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Earthquake resistant limit state design for cylindrical liquid storage tanks

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ABSTRACT: Typical collapse modes of cylindrical stronge tanks subjected to horizontal seismic motions are elephant-foot bulge at the lowest part of cylindrical wall and the fracture of bottom plate at the corner associated with the up-lift of bottom plates.
The essential loading effect of earthquakes upon structures can be expressed in terms of energy.
By equating the energy input due to earthquakes to the energy absorption capacity of structure, a design criteria for unanchored cylindrical tanks can be derived on the basis of the ultimate strength.

1 ELEPHANT-FOOT BULGE OF TANK WALLS

A typical collapse mode of cylindrical tanks in the event of strong earthquakes is the elephant-foot bulge which develops at the lowest circumference of tank wall. The elephant-foot bulge is a phenomenon similar to buckling taking place under a combined action of normal compresive stress associated with the overturning moment acting on the cylinder and the hoop stress caused by the internal liquid pressure.
A series of scale-model tests was carried out on the steel cylindrical shells with fixed bases, applying horizontal forces at the top. The load-carrying capacity and the restoring-force characteristics of cylindrical shells under the development of elephant-bulge mode of deformation were investigated. Fig.1 illustrates an example of elephant-foot bulge. Dominant parameters in the elephant-foot bulge is the ratio of hoop stress σ_h to the yield stress of cylidrical wall σ_Y. The maximum compressive value ${}_b\sigma_m$ of the extreme fiber stress ${}_b\sigma_m$ was found to be strongly correlated to σ_h/σ_Y as shown in Fig.2, in which the filled circle indicates the case of elephant-foot bulge modes of failure. As is indicaled in the figure, the condition which governs the occurrence of elephant-foot bulge can be given by $\sigma_h/\sigma_Y>0.3$. Under the condition of $\sigma_h/\sigma_Y<0.3$, the buckling stresses are enhanced due to a restraining effect of internal pressure against buckling, while under $\sigma_h/\sigma_Y>0.3$, the yielding promoted by the hoop stress prompts the occurrence of the elephant-foot bulge. Solid lines in the figure indicates the empirical formula. The second segment with a negative slop is

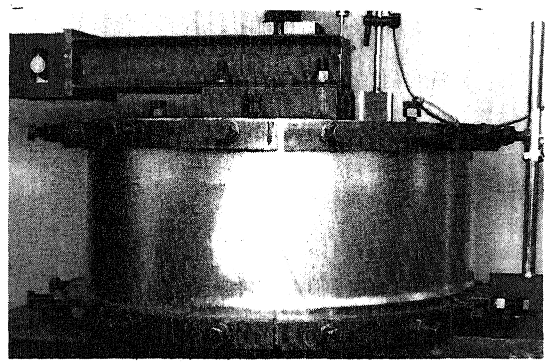

Figure 1. An example of elephant-foot bulge.

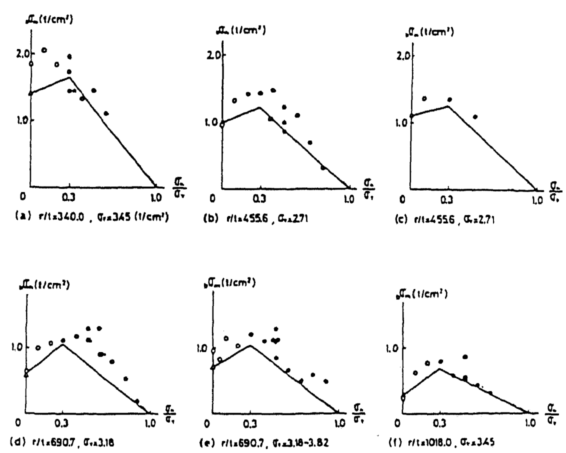

Figure 2. Maximum compressive stresses.

expressed by the following equation

$$ {}_b\sigma_m = \frac{0.8E}{\sqrt{3(1-\nu^2)}}\left(\frac{t}{r}\right)\left(1-\frac{\sigma_h}{\sigma_Y}\right) \qquad (1)$$

where E : Young's modulus
ν : Poisson's ratio
t : thickness of wall plate
r : radius of cylinder

Under a dynamic loading such as an earthquake motion, the occurrence of buckling does not necessarily mean the arrival of collapse for a structure. It is necessary to know the post-buckling behavior under an arbitrarily changing deformation history in order to evaluate the structural resistance in terms of energy absorption capacity of structure. On the basis of experiments, the hysteresis rule governing the post-buckling behavior was found. It was ascertained that cylindrical shells can develop a considerable energy absorptoion capacity without undergoing any breakage of wall within the range of θ less than 0.02.

2 UP-LIFT OF BOTTOM PLATE

The vertical load on the side wall of the cylindrical tank is only its own weight. Therefore, when tanks are not anchored at the base, the base of side wall can easily lift up under the horizontal seismic loading. Since the cylindrical part has high vertical stiffness, the tank shows such an overall displacement as showin in Fig.3 which is characterized by a rotational displacement of tank wall as a rigid body around the point-A and a swelling deformation of flexible bottom plate. In this mode of displacement the bottom plate provides tensile reactional forses on the side wall in the vertical direction. The reactional force per unit length along the circumference of the wall base q can be related to the up-lift displacement δ as shown in Fig.4. q_Y is the yield strength as for the reactional force,

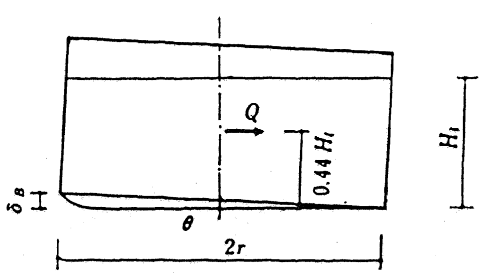

Figure 3. Up-lift of bottom plate.

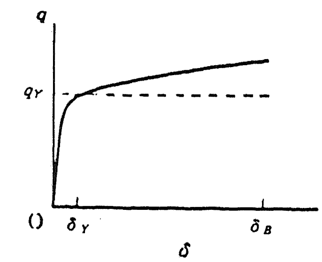

Figure 4. q-δ relationship.

and the displacement which corresponds to q_Y is denoted by δ_Y . As a result of simple inelastic analysis on a strip of bottom plate, q_Y and δ_Y are obtained as follows.

$$\left.\begin{aligned} q_Y &= \frac{2t\sqrt{1.5p\,\sigma_Y}}{3} \\ \delta_Y &= \frac{3t\,\sigma_Y^2}{8Ep} \end{aligned}\right\} \qquad (2)$$

where σ_Y: yield strength of the bottom plate
p : liquid pressure acting on the bottom plate
E : Young's modulus of the bottom plate
t : thichness of bottom plate

The $q-\delta$ relation can be simplified by the rigid-plastic relationship as shown by the broken line in Fig.4.

When one cycle of horizontal force is applied to the tank with an amplitude of up-lift displacement δ_B at the extreme end, the cumulative inelastic displacement of the bottom plate develops uniformly along the circumference of the wall bottom with the amount of δ_B. Therefore, the total energy absorption due to the plastification of bottom plate becomes

$$W_{pl} = 2\pi r q_Y \delta_B \qquad (3)$$

Scale-model tests on the overall behavior of tanks governed by the up-lift of bottom plate were conducted and the relationship between the horizontal force Q and the inclination angle θ was found to take a pattern as shown in Fig.5.

The Q-θ relationship can be simplified as shown by the broken lines in the figure and is characterized by its origin oriented nature.

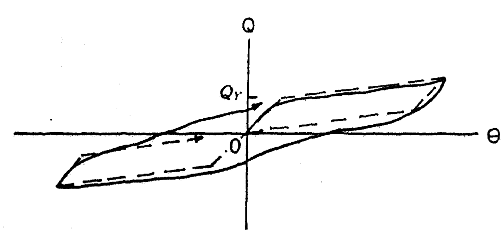

Figure 5. Q-θ relationship.

The inelastic behavior of structures with various restoring force characteristics has been made clear already, and the effective number of cycles encountered in an earthquake with the maximum displacement δ_B was found to be about two for the structure with the origin oriented type of restoring force characteristics.

Therefore, the cumulative inelastic strain energy stored in the tank under the up-lift behavior is expresssed as

$$W_p = 2W_{p1} = 4\pi r q_Y \delta_B \tag{4}$$

3 SIMPLIFICATION OF TANK

Compared to the weight of contained liquid, the own weight of tank is sufficiently small, so can be negligible.

When the stractural safety of tank is discussed under horizontal ground motions, the impulive respose of liquid is of primary importance. Therefore, the sloshing behavior of liquid can be also disregarded.

In this context, the behavior of tank during a horizontal ground excitation can be grasped as a vibrational response of a single-degree of freedom system with an effective mass M related to the impulsive hydraulic pressure.

As a result of the shaking table test on a full-scale tank, the height of the resultant force produced by the impulsive pressure Q was found to be approximately expressed as follows,

$$H_e = 0.44 H_l \tag{5}$$

where H_l: depth of contained liquid

Then, the load carrying capacity of the tank determined by the elephant-foot bulge is expressed in terms of Q as follows

$$Q_Y = \frac{\pi r^2{}_b \sigma_m t_o}{0.44 H_l} \tag{6}$$

As is seen in Eq.(1), the elephant-foot bulge is governed by the hoop stress. The hoop stress consists of two components.

One is produced by the static liquid pressure, and the other is produced by the impulsive pressure exerted by the horizontal ground motion.

The maximum hoop stress attained by the ground motion ${}_Q\sigma_h$ can be related to the resultant horizontal force Q by the following empirical formula.

$${}_Q\sigma_h = \frac{Q}{2.5 H_l t_o} \tag{7}$$

where t_o: thickness of side wall at the bottom

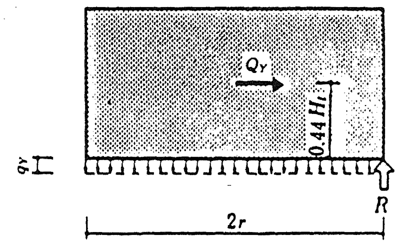

Figure 6. Simplified distribution of reactional forces.

Referring to the simplified distribution of reactional forces in the event of up-lift which is characterized by the uniform distribution of reactional force q_Y along the edge of side wall and a compresive concentrated reactional force at the extreme edge R as shown in Fig.6, the load carrying capacity of the tank determined by the up-lift of bottom plate is obtained as

$$Q_Y = \frac{Rr}{0.44 H_l} = \frac{2\pi r^2 q_Y}{0.44 H_l} \tag{8}$$

4 CRITERIA FOR SEISMIC RESISTANCE

The structural behavior of a structure during an earthquake can be expressed in terms of energy as follows

$$W_e + W_p + W_h = E \tag{9}$$

where W_e: the elestic vibrational energy of the structure
W_p: the cumulative inelastic strain energy in the structure
W_h: the energy absorption due to damping in the structure
E: the total energy input due to earthquake

It was found by Akiyama(1985) that the total energy input depends mainly on the total mass of structure M and the fundamental natural period of structure T and scarcely depends on the other structural properties such as the strength, the strength distribution and the restoring-force characteristics.

E is expressed by using the equivalent velocity V_E as follows

$$E = \frac{M V_E^2(T)}{2} \tag{10}$$

$W_e + W_p$ corresponds to the strains of structure. Denoting the damping constant by h $E - W_h$ can be approximated by

$$E - W_h = \frac{E}{(1 + 3h + 1.2\sqrt{h})^2} \tag{11}$$

Introducing the yield shear force coefficient α, the yield strength of structure is expressed as follows,

$$Q_Y = \alpha M g \tag{12}$$

where g: the acceleration of gravity

W_e is expressed as

$$W_e = \frac{Q_Y \delta_Y}{2} \tag{13}$$

where δ_Y: the elastic depormation which coresponds to Q_Y

Denoting the elastic spring constant by k and knowing $\delta_Y = Q_Y/k$ and $k = 4\pi^2 M/T^2$, the following equation is obtained.

$$W_e = \frac{Mg^2T^2}{4\pi^2} \cdot \frac{\alpha^2}{2} \tag{14}$$

Substituting Eqs.(11) and (14) into Eq.(9), the next equation is obtained.

$$\alpha = \frac{1}{\sqrt{1+\frac{W_p}{W_e}}} \cdot \frac{1}{1+3h+1.2\sqrt{h}} \cdot \frac{2\pi V_E}{Tg} \tag{15}$$

α means the required minimum strength for a structure to resist an earthquake.

Denoting the shear force coefficient of the undamped elastic system by α_e, α_e can be expressed as

$$\alpha_e = \frac{2\pi V_E}{Tg} \tag{16}$$

Therefore, Eq.(15) is expressed as

$$\alpha = D_\eta \cdot D_h \cdot \alpha_e \tag{17}$$

where D_η: response reduction factor due to the plastification of structure
D_h: response reduction factor due to due to damping

D_η and D_h are given by

$$D_\eta = \frac{1}{\sqrt{1+\frac{W_p}{W_e}}} \quad , \quad D_h = \frac{1}{1+3h+1.2\sqrt{h}} \tag{18}$$

In the case of cylindrical storage tanks, the source of damping is mainly attributed to the structure-soil interaction. The estimate of D_η depends on the collapse modes.

As for the elephant-foot bulge, numerous response analyses were made using the restoring force characteristics consistent to the experimental results.

Under the condition that the maximum deflection should be limited within the five times of the deflection at the initiation of elephant-foot bulge the D_η-value was obtained by Akiyama (1991) as follows

$$D_\eta = 0.5 \tag{19}$$

As for the collapse mode associated with the lift-up of bottom plate, the cumulative strain energy is given by Eq.(4).

The inelastic deformation capacity of the bottom plate mainly depends on the yield ratio of the material (the ratio of the yield strength to the tensile strength).

According to the yield ratios, the value of δ_B in Eq.(4) can be taken as follows.

For the yield ratio less than 0.8, $\delta_B = 14\delta_Y$

For the yield ratio greater than 0.8, $\delta_B = 4\delta_Y$

Then, the D_η-value for the up-lift of bottom plate is obtained as follows

For the yield ratio less than 0.8,

$$D_\eta = \frac{1}{\sqrt{1+84\left(\frac{T_1}{T_e}\right)^2}} \tag{20}$$

For the yield ratio greater than 0.8,

$$D_\eta = \frac{1}{\sqrt{1+24\left(\frac{T_1}{T_e}\right)^2}} \tag{21}$$

where T_1: the natural period of structure calculated by considering only the elastic deformation of bottom plate
T_e: the natural period of structure calculated by considering the elastic deformation of side wall and bottom plate.

5 CONCLUSION

The lateral resistant strength of storage tank Q_Y must satisfy the following criterion.

$$Q_Y \geq D_\eta D_h \alpha M g \tag{22}$$

Considering the structure-soil interaction, the reduction factor D_h can be taken as a common value applicable to any collapse mode.

The reduction factor D_η depends on the collapse modes.

Based on the experiments and the inelastic response analyses, D_η-values for the elephant-foot bulge of side wall and the uplift of bottom plated were obtained as shown by Eqs.(19),(20) and (21).

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