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Free vibrations of submerged floating cylindrical shells

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ABSTRACT: The free vibrations of an elastically or rigidly point–supported circular cylindrical shell in a fluid medium are studied theoretically. The fluid is assumed to be inviscid and incompressible. The Ritz method is applied to the prediction of the natural frequencies. The spatial variations of the shell deflection are written in a series of appropriate beam functions. For a rigidly–supported shell, the solution is obtained using the Ritz procedure in conjunction with Lagrange multipliers.

1. INTRODUCTION

This paper presents an analysis of the free vibration of a submerged floating cylindrical shell elastically of rigidly supported at several points.

In the case of very deep straits, conventional bridges and tunnels are expensive. This has lead to the development of alternative concepts for strait crossings. The most significant concept at the time being is the submerged floating shell. The submerged floating shell represents a new type of marine construction. By its length and slenderness it raises technological challenges, especially regarding hydrodynamic loads and shell–fluid interaction. The dynamic behaviour is of particular importance, since this determines the safety of the shells as well as the comfort of the travellers. However, little information can be found in the literature on the behaviour of such shells subjected to environmental effects like earthquakes and sea waves.

The natural frequency of a vibratory system is perhaps the first item of interest in dynamic analysis. The objectives of this paper are: (1) To present a simple and effective solution procedure for the free vibration analysis; and (2) to study the frequency characteristic of a point–supported cylindrical shell in an infinite fluid medium.

For the purpose of this study, the solution is obtained using the Ritz procedure in conjunction with Lagrange multipliers.

A series of the product of the deflection functions of beam and the trigonometric functions of circumferential coordinates are used as the deflection displacements of the shell–fluid interaction probelm. The common "added mass" approach to account for effect of surrounding fluid is used (see Baron and Skalak(1962)). The strain energy of the supports is taken into account for an elastically point–supported shell, while the Lagrange multipliers are introduced for expressing the unknown reaction forces at the supports for a rigidly point–supported shell.

2. METHOD OF ANALYSIS

The submerged floating shell under consideration is shown in Figure 1. The shell is of uniform thickness h, radius a and length ℓ, made of homogeneous, isotropic material with an elasticity modulus E, Poisson's ratio ν and shell density ρ_s. The radial, circumferential, and axial coordinates are denoted r, θ, and x, respectively, and the corresponding displacement components of a point on the shell middle surface are denoted by w, v, and u, respectively. The fluid is assumed to be incompressible and inviscid and the fluid motion irrotational so that the flow can be described by a velocity potential, ϕ, which satisfies the Laplace equation.

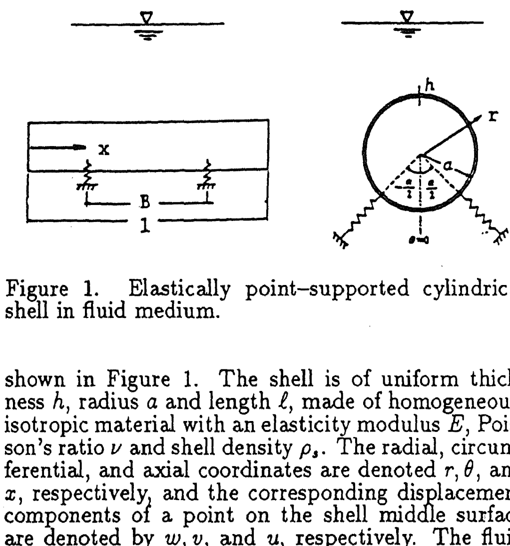

Figure 1. Elastically point–supported cylindrical shell in fluid medium.

2.1 Energy expressions

In the Ritz method it is essential to obtain expressions for the strain energy U and the kinetic energy T.

Based on the strain–displacement relations derived by Novozhilov (1970), the expression for the strain energy may be written in terms of modal displacements and their derivatives

$$\begin{aligned} U &= K \int_0^{\ell}\int_0^{2\pi} \Big[u_{,x}^2 + (v_{,\theta} + w)^2/a^2 \\ &+2\nu u_{,x}(v_{,\theta} + w)/a \\ &+(1-\nu)(v_{,x} + u_{,\theta}/a)^2/2 \end{aligned}$$

$$+\varepsilon\{a^2w_{,xx}^2+(w_{x,\theta\theta}+v_{,\theta})^2/a^2$$
$$+2\nu w_{,xx}(w_{,\theta\theta}-v_{,\theta})$$
$$+2(1-\nu)(w_{,x\theta}-v_{,x})^2\}\Big]ad\theta dx$$
$$+\sum_{s=1}^{S}\Big\{k_xu^2(x_s,\theta_s)+k_\theta v^2(x_s,\theta_s)$$
$$+k_rw^2(x_s,\theta_s)\Big\}/2 \quad (1)$$

where the subscripts following a comma indicate differentiation, $K=Eh(1-\nu^2)$= the extensional rigidity, and $\varepsilon=(h/a)^2/12$. The constants (k_x,k_θ,k_r) are the stiffness of the point supports in the (x,θ,r) coordinate directions, and (x_s,θ_s) are the coordinates of the point supports.

The kinetic energy T of the shell, including the effect of interaction with surrounding fluid, can be written as

$$T = \frac{\rho_s h}{2}\int_0^\ell\int_0^{2\pi}(u_{,t}^2+v_{,t}^2+w_{,t}^2)ad\theta dx + \frac{\rho_f}{2}\int_S \phi\frac{\partial\phi}{\partial n}dS \quad (2)$$

where ρ_f is the mass density of fluid, n is the outward normal vector, and S is the surface of the shell. The second term on the right hand–side of equation (2) can be evaluated from the expression of the velocity potential function, ϕ, which satisfies the Laplace equation and the appropriate boundary condition (i.e. $\phi_{,r}|_{r=a}=w_{,t}$) at the fluid–shell interface. After some algebraic manipulations, the velocity potential function can be expressed as

$$\phi = \sum_i\sum_n K_n(i\pi a/\ell)\sin(i\pi x/\ell)\cdot (A_{in}\cos n\theta + B_{in}\sin n\theta) \quad (3)$$

where $K_n(\ )$ is the modified Bessel function of the order n of the second kind. The integration constants for cosine(sine) components of the shell motion are given by

$$A_{jn}(B_{jn}) = \frac{2}{\ell K_n'(j\pi a/\ell)}\sum_m\sum_n \cdot \int_0^\ell\int_0^{2\pi}\sin(j\pi x/\ell)w_{mn,t}d\theta dx \quad (4)$$

where $K_n'(\ )$ is the derivative of the modified Bessel function.

2.2 Derivation of frequency equation

Because of the nature of the Ritz energy procedure, it is not necessary to enforce the natural boundary conditions. A general relation for the displacements in any mode can be written in the following form for any n

$$u = \sum_m^M\sum_n^N X_m(x)(a_{mn}\cos n\theta + a_{mn}^*\sin n\theta)\sin\omega t$$
$$v = \sum_m^M\sum_n^N Y_m(x)(b_{mn}\cos n\theta + b_{mn}^*\sin n\theta)\sin\omega t \quad (5)$$
$$w = \sum_m^M\sum_n^N Z_m(x)(c_{mn}\cos n\theta + c_{mn}^*\sin n\theta)\sin\omega t$$

where ω is a natural frequency of vibration, X_m, Y_m and Z_m are axial mode functions corresponding to axial, circumferential, and radial displacements, respectively. The a_{mn}, etc. are the unknown coefficients.

For the free vibration of elastically supported shell, the variational functional can be expressed as

$$L = U_{max} - T_{max} \quad (6)$$

where U_{max} and T_{max} are the maximum of the strain energy and the kinetic energy, respectively. After inserting equations (5) into equations (1) and (2), the Ritz method requires the minimization of the functional L which is accomplished by setting

$$\begin{aligned}\partial L/\partial a_{mn} &= 0, \partial L/\partial a_{mn}^* = 0\\ \partial L/\partial b_{mn} &= 0, \partial L/\partial b_{mn}^* = 0\\ \partial L/\partial c_{mn} &= 0, \partial L/\partial c_{mn}^* = 0\end{aligned} \quad (7)$$

Equations (7) are simulataneous, linear, algebraic equations in the unknowns a_{mn}, a_{mn}^*, etc. The frequency equation can be written in the form:

$$\Big\{[K_s]-\omega^2([M_s]+[DM])\Big\}\Big\{d\Big\}=\Big\{0\Big\} \quad (8)$$

where $[K_S]$ is the elastic stiffness matrix obtained from U_{max}, and $[M_S]$ and $[DM]$ are the consistent mass and added mass matrices obtained froma T_{max}. The vector $\{d\}$ is defined in the following form:

$$\{d\} = \{a_{mn}, a_{mn}^*, b_{mn}, b_{mn}^*, c_{mn}, c_{mn}^*\}^T \quad (9)$$

Equation (8) represents an eigenvalue problem. For a non–trivial solution, the determinant of the coefficient matrix is set equal to zero.

For the free vibration of rigidly point–supported shell, the solution is obtained by minimizing the variational functional

$$L = U_{max} - T_{max} - \sum_s^S\{\lambda_s u(x_s,\theta_s) + \beta_s v(x_s,\theta_s) + \gamma_s w(x_s,\theta_s)\} \quad (10)$$

where the λ,β,γ are Lagrange multipliers. These Lagrange multipliers are related to the unknown forces at the point supports. Omitting the strain energy of the supports appearing equation (1) and minizization with respect to $a_{mn},\cdots,c_{mn}^*$ leads to an expression similar to equation (8) as follows:

$$\Big\{[K_s]-\omega^2([M_s])\Big\}\Big\{d\Big\}+\Big[K_r\Big]\Big\{\delta\Big\}=\Big\{0\Big\} \quad (11)$$

where $\{\delta\}$ is the $3S$–dimensional vector whose elements are the Lagrange multipliers, $\lambda_s,\beta_s,\gamma_s$. Minimization of equation (10) with respect to λ_s,β_s, and γ_s leads to the following equation:

$$[K_r]^T\{d\} = \{0\} \quad (12)$$

When equation(11) is solved for $\{d\}$ and the result is substituted into equation(12), we obtain the frequency equation of rigidly point-supported cylindrical shell, i.e.

$$[S]\{\delta\} = \{0\} \tag{13}$$

where

$$\left[S\right] = \left[K_r\right]^T \left[[K_s] - \omega^2[M_s]\right]^{-1} \left[K_r\right] \tag{14}$$

The coefficients of matrix $[S]$ are the transcendental function of the frequency. The necessary and sufficient condition for equation (13) to have a nontrival solution is

$$D = \left|[S(\omega)]\right| = 0 \tag{15}$$

The frequencies are characteristic roots of this determinant. However, in the numerical calculation the eigenvalues can only be determined as parameters that attain minima of the value of the determinant D as a function of the frequency ω.

3. NUMERICAL EXAMPLES

In this section application of the method detailed above is made to cylindrical shell elastically or rigidly supported at symmetrically located four points as shown in Figure 1.

The symmetrical and antisymmetrical modes with respect to the planes of symmetry are described by a symbolism. The S and A modes are referred as the symmetrical and antsymmetrical vibrations with respect to a plane of symmetry passing through the meridional axis, respectively, while the S' and A' modes are referred as the symmetrical and antisymmetrical vibrations with respect to the central plane perpendicular to the meridional axis.

The results presented in this section are defined in terms of a frequency parameter Ω given by

$$\Omega = \omega\sqrt{\rho_s h a^2/K} \tag{16}$$

It is convenient to introduce the following nondimensionalized parameters: $h^*=h/a$; $\ell^*=\ell/a$; and $(k_x^*, k_\theta^*, k_r^*)=(a/K\ell)\ (k_x, k_\theta, k_r)$, where an asterisk superscript denotes a nondimensional parameter. Unless otherwise stated, the following shell properties were used: $\nu=0.3$; $\ell^*=3$; $k_x^*=0$; $k_\theta^*=k_r^*=0.1$; $\alpha=90°$; $b=2/3$, where $b(=B/\ell)$ and α denote the axial distance and angle between axially and circumferentially located two supports, respectively.

3.1 Convergence

The convergence of the proposed method depends on the number of terms $(M \times N)$ in the polynomial series for the displacements, as well as, the number of terms I in the series expansion of the velocity potential.

Figure 2 shows a convergence study for various modes. Calculations were performed by using $I=12$ and by varying $(M \times N)$. The convergence of the solutions is reasonable even with a relatively small

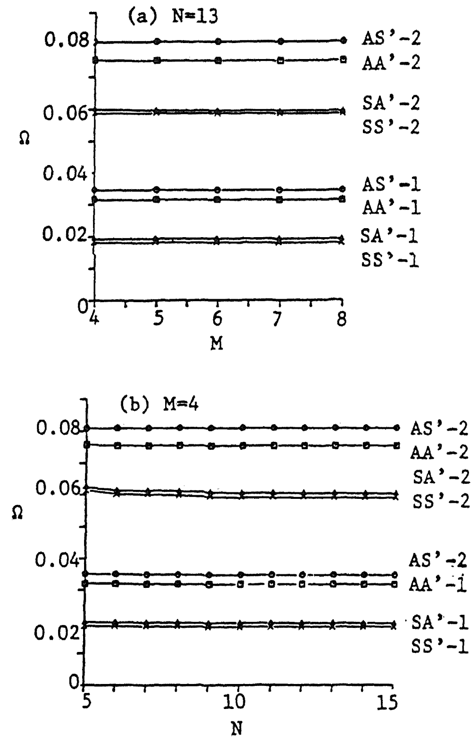

Figure 2. Convergence of natural frequency Ω.

number of terms $(M \times N)$. For $M \geq 5$ and $N \geq 10$ good convergence characteristics are achieved. Similar studies (not shown here) were made for various values of I (say $I \geq 10$) and the results converged rapidly on steady values. All the following results have therefore been calculated with $M=6$, $N=13$ and $I=12$.

3.2 Frequency characteristic

For a elastically point-supported shell, the effect of the stiffness parameter $k_\theta^* = k_r^*$ on the freqencies is shown in Figure 3. The results obtained for a rigidly point-supported shell, which are denoted by small solid circles, are also presented in the figure at the right. As can be seen from Figure 3, the frequencies monotonically increase with the increase in $k_\theta^* = k_r^*$, and when $k_\theta^* = k_r^* \geq 1$, the frequencies for the two shells become almost the same.

The effect of the angle α on the frequencies is illustrated in Figure 4. for a elastically point-supported shell. The effect of the angle is more pronounced for the $AS'-$, $AA'-$, $SA'-2$ and $SS'-2$ modes. The frequencies of both the $SS'-1$ and $SA'-1$ modes are monotonically increasing with the increase of the angle.

The effect of the thickness-to-radius ratio h^* on the frequencies is presented in Figure 5, for a elastically point-supported shell. As expected, an increase in the parameter h^* will result in an increase in the natural frequencies of the shell.

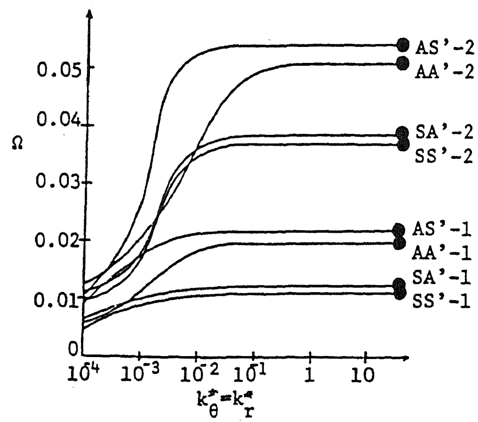

Figure 3. Varaiation of natural frequency Ω with stiffness $k_\theta^*=k_r^*$.

3.3 Added mass factor

From the authors' past work (see Mikami and Yoshimura (1990)), it can be shown that the frequency of the fluid–shell system is equal to that of the shell in air whose density has been replaced by ρ,

$$\rho = \rho_s + \rho_a \tag{17}$$

where

$$\rho_a = \rho_f (a/h) C \tag{18}$$

where ρ_a is the added mass of a fluid–shell system, and depends on the fluid density, radius–to–thickness ratio, etc. C is the added mass factor.

The effect of variation in the three parameters $k_\theta^*(=k_r^*)$, α and h^* on the added mass factor C are depicted in Figures 6,7 and 8, for the first four modes of a elastically point–supported shell.

The following observations can be made from these figures:

1. For values of $k_\theta^*(=k_r^*) \leq 0.1$, the added mass factor, C, gradually increases as $k_\theta^*(=k_r^*)$ increases. However, for larger values of $k_\theta^*(=k_r^*)$, C is almost constant and is the range between about 0.25 to 0.30.
2. The added mass factor C is almost independent of the angle α; C is about 0.2 to 0.3.
3. The added mass factor C is highly dependent on the thickness–to–radius ratio h^*, especially for smaller values of h^*. Its value ramains almost same except for the $SS'-$ mode.

4. CONCLUDING REMARKS

A simple numerical procedure based on the Ritz method is developed herein for the free vibration analysis of an elastically point–supported cylindrical shell in an fluid medium. Furthermore, the Ritz procedure in conjunction with Lagrange multipliers is presented to study the free vibrations of a rididly point–supported shell. The Lagrange multipliers are

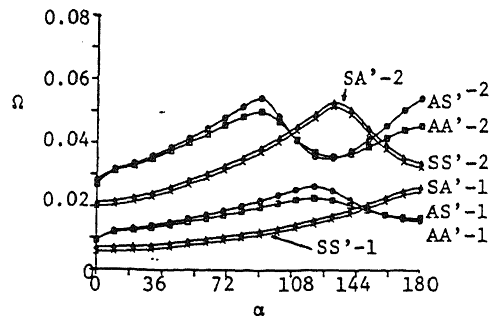

Figure 4. Variation of natural frequency Ω with angle α.

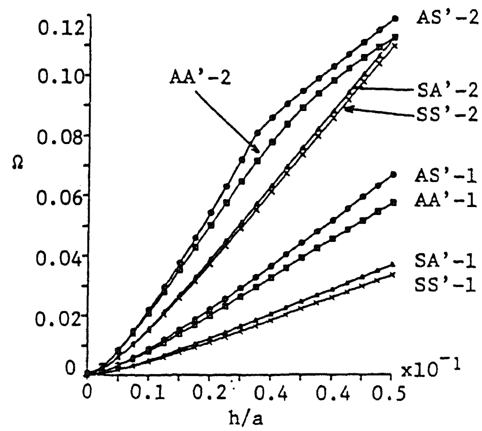

Figure 5. Variation of natural frequency Ω with thickness–to–radius raito (h/a).

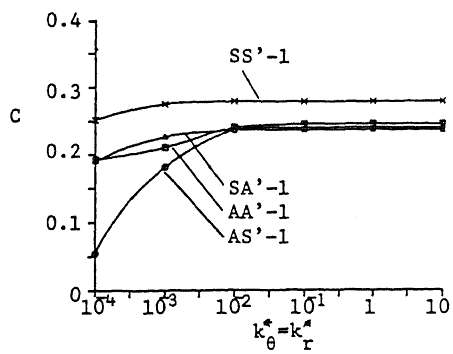

Figure 6. Variation of added mass factor C with $k_\theta^*=k_r^*$.

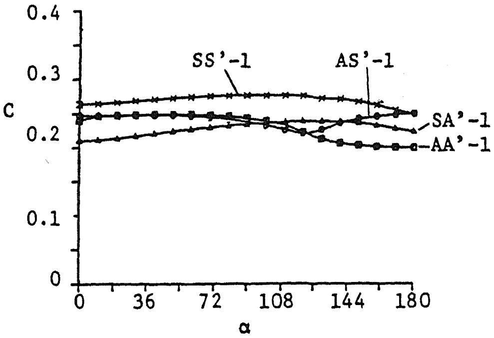

Figure 7. Variation of added mass factor C with angle α.

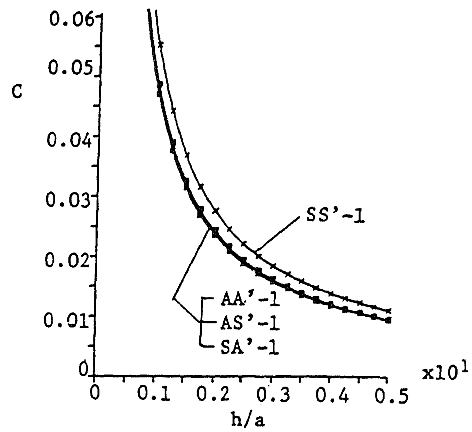

Figure 8. Variation of added mass factor C with thickness-to-radius ratio (h/a).

used to express the unknown raction forces at the supports.

The natural frequencies of vibration are calculated numerically, and the effects of shell geometric and stiffness parameters on the frequency characteristic are investigated. Finally, The added mass factor which is useful for a better understanding of the vibration characteristic is also calculated numerically.

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