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Seismic design response of rotating machines

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ABSTRACT: An efficient response spectrum approach is proposed for calculating the seismic response of systems with general matrices such as rotating machines. The approach requires the left and right eigenproperties of the system. It is, however, not necessary to calculate the complete set of eigenvalues and eigenvectors as the eigenproperties associated with high frequencies are not explicitly used; their contribution to the response is included by a simple pseudo static analysis. The validity of the proposed response spectrum approach is established by a numerical example where the results obtained by the proposed approach are compared with those obtained for an ensemble of time histories.

1 INTRODUCTION

Rotating machines such as motors, generators, turbines, pumps, etc. are among the most common mechanical components of an industrial facility such as a nuclear power plant, chemical plants and refineries. They usually serve critical operating functions in these facilities. Therefore, they must be carefully designed and evaluated if seismic motions are expected to occur at the sites of the facilities. In the last decades, several studies have been performed to examine the effect of seismic motions on rotating machines. See, for example, Asmis and Duff (1978), Hori (1988), Iwatsubo et al. (1979), Srinivasan and Soni (1984).

The structures as well as stationary mechanical components of important facilities such as nuclear power plants are usually designed for seismic motion defined by ground response spectra such as those prescribed by the Regulatory Guide No. 1·60 (U.S. Nuclear Regulatory Commission, 1975). To obtain the design response for seismic input defined in this form, response spectrum approaches have been developed and are now commonly used. The response spectrum approaches have, however, not been available for rotating machines until very recently. It is primarily because the rotating systems do not permit their analysis by a conventional mode superposition approach, as their system matrices are asymmetric. Utilizing a generalized modal analysis approach, the writers (Singh et al. 1991) have recently developed a response spectrum approach for calculating the design response of these machines for inputs defined by design response spectra. This recently proposed approach, however, can be further improved, especially in view of the fact that the rotating machines usually have several modes with rather quite high frequencies. The effect of these high frequency modes can be included through a simple static type of analysis. A method to do this was proposed by Maldonado and Singh (1991) for conventional structural systems. Here, it is shown that this method can also be extended for systems with general matrices such as rotating machines. A response spectrum method utilizing the improvement is developed. The method considers the correlated multiple components of the base input motion in the calculation of the maximum design response.

2 ANALYTICAL FORMULATION

Using the variational methods, the equations of motion for rotor–disk–bearing systems have been derived by Suarez, et al. (1991). These equations can be written in the standard form as:

$$[M]\{\ddot{x}\} + [C]\{\dot{x}\} + [K]\{x\} = \{F(t)\} \qquad (1)$$

where [M], [C], and [F] are the mass, damping, and stiffness matrices; {x} is the displacement vector, measured relative to base and; {F(t)} is the forcing function vector. The mass matrix is similar to the mass matrices in conventional structural systems. The damping and stiffness matrices are, however, quite different from those of the conventional structures. The damping matrix contains usual symmetric part. The damping terms coming from the fluid film bearing are also symmetric. They depend on the size of the shaft, bearing, clearance, rotational speed and viscosity of the fluid in the bearing. These terms are usually large, and provide motion stability to the rotor–shaft system. There are also skew symmetric terms which occur because of the shaft rotation. In addition to the above symmetric and skew symmetric terms, the damping matrix also contains terms which depend upon the rotational input at the base. These input dependent terms are referred to as parametric terms. Because of these terms, the equations of motion become nonlinear and difficult to solve. However, a numerical study by Suarez et al. (1991) shows that these terms can be neglected even for very high level of base rotations.

The constitution of the stiffness matrix also is very much similar to that of the damping matrix, except in the following two aspects: (1) there are no skew symmetric terms and (2) the stiffness contribution from the fluid film bearings are asymmetric. These asymmetric terms tend to cause instability in the motion if a machine is operated at high speeds. Also, there are rotational input dependent parametric terms in the stiffness matrix as well. Again they can be neglected without causing any inaccuracies in the calculated response, even if the rotational inputs are very strong. Neglecting the parametric terms from the damping and stiffness matrices renders the equation of motion linear, thus permitting their modal solution.

The forcing function terms on the right hand side are also quite complex. They have terms which depend upon the components of base accelerations — — both translation and rotational — — similar to what one obtains in civil structures. In addition to these, there are terms which appear as the product of velocities as well as terms which depend upon the rotational base velocities. All these can be included in the random vibration analysis of a rotor system, but they cause complications if it is desired to develop a response spectrum approach. A random vibration study (Chang 1992) of the effect of these terms on the rotor response has, however, clearly demonstrated that the nonlinear velocity–dependent terms can be conveniently deleted without affecting the accuracy of the calculated response.

The aforementioned simplifications and deletions of the nonlinear parametric and forcing function terms make it possible to develop a seismic response spectrum approach for rotating machines. However, since the damping and stiffness matrices of these systems lack any symmetry, the conventional modal analysis approach can not be used. It is shown by Singh et al. (1991) that one can still use a generalized modal analysis approach to develop a response spectrum approach for calculating the seismic design response for seismic design input defined by smoothed ground response spectra. Here it is proposed to modify this approach to make it more efficient. The basic elements of this modal analysis approach are briefly outlined below.

To solve equation (1) for any arbitrary forcing function and to develop a response spectrum approach, a generalized modal analysis approach is used. In this approach, equation (1) is rewritten in the state form, by using an auxiliary equation, as follows:

$$[A]\{\dot{y}\} + [B]\{y\} = \{\hat{F}(t)\} \tag{2}$$

where

$$[A] = \begin{bmatrix} M & 0 \\ 0 & K \end{bmatrix}, \quad [B] = \begin{bmatrix} C & K \\ -K & 0 \end{bmatrix}, \quad \{y\} = \begin{Bmatrix} \dot{x} \\ x \end{Bmatrix}, \quad \{\hat{F}(t)\} = \begin{Bmatrix} F(t) \\ 0 \end{Bmatrix} \tag{3}$$

Equation (2) is uncoupled into the equations of the principal coordinates by using the right and left eigenvectors of the following adjoint eigenvalue problems:

$$[B]\{\phi_j\} = -\lambda_j[A]\{\phi_j\}\ ; \qquad [B]^T\{\psi_j\} = -\lambda_j[A]^T\{\psi_j\} \tag{4}$$

These eigenvectors satisfy the following bi–orthonormality conditions

$$\{\psi_j\}^T[A]\{\phi_k\} = \delta_{jk}\ ; \qquad \{\psi_j\}^T[B]\{\phi_k\} = -\lambda_j\delta_{jk} \tag{5}$$

where δ_{jk} is the Kronecker delta. In terms of these eigenvectors, one can write the response vector $\{x\}$ as

$$\{x(t)\} = \sum_{j=1}^{2n}\{\bar{\phi}_j\}q_j(t) \tag{6}$$

where n is the degrees of freedom of the system, $\{\bar{\phi}_j\}$ is the lower half of the eigenvector $\{\phi_j\}$, and the principal coordinate $q_j(t)$ is defined by the following first order equation:

$$\dot{q}_j - \lambda_j q_j = \{\bar{\psi}_j^T\}\{F(t)\} \tag{7}$$

where $\{\bar{\psi}_j\}$ is the upper half of $\{\psi_j\}$. A response quantity s(t) linearly related to $\{x\}$ as $s(t) = \{T\}^T\{x\}$ can also be expressed as:

$$s(t) = \sum_{j=1}^{2n}\bar{\rho}_j q_j(t) \tag{8}$$

where $\bar{\rho}_j = \{T\}^T\{\bar{\phi}_j\}$ and $\{T\}$ is a transformation vector which is defined in terms of geometric and mechanical properties of the system, and which when premultiplied by the displacement vector $\{x\}$ provides the response quantity of interest.

Equation (8) was used to develop a response spectrum approach for calculating the design response (Singh, et al. 1991). In the calculation of the response it was, however, observed that the rotor systems usually have several eigenvalues which are associated with high frequency modes. For earthquake loadings, these high frequency modes contribute only statically. In such cases, it is advantageous to adopt an alternative formulation, where the high frequency modes are not explicitly considered and their effect is included by a simple state analysis. This leads to a more efficient response spectrum approach.

To formulate this approach, we rewrite equation (8), by utilizing equation (7) as:

$$s(t) = -\sum_{j=1}^{2n} \frac{\rho_i}{\lambda_j} \{\psi_j\}^T\{F\} + \sum_{j=1}^{2n} \frac{\rho_i}{\lambda_j} \dot{q}_j \tag{9}$$

It can be shown that the first term is nothing but the pseudo static response which can be written as $\{T\}^T[K]^{-1}\{F\}$. Thus equation (9) can be rewritten as

$$s(t) = \{T\}^T[K]^{-1}\{F\} + \sum_{j=1}^{2r} \frac{\rho_i}{\lambda_j} \dot{q}_j \tag{10}$$

where now in equation (10), the summation is carried out for only the first r complex and conjugate modes. The modes with higher frequencies are completely omitted from further consideration. It is shown numerically that this truncation of modes does not cause any significant error in the calculation of response. To identify the high frequency modes, the frequency corresponding to an eigenvalue λ_j is defined as $\omega_j = |\lambda_j|$.

If we proceed further with equation (10) to develop a response spectrum approach we will need to defined seismic input in terms of the relative acceleration and relative velocity response spectra. To avoid the use of the relative acceleration spectra, which are rarely used in seismic design practice, we use equation (7) again to substitute for $\dot{q}_j$ in terms of q_j in equation (10) which after some rearrangement of terms leads to the following equation

$$s(t) = \{u_s\}^T\{F(t)\} + \sum_{j=1}^{2r} \bar{\rho}_j q_j(t) \tag{11}$$

where

$$\{u_s\}^T = \{T\}^T[K]^{-1} + \sum_{j=1}^{r} \lambda_j^{-1} \bar{\rho}_j \{\bar{\psi}\}^T \tag{12}$$

Substituting the solution of equation (7) for $q_j(t)$ in equation (11) we obtain

$$s(t) = \{u_s\}^T\{F(t)\} + \sum_{j=1}^{2r} \bar{\rho}_j \{\psi_j^T\} \int_0^T \{F(\tau)\}\, e^{\lambda_j(t-\tau)}\, dt \tag{13}$$

The forcing function $\{F(t)\}$ depends upon the base excitation. For a machine sitting on ground, the rotational components will not be significant. Also a machine situated on the floor of a building is not likely to experience much rotational base input, unless the building has strong torsional response. Therefore, here we will consider only the translational acceleration input at the base to define $\{F(t)\}$, although the rotational inputs can also be considered without any special problem.

For the three translational components applied at the base, we now present a response spectrum approach to calculate the maximum response which can occur in the system irrespective of the directions in which the three orthogonal components are applied. The method presented here for calculating this maximum response follows the approach presented by Singh and Ghafory–Ashtiany (1984) for conventional structural systems subjected to multi–component earthquakes.

The input earthquake motions are represented by three orthogonal principal components $E_1(t)$, $E_2(t)$ and $E_3(t)$ which are uncorrelated with each other. The direction cosines of the m^{th} principal component with respect to the n^{th} coordinate axis of the machine is denoted by d_{mn} which is an element of the direction cosine matrix [D]. In terms of these principal components and the direction cosine matrix, the input forcing function $\{F(t)\}$ can be defined by

$$F(t) = [r][D]^T\{E(t)\} \tag{14}$$

where [r] is the matrix of influence coefficient and $\{E(t)\}$ is the vector of the principal excitation components. Substituting for the forcing function defined in equation (14) in equation (13), we obtain

$$s(t) = \{\rho_s\}^T[D]^T\{E(t)\} + \sum_{j=1}^{2r}\{\rho_j\}^T[D]^T \int_0^t \{E(\tau)\} e^{\lambda_j(t-\tau)} d\tau \tag{15}$$

where

$$\left.\begin{aligned} \{\rho_s\}^T &= \{u_s\}^T[r]; \\ \{\rho_j\} &= \bar{\rho}_j \{\psi_j\}^T[r] \end{aligned}\right\} \tag{16}$$

We use equation (15) to obtain the mean square value of s(t), which after some lengthy manipulations, can be written in the following form:

$$E[S^2(t)] = \sum_{\ell=1}^{3} \{d_\ell\}^T[R_\ell]\{d_\ell\} \tag{17}$$

where $\{d_\ell\}$ is the vector of the direction cosines of the ℓ^{th} principal component. $[R_\ell]$ is a 3×3 matrix, a typical element $R_{mn\ell}$ of which can be defined as follows:

$$\begin{aligned} R_{mn\ell} = {} & \rho_{sm}\rho_{sn} I_\ell + 2\sum_{j=1}^{r}\Big[\Big[\sigma_{jmn} + \sigma_{jnm}\Big]\omega_j^2 I_{o\ell_j} \\ & + \Big\{2\beta_j\omega_j\Big[u_{jmn}+u_{jnm}\Big] - \Big[\sigma_{jmn}+\sigma_{jnm}\Big]\Big\} I_{2\ell_j}\Big] \\ & + 4\sum_{j=1}^{r}\Big[Z_{jm}Z_{jn}I_{o\ell_j} + a_{jm}a_{jn}I_{2\ell_j}\Big] \\ & + 4\sum_{j=1}^{r-1}\sum_{k=j+1}^{r}\Big[A_{jk}I_{o\ell_j} + B_{jk}I_{2\ell_j} + C_{jk}I_{o\ell_k} \\ & + D_j I_{2\ell_k}\Big] \end{aligned} \tag{18}$$

where

$$\left.\begin{aligned}&\omega_j=|\lambda_j|,\quad \beta_j=-\mathrm{Real}(\lambda_j)/\omega_j\\&Z_{jm}=(a_{jm}\beta_j-b_{jm}\sqrt{1-\beta^2})\omega_j\\&\rho_{jm}=a_{jm}+ib_{jm}=m^{th}\text{ element of }\{\rho_j\}\\&\sigma_{jmn}=Z_j\quad\rho_{sn};\quad u_{jmn}=\mathrm{Real}\ (\rho_{jm}\rho_{sn})\end{aligned}\right\}\tag{19}$$

The coefficients of the partial fraction A_{jk}, B_{jk}, etc. are defined by Singh et al. (1992). The frequency integrals are: I_ℓ = mean square value of the acceleration of the ℓ^{th} component and, $I_{o\ell j}$ and $I_{2\ell j}$, respectively, are the mean square values of the relative displacement and relative velocity responses of an oscillator of frequency ω_j and damping ratio β_j excited at its base by the acceleration of the ℓ^{th} excitation component. In terms of the spectral density function of the ℓ^{th} component, these integrals are defined as

$$\left.\begin{aligned}&I_\ell=\int_{-\infty}^{\infty}\Phi_\ell(\omega)d\omega\ ;\\&I_{o\ell j}=\int_{-\infty}^{\infty}|H_j|^2\Phi_\ell(\omega)\,d\omega\\&I_{2\ell j}=\int_{-\infty}^{\infty}\omega^2|H_j|^2\Phi_\ell(\omega)d\omega\end{aligned}\right\}\tag{20}$$

In the above equations $\Phi_\ell(\omega)$ is spectral density function of the ℓ^{th} component and $H_j=(\omega_j^2-\omega^2+2i\beta_j\omega_j\omega)^{-1}$ is the frequency response function. This assumes that the seismic motions are stationary and act for a long time. The assumption of stationarity in a strict sense is not correct. But this enables us to develop a convenient expression for response spectrum calculations.

To obtain the maximum of $E[s^2(t)]$, irrespective of the excitation direction, it is only necessary to calculate the eigenvalues of the matrix $[R_\ell]$, as described by Singh and Ghafory–Ashitany (1984). This will provide a global maximum of the mean square response. However, if one is interested in calculating the global maximum of the design response, then the response represented by $R_{mn\ell}$ must be the design response value. This can be done by calculating $R_{mn\ell}$ for inputs defined by response spectra. Equation (18) can still be used to obtain the design value of $R_{mn\ell}$ if I_ℓ, $I_{o\ell j}$ and $I_{2\ell j}$, respectively, are replaced by the squared values of the maximum ground acceleration, displacement response spectrum value and relative velocity response spectrum value. The displacement response spectrum values can also be directly obtained from the pseudo acceleration response spectrum value. These response spectrum values are for an oscillator of frequency ω_j and damping ratio β_j. Replacing the frequency integrals by the corresponding response spectrum value to obtain design response, implies that peak factors for various response quantities appearing in equation (16) are the same. This is not strictly true, but this assumption enables us to simplify the expression for calculating the design response. The practical applicability of this assumption as well as the assumption of the stationarity of the input are verified by the numerical study, the results of which are given in the following section.

3 NUMERICAL RESULTS

Equations (17) and (18) were used to obtain the global maximum values of the displacement and bearing force responses of the rotor–disk–bearing shown in Figure 1. The same system was used in an earlier study by by Suarez et al. (1992). The seismic input at the base was defined by pseudo–acceleration and relative velocity response spectra, shown in Figure 2. The maximum acceleration values for the three components were .2g, .2g, and 0.134g. The spectral shape for the three components were assumed to be similar.

The global maximum values of the response obtained for these inputs are shown in Tables 1 and 2. Table 1 is for the input defined by the mean spectra whereas table 2 is for the input defined by the mean–plus–one–standard deviation spectra. In column (2) of the Tables are shown the response values obtained by the time history analysis for an ensemble of 50 set of time histories. The time histories in the ensemble were the same time histories for which the input response spectra were generated. In Table 1, the time history results are the mean of the maximum responses obtained for the ensemble of time histories, whereas those in Table 2 are the mean–plus–one–standard deviation values of the maximum response. In column (3) are shown the values obtained when all system modes are considered in the analysis. In Column (4) are the response values obtained with only the first four modes utilized in the approach proposed in the paper.

The comparison of the response values given in Columns (3) and (4) of Table 1 shows that proposed approach is effective in predicting the response accurately even with only 4 modes. Also, comparison of the response spectrum results with the time history results validates the spectrum approach in spite of the assumption of stationarity of the input and response and equality of the peak factors of various response quantities appearing in equation (16). Similar favorable comparisons are also seen among the various results in Table 2. Thus, the proposed response spectrum approach is also acceptable for the mean–plus–one–standard deviation input spectra, which are commonly used in seismic design practice.

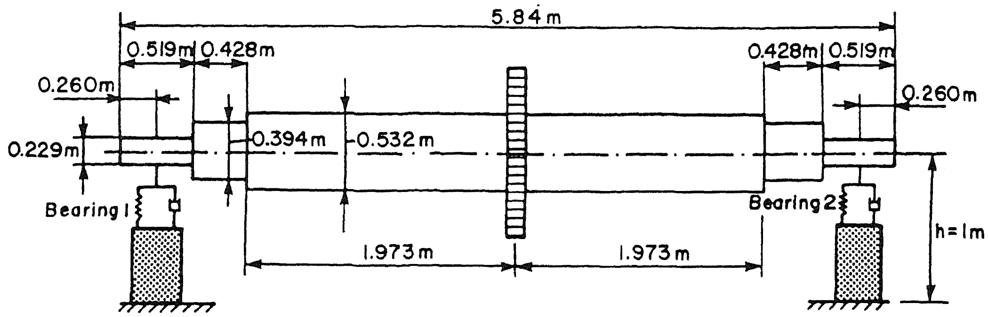

Figure 1: Rotor–Disk–Bearing System considered in the study.

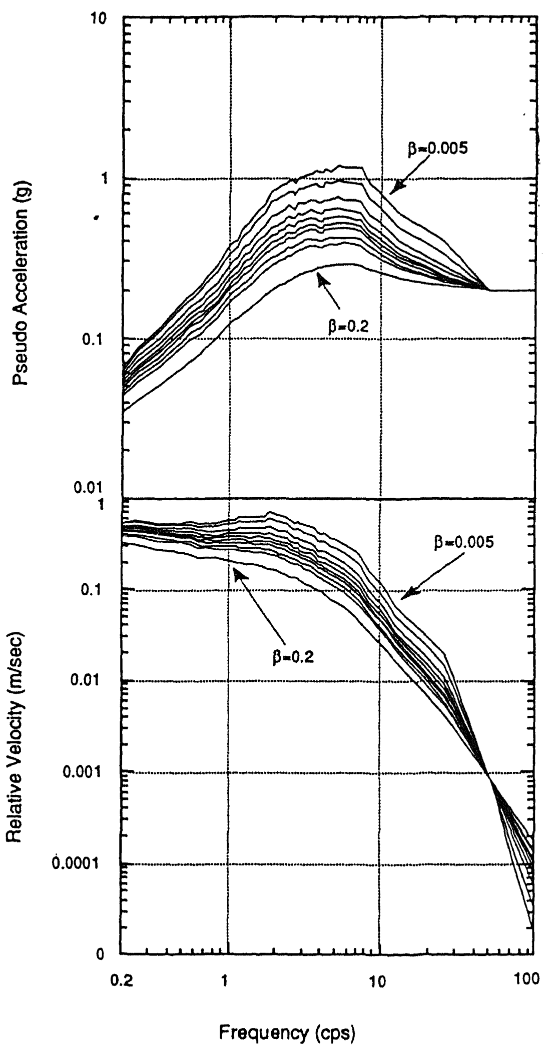

Figure 2: Mean pseudo acceleration and relative velocity response spectra for the ensemble of ground motions considered in the study. Damping ratios: .005, .01, .02, .03, .04, .05, .06, .08, .1 and .2.

Table 1: Comparison of the mean of the maximum responses obtained by the proposed response spectrum approach and the time history analysis.

Response (1)	Time History (2)	Spectrum Approach 4 modes (3)	Spectrum Approach All mode (4)
Displacements, 1 mm			
X	0.1430	0.1425	0.1428
Y	0.1440	0.1491	0.1496
Bearing Forces, kN			
X	15.65	15.23	14.34
Y	18.94	19.34	18.40

Table 2: Comparison of the mean–plus–one–standard deviation responses obtained by the proposed response spectrum approach and the time history analysis.

Response (1)	Time History (2)	Spectrum Approach 4 modes (3)	Spectrum Approach All mode (4)
Displacements, mm			
X	0.1527	0.1512	0.1514
Y	0.1595	0.1685	0.1695
Bearing Forces, kN			
X	16.75	16.13	15.15
Y	20.84	21.86	20.85

5 CONCLUDING REMARKS

An efficient response spectrum approach is presented to obtain the global maximum response of rotating machines subjected to multi–component base motions. The approach needs only a first few eigenvalues and eigenvectors and the input defined in terms of pseudo acceleration and relative velocity response spectra. The eigenvalues and eigenvectors corresponding to high frequency modes are not explicitly required as their contribution to the system response can be included through a simple static analysis. The required eigenvalues and eigenvectors are obtained from the solution of two adjoint eigenvalue problems. The applicability of the proposed response spectrum analysis is verified through a numerical example solved for an ensemble of base acceleration time histories. It is shown that the response results obtained by proposed spectrum approach match very well with the time history results.

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