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Importance of local modes in seismic response of bridges

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ABSTRACT: The paper describes a simple approach to study the importance of local modes in the dimensioning load of bridge columns.

1 INTRODUCTION AND OBJECTIVES

The seismic analysis of bridges is a subject in fashion after the spectacular failures in recent eartquakes and the continuous development of infrastructure works in seismic regions.

The dynamic problems presented by bridges are qualitatively different from those of the ordinary regular buildings even for the simplest case of longitudinal vibrations of straight overcrossings. In spite of that the current regulations propose an "equivalent force" approach based on the application of the Rayleigh method which is not of automatic application to the computation transverse or vertical displacements.

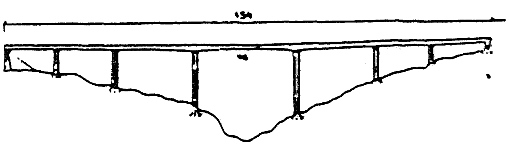

Figure 1

In addition, as will be shown below, the introduction of elastic supports in-between deck and columns can induce local modes that are fundamental to the evaluation of colum resistance and that the analyst can inadvertendly forget if a blind application of the rules for mode truncation is applied (for instance the 90 % mobilised mass criterion). The motivation for the study was the

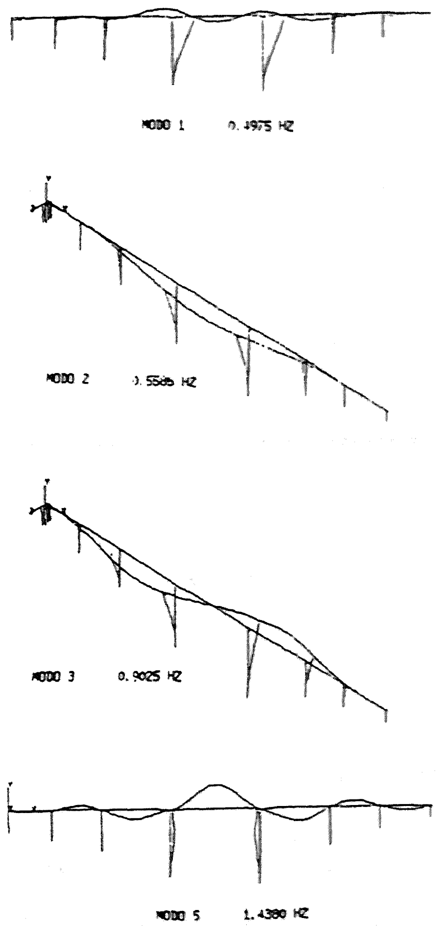

Figure 2a

analysis of the bridge represented in figure 1. The base shear needed a large amount of modes to be represented with a certain degree of accuracy. Fig. 2a shows some of the first modes, while figure 2b compares the modal shapes in the first mode and two local modes that are fundamental to represent the column behaviour.

The objective of the paper is to develop a simplified two-degrees of freedom model in which the importance of the colum vibration is showed following an asymptotic approach developed by Kelly (1988) in other context.

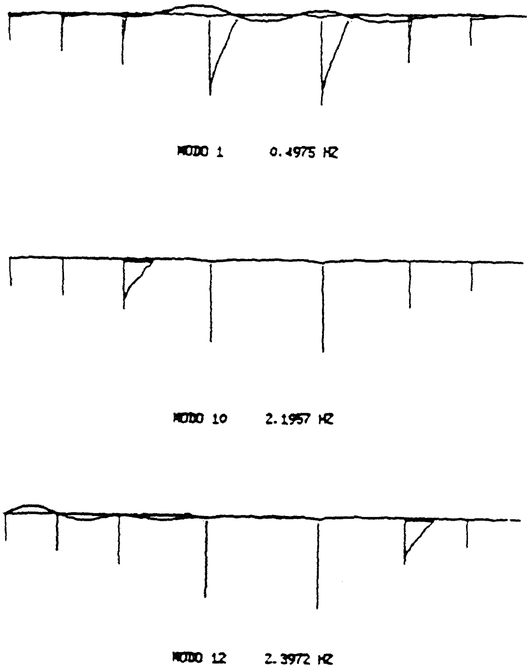

Figure 2b

2 SIMPLIFIED MODEL

Bridge, column and elastic connections are modelled by a two-D.O.F. system as shown in figure 3.

The mean mass M represents the deck, while the secondary mass m is that part of the column one associated to its displacement shape. The stiffness is modelled by k_p for the column and k_n for the elastic connections.

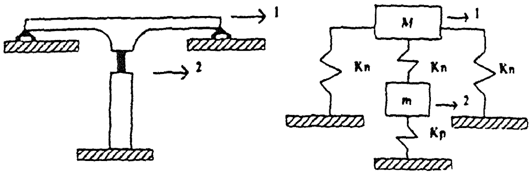

Figure 3

The analysis is conducted through two parameters ωs and ωp defined by the relations

$$\omega_s^2 = \frac{\alpha\, k_n}{M} \tag{1}$$

$$\omega_p^2 = \frac{k_n + k_p}{m} \tag{2}$$

where α is the number of equivalent elastic connections on which the deck is supported. Two characteristics ratios are

$$\varepsilon = \frac{\omega_s^2}{\omega_p^2} \tag{3}$$

$$\gamma = \frac{M}{m} \tag{4}$$

relating frequency and mass properties of deck and column.

2.1 General approach

The free vibration analysis is based on the following equations

$$\begin{bmatrix} M & 0 \\ 0 & m \end{bmatrix} \begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{Bmatrix} + \begin{bmatrix} \alpha k_n & -k_n \\ -k_n & k_n + k_p \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \underline{0}$$

or

$$\underline{m}\ddot{\underline{x}} + \underline{k}\underline{x} = \underline{0} \tag{5}$$

that allows the determination of modes and natural frequencies in the form

$$\begin{bmatrix} \omega_s^2-\omega^2 & -\frac{\omega_s^2}{\alpha} \\ -\frac{\omega_s^2}{\gamma\alpha} & \omega_p^2-\omega^2 \end{bmatrix} \begin{bmatrix} \phi_1 \\ \phi_2 \end{bmatrix} = \underline{0} \quad (6)$$

In this way

$$\left[\frac{\omega_1}{\omega_s}\right]^2 = \frac{1}{2\varepsilon}\left[(1+\varepsilon) - \sqrt{(1+\varepsilon)^2-4\varepsilon(1-\frac{\varepsilon}{\gamma\alpha^2})}\right]$$

$$\left[\frac{\omega_2}{\omega_p}\right]^2 = \frac{1}{2}\left[(1+\varepsilon) + \sqrt{(1+\varepsilon)^2-4\varepsilon(1-\frac{\varepsilon}{\gamma\alpha^2})}\right] \quad (7)$$

and the modal matrix is

$$\Phi = \begin{bmatrix} 1 & 1 \\ \alpha\left[1-(\frac{\omega_1}{\omega_s})^2\right] & \alpha\left[1-(\frac{\omega_2}{\omega_s})^2\right] \end{bmatrix} \quad (8)$$

For an earthquake acceleration acting at the base

$$\underline{m}\ddot{x} + kx = -\underline{mJ}\,\ddot{x}_s \quad (9)$$

where $\underline{J}$ is the influence vector.

The participation factors are

$$\Gamma_i = \frac{\underline{\phi}_i^T \underline{mJ}}{\underline{\phi}_i^T \underline{m}\underline{\phi}_i} = \frac{1+\alpha\gamma\left[1-(\frac{\omega_i}{\omega_s})^2\right]}{1+\gamma\alpha^2\left[1-(\frac{\omega_i}{\omega_s})^2\right]^2} \quad (10)$$

$i = 1,\ 2$

and the mobilized masses

$$M_i = \Gamma_i^2\,\underline{\phi}_i^T\,\underline{M}\,\underline{\phi}_i = M\,\frac{\left[1+\alpha\gamma\left[1-(\frac{\omega_i}{\omega_s})^2\right]\right]^2}{1+\alpha^2\gamma\left[1-(\frac{\omega_i}{\omega_s})^2\right]^2}$$

$i = 1,\ 2$

(11)

while the equivalent loads for every mode are

$$\underline{F}^i = M\,\Gamma_i \begin{bmatrix} 1 \\ \alpha\gamma\left[1-(\frac{\omega_i}{\omega_s})^2\right] \end{bmatrix} PSA(\omega_i;\zeta_i)$$

$i = 1,\ 2$

(12)

where PSA(ω_i; ζ_i) is the value of the pseudoacceleration for frequency ω_i and damping ratio ζ_i.

2.2 Asymptotic analysis

In some practical applications the stiffness of the column is at least one order of magnitude higher than that of the elastic bearing and the same happens between the deck and column masses. Then it is reasonable to assume values

$$\varepsilon \sim \frac{\alpha}{100} \qquad \gamma \sim \frac{1}{10} \quad (13)$$

to analyze what happens in the previous formula.

Operating in formula (7) we obtain

$$(\frac{\omega_1}{\omega_s})^2 = \frac{1}{2\varepsilon}\left[1+\varepsilon-(1-\varepsilon)\sqrt{1+\frac{4\varepsilon^2}{\gamma\alpha^2(1-\varepsilon)^2}}\right] \quad (14)$$

and

$$(\frac{\omega_2}{\omega_p})^2 = \frac{1}{2}\left[(1+\varepsilon)+(1-\varepsilon)\sqrt{1+\frac{4\varepsilon^2}{\gamma\alpha^2(1-\varepsilon)}}\right] \quad (15)$$

Using the limitations (13) it is easy to show

$$(\frac{\omega_1}{\omega_s})^2 \approx 1 - \frac{1}{\gamma\alpha^2}\,\varepsilon \qquad (\frac{\omega_2}{\omega_p})^2 \approx 1 - \frac{1}{\gamma\alpha^2}\,\varepsilon^2 \quad (16)$$

What means that $\omega_2 = \omega_p$ with a high precission.

Using those approximations in eq. 10 the following participation factors are obtained

$$\Gamma_1 = \frac{1+\frac{\varepsilon}{\alpha}}{1+\frac{\varepsilon^2}{\gamma\alpha^2}} \sim 1+\frac{\varepsilon}{\alpha}$$

$$\Gamma_2 = \frac{1+\alpha\gamma-\frac{\gamma\alpha}{\varepsilon}}{1+\gamma\alpha^2\left[1-\frac{1}{\varepsilon}\right]^2} \sim -\frac{\varepsilon}{\alpha} \qquad (17)$$

and the mobilized masses

$$M_1 \sim M\left[1+\frac{2\varepsilon}{\alpha}\right]$$
$$M_2 \sim \gamma M = m \qquad (18)$$

If γ is of the order of 10 % it is clear that a rule of the type "90 % of the mobilized mass" will preclude the use of the second mode. On the other hand the base shear contribution of that mode is very important. Using eq. 12 the equivalent forces of the first mode are

$$F^{(1)} = (PSA)\begin{bmatrix} M\left(1+\frac{\varepsilon}{\alpha}\right) \\ M\frac{\varepsilon}{\alpha}\left(1+\frac{\varepsilon}{\alpha}\right)\end{bmatrix} \qquad (19)$$

that is taking into account the α supports the base shear is

$$Q^{(1)} = M(PSA)_1\frac{1+\varepsilon}{\alpha}\left(1+\frac{\varepsilon}{\alpha}\right) \qquad (20)$$

while for the second mode

$$F^{(2)} = (PSA)_2\begin{bmatrix} -\frac{\varepsilon M}{\alpha} \\ M\gamma(1-\varepsilon)\end{bmatrix} \qquad (21)$$

producing

$$Q^{(2)} = M(PSA)_2\left[\gamma(1-\varepsilon)-\frac{\varepsilon}{\alpha^2}\right] \qquad (22)$$

Using, for instance

$$\frac{\varepsilon}{\alpha} \sim \frac{1}{100}$$
$$\gamma \sim \frac{1}{10} \qquad (23)$$
$$\alpha \sim 8$$

$$Q^{(1)} = M\,(PSA)_1\ 0{,}1362$$
$$Q^{(2)} = M\,(PSA)_2\ 0{,}1098$$

that clearly points out the importance of the second mode to analyse the column. That importance can grow if the second frequency is much higher than the first, so that spectral values can be in the flat of the spectrum.

3 PRACTICAL APPLICATION

On what follows we are going to discurs the application of the above mentioned formula to the bridge shown in fig.1.

Once the column to be analyzed has been selected, the first point is to establish the number α of equivalent elastic connections. As the frequency ω_s (eq.1) is established by fixing the column and displacing the deck, the "equivalent" stiffness is a combination of that of columns and their corresponding connections. It is easy to show that

$$\alpha = 1+\frac{1}{k_m^i}\sum_{j=1}^{n}\frac{k_p^j k_n^j}{k_p^j+k_n^j} \qquad (24)$$

where k_n means the elastic-support stiffness and k_p the column stiffness while i refers to the column under study and j to the other columns of the bridge.

The fixation of the column stifness depends obviously on the boundary conditions. Specially in relation with the scheme of colocation of the elastic supports in the free end of the column. In the above mentioned case it was found that a weighted combination of the fixed-free and fixed-fixed stiffness of the column such

$$k_p = \frac{3}{4}k_p^{fix\text{-}free}+\frac{1}{4}k_p^{fix\text{-}fixed} \qquad (25)$$

produced the best model.

Table I allows the comparison of the results obtained with several approaches. Row 1 collects the results obtained for the base shear without taking into account the local modes. As can be seen tthe 3rd row obtained using eq.(20,22) compares very well with those results and the same happens when the comparison is done between a more axact (30 modes) mode superposition and the proposed approach.

Table 1

	Column number			
	2	3	6	7
MODAL ANALYSIS				
9 Modes	89	88	88	92
30 Modes	224	238	253	200
SYMPLIFIED ANALYSIS				
1st mode contrib	86	82	83	89
1st + 2nd	202	209	222	181

For instance in colum 6 it was found

$$\alpha = 13{,}95$$
$$\gamma = 3{,}69 \ . \ 10^{-2}$$
$$\varepsilon = 3{,}98 \ . \ 10^{-2}$$
$$M = 1{,}12 \ . \ 10^{7} \ Kg$$
$$m = 0{,}414 \ . \ 10^{6} Kg$$
$$(PSA)^{1} = 1{,}1139 \ \frac{m}{seg^{2}}$$
$$(PSA)^{2} = 5{,}3663 \ \frac{m}{seg^{2}}$$

which produce the following contributions to the base shear

$$Q^{1} = 0{,}82 \ . \ 10^{6} \ N$$
$$Q^{2} = 2{,}06 \ . \ 10^{6} \ N$$

whose composition by the SRSS rule produces the above mentioned value of 222 ton.

CONCLUSIONS

A simple approach to detect the contribution of local modes in bridges has been presented. It is interesting to see that even if an accusare model is not produced the method allows a qualitative estimate of the importance of those modes, whose contribution to the strength assesment of the columns can be definitive.

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