating structure

$$\underline{x}(t) = \begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix} = \begin{pmatrix} Y(t) \\ \dot{Y}(t) \end{pmatrix} \qquad (3)$$

is introduced, the state-space open loop

dynamics is:

(LC): $\dot{\underline{x}}(t) = A \underline{x}(t) + \underline{b} u(t)$ (state motion)

$$w(t) = \underline{c}^T \underline{x} \qquad \text{(output transformation)} \qquad (4)$$

$\forall t \in [t_o, t_f]$; $\dot{\underline{x}}, \underline{x} \in R^2$; $u, w \in R$; $A \in R^{2 \times 2}$

with $\underline{b} = (0 \ B)^T$, $\underline{c}^T = (c_1 \ c_2)$ vector of sensor's gains and $a_{11} = 0$, $a_{12} = 1$, $a_{21} = -(\omega^2 + C)$, $a_{22} = -2\xi\omega$.

The control action $M_a(t)$ should shift the eigenvalues $\lambda_{1,2} = f(\omega, \xi)$ to a set of reference values $\lambda_{1,2}^* = f(\omega^*, \xi^*) \in \Sigma^*$, Σ^* being a pre-established region of the left complex half plane, which ensures no resonance and that stability conditions are met.

The control $u(t)$ must also minimize a suitable cost functional J (J-minimum control $\hat{u}(t)$, $\forall t \in [t_o, t_f]$ which steers (L) to (L*) and then keeps the deviation of (LC) from (L*) close to zero in $[t_o, t_f]$

CONTROL-LAW PREDESIGN

Let $\underline{x}^*(t) \in R^2$ and $A^* \in R^{2 \times 2}$, with the same characteristics of $\underline{x}(t)$ and A in (3) and (4), be the state vector and the natural matrix of the (L*) dynamics. The two dynamics (LC), given in (4), and (L*):

(L*) : $\dot{\underline{x}}^*(t) = A^* \underline{x}^*(t)$, $\forall t \in [t_o, t_f]$,

$$\underline{x}^*(t_o) = \underline{x}_o^* \qquad (5)$$

can be summarized in a single extended dynamics $\tilde{\Gamma}$ with state vector $\underline{v}(t) = \underline{x}^*(t) \cup \underline{x}(t)$ and output transformation

$\underline{e}(t) = \tilde{I} \underline{v}(t)$ with $\tilde{I} = [I_2 \ -I_2]$ ($I_2 \in R^2$

being the 2nd order unit matrix); the open loop (LC) dynamics is:

$$\tilde{(LC)} : \quad \dot{\underline{v}}(t) = \tilde{H} \underline{v}(t) + \tilde{\underline{h}} u(t) \qquad (6)$$

$$\underline{e}(t) = \tilde{I} \underline{v}(t) \qquad (6')$$

being:

$$\tilde{H} = \begin{pmatrix} A^* & 0 \\ \hline 0 & A \end{pmatrix} \in R^{4 \times 4} \ ; \quad \tilde{\underline{h}} = \begin{pmatrix} \underline{o} \\ \hline \underline{b} \end{pmatrix} \in R^4 \ ;$$

$0 \in R^{2 \times 2}$, $\underline{o} \in R^2$ $\forall t \in [t_o, t_f]$, $\underline{v}(t_o) = \underline{v}_o$

The dynamic problem can be traced back to that of minimizing the functional:

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$$2J = \int_{t_o}^{t_f} [\underline{v}^T(\widetilde{I}^T\widetilde{QI})\underline{v} + RR\, u^2(t)\,]\, dt \qquad (7)$$

with the constraints (6) and (6'), in the time interval $(t_f \cdot t_o)$, with $t_f < \infty$.

From the minimum conditions on the hamiltonian HH of the system ($HH_{/u} = 0$ and $HH_{/uu} \geq 0$) it follows the HH-minimum control (see Carotti, 1983):

$$\overset{*}{u}_c(\underline{v}, \underline{\mu}|t) = -(1/RR)\,\widetilde{\underline{h}}^T\underline{\mu} \qquad (8)$$

From (8), the finite-time optimal control is

$$\hat{u}_c(t) = -\underline{l}^T(t) \cdot \underline{v}(t) \qquad \text{with}$$

$$\underline{l}^T(t) = (1/RR)\,\widetilde{\underline{h}}^T P(t,0,t_f), \qquad (9)$$

a linear combination of the auxiliary state-variables $v_i(t)$ with time-dependent coefficients.

The strategy $(t_f \to \infty)$ J-minimum control $\hat{u}_r(t)$ is obtained by utilizing in (9) the backward steady state solution of the Riccati matrix eqtn for the $t_f < \infty$ case, with a positive definite Q (see Carotti, 1983):

$$\hat{u}_r(t) = -\underline{l}^\bullet \cdot \underline{v}(t) \quad \text{with } \underline{l}^{\bullet T} =$$

$$= (1/RR)\,\underline{h}^T P^\bullet; \quad \underline{l}^\bullet \in R^4 \qquad (10)$$

Finally (10) becomes: $\hat{u}_r(t) = -\underline{K}^{*T} \cdot \underline{x}^*(t) - \underline{K}^T \cdot \underline{x}(t)$, which is made up of two signals: a linear combination of $\underline{x}^*(t)$ elements by time independent vector $\underline{K}^* \in R^2$, and a linear combination of the state of the vibrating structure by the time independent vector $\underline{K} \in R^2$.

NUMERICAL RESULTS

Units adopted: [m] , [s] , [kg] ; [N]. When we assume:
(L) : $\omega = 5.66$ rad/s, f=.901 Hz,
    T =1.11 s, ξ =.055

(L*): ω*= 4.46 rad/s, f*=.71 Hz,
    T*= 1.4 s, ξ*=.012
the (L) and (L*) responses respectively are:

$$x_1(t) \approx .127\, e^{-.028t} \cos 5.66t \quad ;$$

$$x_2(t) \approx .72\, e^{-.028t} \sin 5.66t$$

$$\overset{*}{x}_1(t) \approx .127\, e^{-.053t} \cos 4.46t \quad ;$$

$$\overset{*}{x}_2(t) \approx -.568\, e^{-.053t} \sin 4.46t$$

The strategies [RR, Q(q,q)] considered has been the following:
1) [RR= 1,Q(10,10)] ; 2) [RR=1,Q(100,100)];

3) [RR= 1,Q(1000,1000)]; 4) [RR=2,Q(10,10)]

a) The (LC)—>(L*) tracking: starting from initial conditions $\underline{x}^*(t_o) = (.127\ \ .0)^T$ each

of the three strategies provides a satisfactory tracking of the model, with a speed of convergence ranging from few seconds up to about ten seconds. With [RR=1, $Q=100\cdot I_2$] the (LC) response is

$$x_1(t) = -.001\, e^{-2.46t} + .0003\, e^{-52.t} +$$

$$+.1279\, e^{-.053t} \cos 4.46t + .004\, e^{-0.53t} \sin 4.46t$$

$$x_2(t) = .003\, e^{-2.46t} - .01\, e^{-52.t} +$$

$$+0.12\, e^{-.53t} \cos 4.46t - .57\, e^{-.053t} \sin 4.46t$$

Strategy [RR=2, $Q=10\cdot I_2$], affords an approximately 10% reduction of the peak values of $\hat{u}_r(t)$, as compared with those required in [RR=1, $Q=10\cdot I_2$] and of almost 20% as compared with those in [RR=1, $Q=100\cdot I_2$].

b) the (DC)—>(L*) tracking. Assumptions:

$$\underline{x}^*(t_o) = (.03\ \ .0)^T$$

$$x^*_1(t) = .03\, e^{-0.53t} \cos (4.46\ t);$$

$$x^*_2(t) = -.13\, e^{-.053t} \sin (4.46\ t)$$

We suppose that the (D) resonating dynamics is observed in $[0, t_o[$ until $x_1(t_o) = x^*_1(t_o) = .03$; at that point the (DC)—>(L*) tracking begins and evolves in $[t_o, t_f]$:

$$x_1(t) =$$

$$= e^{-.028t}\,(.58 \cos 5.65t + .07 \sin 5.65t) +$$
$$-.55 \cos 5.66t - .07 \sin 5.66t$$

(D) .

$$x_2(t) =$$

$$e^{-.028t}\,(.38 \cos 5.65t - 3.28 \sin 5.65t) +$$
$$-.38 \cos 5.66t + 3.11 \sin 5.66t$$

$$x_1(t) = .03 \ e^{-.053t} \cos 4.46t$$

(DC): $\qquad$ with [RR=1; Q=100·I_2]

$$x_2(t) = -.13 \ e^{-.053t} \sin 4.46t$$

This approach allows control energy to be saved which would otherwise be spent in steering $\underline{x}(t=0)$ closely approximate to $\underline{x}^*(t_o)=(.03 \ .0)^T$ and the times required for convergence with (L*) are of the order of those found for (LC) —> (L*) tracking, where the initial conditions were much less favorable.

Once ω has been transferred to ω^* the control peak levels on the no longer resonating structure, are much lower than the initial ones.

Finally, power absorption on the onset of control was estimated to be rapidly decreasing. It goes without saying that if the most advantagenous (L*) model and the [RR,Q] strategy are developed for each case, the levels thus far considered can be reduced considerably though still respecting the rapid convergences to (L*).

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