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Damage assessment of existing bridge structures with system identification

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ABSTRACT: A method to estimate the damage of existing bridge structures is developed using results of system identification. Dynamic behavior of damaged structures is represented by a nonlinear hysteretic moment model. Structural properties can be evaluated through system identification. To incorporate the variability of the structural properties and the effects of stochastic excitations, response statistics are obtained through random vibration and damage is represented as random quantities. A numerical example is illustrated for a bridge structure under different ground excitations.

1 INTRODUCTION

Damage has been observed in many bridge structures during recent earthquakes. Damage, however, is still determined by intuition, experience and judgement of engineers or as a function of simple quantities, such as maximum deformation or change of stiffness. Considering that damage is a nonlinear function of the excitation, a more systematic approach is essential in damage estimation, that includes in particular the nonlinear characteristics of a structure.

A method for damage estimation incorporating system identification is suggested. Structural response is analyzed through random vibration under earthquake excitations. Structural properties of an existing structure and their respective uncertainties are determined through system identification. In particular, the extended Kalman filtering algorithm is used to identify the structural parameters. For this purpose, a hysteretic model is developed to describe the nonlinear behavior of a moment-resisting frame, and the corresponding parameters of a system are obtained through filtering using measured excitation and response. With these identified parameters, a structure is analyzed and damage is assessed at locations where damage hinges are expected to occur. Damage is represented in terms of damage indices at the hinges from which the overall damage index of a bridge is determined.

The proposed method is applied to a horizonatlly circular bridge as a numerical example. The structural properties were determined from the identified parameters; degradation of properties relative to those of the undamaged structure is included. The calculated damage index is compared with the observed earthquake damage. Also, fragility curves are obtained to illustrate the potential damage of similar structures under earthquakes of different ground intensities.

2 DAMAGE MODEL FOR BRIDGE STRUCTURES

2.1 *Discretization of Bridge for damage analysis*

Bridges are usually constructed of flexural members or frame systems. Moment-curvature relations govern the nonlinear behavior of a structure and, therefore, may be modeled as moment-resisting frames for damage assessment purposes. Damages may occur anywhere along a member or structure. However, for mathematical simplicity, damages may be idealized as concentrated at appropriate nodes where damages are likely to be high. Accordingly, a discretized model for bridge damage analysis can be represented as nodes and beam elements as shown in Fig. 1, and damages are concentrated and defined at the nodes.

For damage analysis, plastic hinges are assumed to occur at the nodes under strong excitations, whereas the beam elements will remain elastic. Accordingly, damages are calculated only at locations where potential hinges may occur. For the bridge shown in Fig. 1, three nodes are defined at each beam-column joint, at which different levels of damage may be observed at these locations.

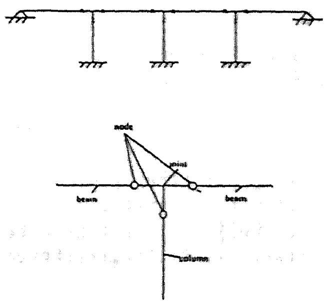

Figure 1. Bridge model for damage analysis

2.2 *Model for structural damage*

Damage of a reinforced concrete member using moment-curvature relation will be expressed in terms of a damage index

$$D_r = \frac{\phi_M}{\phi_U} + \frac{\beta_E}{M_y \phi_U} \int dE \qquad (1)$$

where, ϕ_M = maximum response curvature under an earthquake; ϕ_U = the ultimate curvature capacity under monotonic loading; M_y = yield moment; dE = incremental dissipated hysteretic energy; β_E is a non-negative constant which is a function of steel ratio, axial force and stirrup ratio given as

$$\beta_E = [0.37 n_0 + 0.36(k_p - 0.2)^2] 0.9^{\rho_w} \qquad (2)$$

in which, k_p is normalized steel ratio and given as $\frac{\rho_t f_y}{0.85 f_c'}$ and n_0 is the normalized axial stress given by $\frac{N}{bd f_c'}$.

To include the structural damage capacity, D_a, damage of a reinforced concrete member can be described as (Park, et al, 1985)

$$D_i = \frac{D_r}{D_a} \qquad (3)$$

where, D_i = damage index ratio for node i of a structure; D_r = structural damage; D_a = ultimate damage capacity with μ_{Da} = 1.0 and σ_{Da} = 0.54. To incorporate the uncertainties in the random response, the maximum curvature and hysteretic energy are represented as random quantities; the mean and variance of damage at each node can be calculated as

$$\begin{aligned} \bar{D}_i &= (1+\sigma_D^2)(\frac{\bar{\phi}_M}{\phi_U} + \frac{\beta_E}{M_y \phi_U} \int_0^t dE) \\ Var[D_i] &= \sigma_D^2 \bar{D}_r^2 + \frac{1}{\phi_U^2} Var[\phi_M] + (\frac{\beta_E}{M_y \phi_U})^2 Var[\int_0^t dE] \end{aligned} \qquad (4)$$

2.3 *Response statistics*

To calculate the damages of the nodes, the means and variances of the respective maximum curvatures and dissipated hysteretic energies are necessary. For this purpose, the following are required: a nonlinear model for bridge structures, a ground motion model, and a method for response analysis.

1. Nonlinear Model -- The equations of motion including the nonlinear restoring forces can be written as

$$[M]\{\ddot{X}\} + [C]\{\dot{X}\} + \{F_{NL}\} = -[M]\{J\}\ddot{x}_g \qquad (5)$$

where, [M] = mass matrix; [K] = stiffness matrix; {J} = direction vector; and $\ddot{x}_g$ = ground acceleration. The nonlinear force vector, $\{F_{NL}\}$, is defined from the moments at the nodes to incorporate the damages and given as,

$$\{F_{NL}\} = [K]\{X\} + (1.0 - \alpha)[T][K_{el}](\{z\} - \{\phi\}) \qquad (6)$$

where, [T] is a matrix transforming moments in local coordinates to forces in global coordinates, and $[K_{el}]$ is the element stiffness matrix. Observe that α = 1.0 in Eq.6 indicates linear restoring force and that nonlinear forces are calculated from the locations where plastic hinges are expected to occur. The curvature at each node, $\{\phi\}$, can be obtained from the modal displacement vector, and the hysteretic component, z, can be described as

$$\dot{z} = \frac{A\dot{\phi} - \nu\{\beta|\dot{\phi}||z|^{n-1}z + \gamma\dot{\phi}|z|^n\}}{\eta} \qquad (7)$$

where, α, β, γ and n are constants related to the hysteretic restoring force characteristics; A, ν and η are parameters related to degradation and are functions of the dissipated energy (Sues, et al, 1985).

In this study, the state vector approach is used to solve the equations of motion and mode superposition is adopted to reduce the number of variables. In such a case, the equations of motion are changed to,

$$\{\ddot{W}\} = [-2\xi\omega]\{\dot{W}\} - [\omega^2]\{W\} - (1-\alpha)\frac{[\Phi^T][T]}{[\Phi]^T[M][\Phi]}[K_{el}](\{z\} - \{\phi\}) - \{\Gamma\}\ddot{x}_g \qquad (8)$$

where, [Γ] is modal participation vector; ξ is damping ratio and ω is the natural frequency of the structure.

2. Modeling of Ground Motion -- Earthquake ground motion is modeled as a zero mean filtered Gaussian shot noise with a Kanai-Tajimi spectrum. To model the non-stationarity in the ground motion, its intensity is modulated by the Amin-Ang type envelope function (Amin and Ang, 1968).

3. Random Vibration Analysis -- Using equivalent linearization (Baker and Wen, 1981), the underlying nonlinear-hysteretic random vibration problem is reduced (Kim and Ang, 1992) to the solution of the following stochastic differential equation:

$$\frac{d}{dt}S = GS + SG^T + B \qquad (9)$$

where: $S = E[y(t)y(t)^T]$ is the covariance of the response,
$B_{ij} = 0$ except $B_{22} = I(t)$
I(t) = intensity function of excitation, and

$$G = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ -\omega_g^2 & -2\xi_g\omega_g & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ \omega_g^2 & 2\xi_g\omega_g & -\alpha\frac{K}{m} & -\frac{c}{m} & -(1-\alpha)\frac{K}{m} \\ 0 & 0 & 0 & C_{eq} & K_{eq} \end{bmatrix}$$

Also,

$$\frac{d}{dt}y = Gy + F \qquad (10)$$

in which,

$$
\begin{aligned}
y &= \{x_g, \dot{x}_g, w, \dot{w}, z\}^T \\
F &= \{0, \ddot{x}_g, 0, 0, 0\}
\end{aligned}
\tag{11}
$$

and, x_g, $\dot{x}_g$ and $\ddot{x}_g$ are displacement, velocity and acceleration of the soil, respectively; w and $\dot{w}$ are modal displacement and velocity of the structure.

4. Evaluation of Response Statistics -- The mean and variance of the maximum curvature can be obtained assuming that the nonstationary peak has a Weibull distribution (Yang and Liu, 1981). The mean hysteretic energy is obtained by solving Eq. 9, whereas its variance requires the solution of another differential equation (Pires, et al, 1983). Approximation can also be obtained (Kwok and Ang, 1987) showing that the C.O.V. is fairly constant around 0.2.

2.4 *Global damage index*

The global damage is necessarily a function of the damages of the nodes. In general, however, this functional relationship may be expressed in terms of probability. Specifically, the event that the global damage is greater than damage level d can be defined as

$$(D_T > d) = \cup(D_i > d) \tag{12}$$

where, D_T is the global damage of the structure; and D_i is the damage index at node i, where $\cup$ stands for the union of events.

The probability of the global damage exceeding the damage level d is then expressed as

$$P(D_T > d) = P[\cup(D_i > d)] \tag{13}$$

The required probability may be evaluated using the geometric mean of its second-order bounds, which can be written as,

$$
\begin{aligned}
P(E_1) + \sum_{i=2}^{k} max[\{P(E_i) - \sum_{j=1}^{i-1} P(E_i E_j)\}; 0] \le P(D_T > d) \\
P(D_T > d) \le \sum_{i=1}^{k} P(E_i) - \sum_{i=2}^{k} \max_{j<i} P(E_i E_j)
\end{aligned}
\tag{14}
$$

where,

$$P(E_i) = P(D_i > d) \tag{15}$$

in which i denotes the ith node where damage hinges are expected to occur.

To calculate $P(E_i)$, the performance function is defined considering the maximum curvature and hysteretic energy. Using the damage model defined in Eqs. 1 and 3, the performance function can be written as

$$g_i(X) = \frac{aX_i + bY_i}{D} - d \tag{16}$$

where, $a = \frac{1}{\phi_U}, b = \frac{\beta_e}{M_y \phi_U}, X_i = \phi_{M_i}, Y_i = \int dE_i, D = D_a$

and, d is the ratio of damage relative to the ultimate damage capacity, D_a.

The safety index for each node is calculated on the basis of Eq. 16 and the correlation coefficients between nodes are also obtained. Assuming log-normal distribution for the global damage, the two parameters λ and ζ of the log-normal distribution can be evaluated using any two probability levels obtained above.

On the basis of the derived lognormal distribution for the global damage, its median and logarithmic standard deviation can then be evaluated.

3 SYSTEM IDENTIFICATION OF STRUCTURAL PARAMETERS

The dynamic equations of motion can be converted into a set of nonlinear state equation as,

$$\{\dot{X}\} = \{f(X, \ddot{x}_g, t)\} + \{w(t)\} \tag{17}$$

where {X} is the state vector, and {w} is the system noise vector with covariance v_w.

An observational vector {Y(t)} is related to the state vector as

$$\{Y(t)\} = [O]\{X(t)\} + \{\eta(t)\} \tag{18}$$

where [O] is a matrix associated with the observations, and $\{\eta(t)\}$ is an observational noise vector with covariance v_η.

The extended Kalman filtering is basically a recursive process for estimating the conditional state vector based on the observed excitation and response. The method has been successfully applied to linear as well as nonlinear parmeter estimations (Hoshiya, 1984). Details of its application to the identification of nonlinear parameters of bridge structures can be found in Kim and Ang (1992).

4 NUMERICAL EXAMPLE

In this study, the Highway 5/14 overcrossing is selected as a numerical example. This bridge is a typical highway overcrossing constructed with piers and has an arch shape plan. The structure collapsed during the 1971 San Fernando earthquake; a 1/30 scale model study was performed (Williams and Godden, 1976) to examine the seismic behavior of this bridge. The dimensions of the actual and model structures are shown in Table 1 and the description of the original actual structure is shown in Fig. 2. Structural properties of the model structure are obtained through extended Kalman filtering (Yun, et al, 1989) using measured time histories and the results are converted to the actual structure using appropriate scaling relations.

Table 1: Description of Model and Real Structure

	real structure	*model*
Total Length	$636ft$	$254.5inch$
Radius of Curvature	$270ft$	$108inch$
Column Height	$90ft$	$36inch$
Deck Section	$30ftX7ft$	$8.5inX2.5in$
Column Section	$10ftX5ft$	$4inX2in$

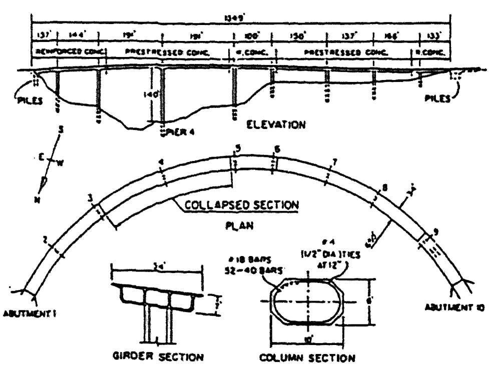

Figure 2. Description of Highway 5/14 overcrossing

The properties of the actual structure needed for damage estimation are summarized in Table 2.

Table 2: Structural Properties of Highway 5/14 Overcrossing

α	β	γ	ξ	ω	ξ_g	ω_g
0.05	$6.09X10^6$	$-2.03X10^6$	0.10	1.15	0.8	15.08

Damage is calculated from the expected maximum curvature and dissipated energy at the locations shown in Fig. 3 and the damage at the corresponding nodes are summarized in Table 3. The global damage of the structure is determined as a function of the damages at the nodes according to Eq. 13, and the results are summarized in Table 3.

Table 3: Calculated Damage Index for Each Node (Mean)

node	$1/6g$	$1/3g$	$1/2g$	$2/3g$	$5/6g$	$1g$
1	0.0107	0.0428	0.0963	0.1714	0.2680	0.3856
2	0.0361	0.1446	0.3257	0.5794	0.9059	1.3032
3	0.0090	0.0361	0.0812	0.1445	0.2259	0.3249
4	0.0050	0.0200	0.0449	0.0800	0.1250	0.1798
5	0.0074	0.0298	0.0671	0.1193	0.1866	0.2684

The coefficient of variation of the global damage is fairly constant at a value of approximately 0.62. For this uncertainty, 0.54 comes from the variability of the structural properties and the remainder can be attributed to the randomness in the structural response. This structure collapsed around 0.87g of ground excitation during the 1971

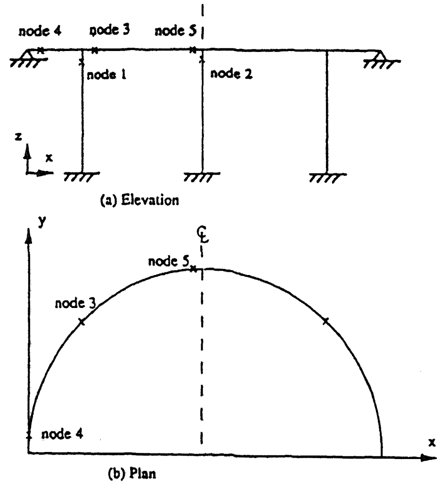

Figure 3. Analytical modeling of Highway 5/14 overcrossing

Table 4: Global Damage Statistics under Different Intensities

	$1/6g$	$1/3g$	$1/2g$	$2/3g$	$5/6g$	$1g$
Global Damage	0.0307	0.122	0.277	0.493	0.770	1.11
C. O. V.	0.620	0.620	0.620	0.620	0.621	0.621

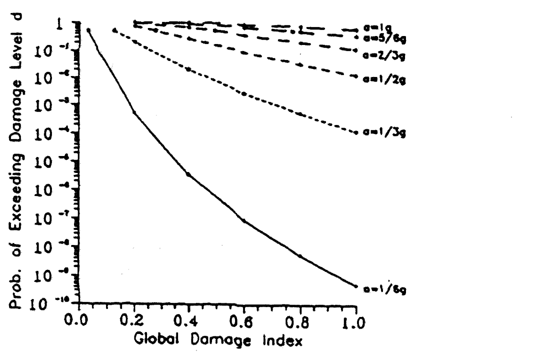

Figure 4. Probability of exceeding damage level

San Fernando earthquake. Collapse appears to be primarily caused by the concentration of damage at the top of the center pier (node 2 in Fig. 3). In accordance with this observation, the structure can be considered to collapse at approximately a global damage index of 0.8 and the probability of exceeding different damage level d is shown in Fig. 4. From these cumulative probability functions, the median and C.O.V. of the global damage are calculated for different earthquake intensities; the results are shown in Fig. 5. Assuming that the probability of collapse is the probability of exceeding damage level 0.8, the resulting fragility curve for this bridge would be as shown in Fig. 6.

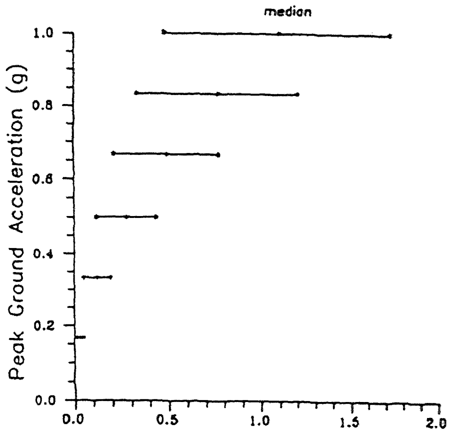

Figure 5. Global damage index

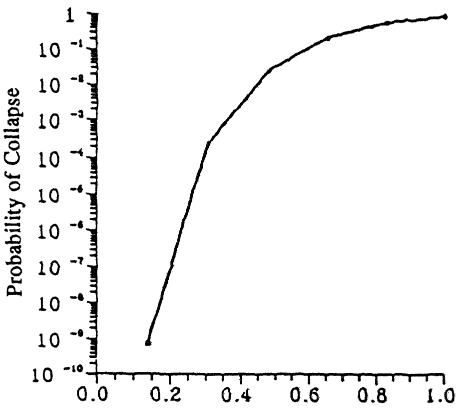

Figure 6. Fragility curve

5 CONCLUSION

A quantitative method for the damage estimation of existing bridge structures is developed using identified structural properties. A numerical example indicates that bridge structures would collapse around a global damage index of 0.8. To increase resistance to future earthquake loadings, a structure can be strengthened in order to avoid damage concentration. The suggested method can be used with structural parameters identified through simple tests. Information provided by the proposed damage assessment method, including the expected global damages at different earthquake intensities as well as the corresponding fragility curves, should be useful for decisions related to needed rehabilitation of bridge structures for seismic resistance.

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