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Dynamic response analysis of three-dimensional strutted rigid-frame bridge with horizontally bent-angles

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ABSTRACT: Natural vibration analysis of a space strutted rigid-frame bridge with horizontally bent-angles is presented by using the lumped and consistent mass methods. In three-dimensional free vibration, it is estimated that there is considerably effect of the flexural rigidity of lateral bracing members and effective widths of floor beam systems. The effect of warping deformation is indicated clearly the eigenvalues of torsional mode shapes of three-dimensional vibrating bridges. In the response spectrum calculations of the complete quadratic combination method, the values of modal cross-correlation coefficients estimated by some higher modes are larger than those estimated by the first few lower modes.

1 INTRODUCTION

Numerous space rigid-frame bridges have been constructed for use as highway bridges in earthquake-prone areas. It is important to take into account the structurally complex superstructures that characterize three-dimensional strutted rigid-frame bridges with horizontally bent-angles. Because of the long spans typically involved, their superstructures are relatively flexible to axially, vertically, transversely and torsionally motions of three-dimensional vibrating bridges. In general, the structural response of space rigid-frame bridges subjected to seismic loads is affected significantly on both the lower and higher mode shapes of vibration. Therefore, it is essential in designing these kinds of bridge structures that the natural frequencies and mode shapes be determined accurately.

Free vibration of three-dimensional framework structures is usually analyzed by using the lumped and the consistent mass matrices based on the displacement method (i.e., finite element approach). However, the effects of lateral bracing members and warping deformations of bridge structures have been neglected in many technical papers for simplicity. The strutted rigid-frame bridge presented in this study has horizontally bent-angles on main girders and is composed of thin-walled beam members. In three-dimensional free vibrations, the effects of the bending stiffness of lateral bracing members, of warping deformations of main girders and of effective widths of floor slab are investigated. The mathematical idealization of bridge structures for three-dimensional free vibrations is discussed, and the accuracy of eigenvalues obtained by the lumped and consistent mass methods is evaluated. For the asymmetrical rigid-frame bridge structures, the closeness of natural frequencies and the complex nature of the mode shapes are realized. Thus, the Complete Quadratic Combination (CQC) method is applied in seismic analysis to combine the modal maxima. The values of modal cross-correlation coefficients are discussed in numerical results of the strutted rigid-frame bridge.

2 NATURAL VIBRATION ANALYSIS

In general, the coordinate systems of structures are divided into two different basic types: distributed coordinate systems and discrete coordinate systems (see Clough and Penzien (1975)). The first type is the distributed-parameter system, which is applied to structures whose physical properties are continuously distributed in three-dimensional space. In this case, the basic relation between end forces and displacements for a beam segment subjected to axial, flexural and torsional vibrations is formulated by using general solutions of differential equations of motion. The above general solutions lead to a dynamic stiffness matrix, which includes natural circular frequencies of vibrating structures. This approach results in exact solutions of vibration and is called the continuous mass method by Ovunc (1974) or the eigen stiffness matrix method by Hayashikawa and Watanabe (1985).

The second type is the lumped-parameter system, which defines forces and displacements at a set of discrete nodal points in terms of components having specified directions. The analytical procedure for this type can be greatly simplified as an eigenvalue problem because the inertia forces are developed only at these points. In the space framework structures, the lumped mass matrix is derived as a diagonal matrix by applying half the mass and its associated mass-moment of each beam segment to the appropriate nodal point. Moreover, the mass influence coefficients are evaluated by a procedure similar to that used to determine the static stiffness coefficients. The resulting matrix is called the consistent mass matrix (see Paz (1980)), and it contains many off-diagonal terms due to the effect of mass coupling.

The typical categories of the prescribed coordinate systems are shown in Figure 1. The choice of the discrete or distributed coordinate systems depends upon the geometry of the bridge structure, the location and nature of constrains, the distribution of applied forces and the necessary information concerning deflections.

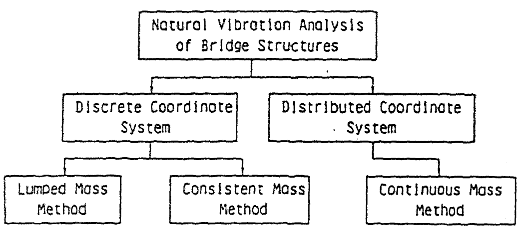

Figure 1. Schematical description of coordinate systems

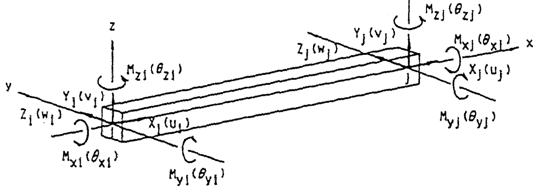

Figure 2. Beam segment of a space frame showing forces and displacements

Figure 2 shows a beam segment of a three-dimensional frame with its 12 nodal coordinates: three translations along the x-, y-, z-axes and three rotations about these axes. The dynamic analysis of three-dimensional rigid-frame structures results in a comparatively longer computer program in general requiring substantially more input data and the availability of a computer with larger storage memory. However, it is not a serious problem as a result of the recent development of high-speed digital computers.

In this study, the discrete coordinate systems are used as a convenient and practical approach to the dynamic response analysis of bridge structures, including assemblages of many members in three-dimensional space.

3 DYNAMIC RESPONSE ANALYSIS

The formulation of the earthquake-response analysis of a discrete coordinate system can be carried out in matrix notation. The dynamic equilibrium equation for a space structural system subjected to a ground acceleration $\ddot{u}(t)$ is written as follows:

$$[M]\{\ddot{U}\}+[C]\{\dot{U}\}+[K]\{U\}=\{Mo\}\ddot{u}(t) \qquad (1)$$

in which [M], [C] and [K] are the mass, damping and stiffness matrices, respectively. The relative displacements, velocities and accelerations of structures are indicated by $\{U\}$, $\{\dot{U}\}$ and $\{\ddot{U}\}$, respectively. The column vector {Mo} contains the components of mass in the x-, y- and z-directions. The solution {U} of equation (1) is found by the mode superposition

$$\{U\}=[\Phi]\{Y\} \qquad (2)$$

in which $[\Phi]$ is the modal matrix, and {Y} represents the normal coordinates. By substituting equation (2) into equation (1) and due to the orthogonal properties of the mode shapes, equation (1) reduces to a set of uncoupled equations for proportional damping

$$\ddot{Y}_i+2\xi_i\omega_i\dot{Y}_i+\omega_i^2Y_i=p_i\ddot{u}(t) \qquad (3)$$

in which ξ_i is the damping ratio for mode i and ω_i is the i-th natural circular frequency of structures. The participation factor p_i is obtained as follows:

$$p_i=\{\phi_i\}^T\{Mo\}/\{\phi_i\}^T[M]\{\phi_i\} \qquad (4)$$

The equation (3) for the i-th mode is independent of those for all other modes. Therefore, it may be integrated directly to yield the time history solution for the normal coordinate $Y_i(t)$. The total structural displacements {U} are calculated from the equation (2).

To evaluate the earthquake response of Multi-Degree-of-Freedom (MDOF) systems at any time involves the computation of the Duhamel integral at that time for each significant response mode. The evaluation of the maximum response requires that each modal response be computed in this way for each time during the earthquake history. This obviously constitutes a major computation effort and makes an approximate analysis based on the ground-motion response spectra an attractive alternative. The complete development of the CQC method is presented by Kiureghian (1979 and 1980). The CQC method requires that all modal response terms be combined for a typical displacement u_k by the following equation

$$u_k=\sqrt{\sum_i \sum_j u_{ki}\rho_{ij}u_{jk}} \tag{5}$$

in which u_{ki} is a typical component of the modal displacement response vector. In general, the cross-modal coefficients ρ_{ij} are functions of the duration and frequency content of the loading and of the natural frequencies and damping ratios of the structure. If the duration of the earthquake is long compared to the periods of the structure, and if the earthquake spectrum is smooth over a wide range of frequencies, it is possible to approximate the coefficients by the following

$$\rho_{ij}=\frac{8\sqrt{(\xi_i\xi_j)}(\xi_i+r\xi_j)r^{3/2}}{(1-r^2)^2+4\xi_i\xi_j r(1+r^2)+4(\xi_i^2+\xi_j^2)r^2} \tag{6}$$

in which $r=\omega_j/\omega_i$. For constant modal damping ξ, the equation (6) reduces to

$$\rho_{ij}=\frac{8\xi^2(1+r)r^{3/2}}{(1-r^2)^2+4\xi^2 r(1+r)^2} \tag{7}$$

If the cross-modal coefficients ρ_{ij} of all natural modes are $\rho_{ij}=1$ for $i=j$ and $\rho_{ij}=0$ for $i\neq j$, then the maximum response of displacements calculated by the CQC method agrees with that by the Square-Root-of-Sum-of-Squares (SRSS) method.

4 NUMERICAL RESULTS

4.1 Numerical example

A numerical example is presented to investigate some dynamic characteristics of three-dimensional vibrating strutted rigid-frame bridges. The bridge geometry and the span lengths are given in Figure 3, and the structural properties are summarized in Table 1. The boundary conditions of the bridge consist of movable supports at A1 and A2, and pin supports at P1 and P2 as shown in Figure 3. The superstructure of the rigid

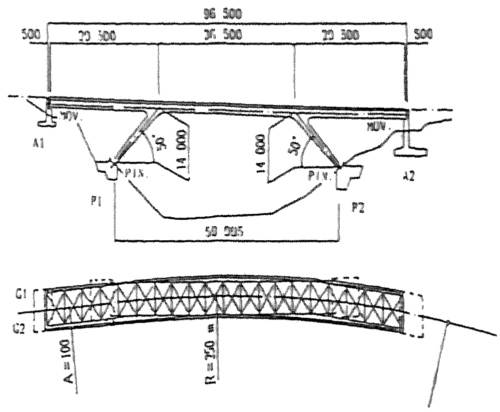

Figure 3. General view of strutted rigid-frame bridge with horizontally bent-angles

frame bridge has horizontally bent-angles at the connecting point between the strut and the main girder. And the radius of curvature on the floor slab is 250m.

4.2 Effect of lateral bracing members

The three-dimensional framework models without and with lateral bracing members are shown in Figures 4(a) and 4(b), respectively. The number of total nodal points of the bridge is 66 unrelated to lateral bracings. In this study, the natural periods and mode shapes are calculated by the lumped and

Table 1. Structural properties of strutted rigid-frame bridge

Members	$A(m^2)$	$I_x(m^4)$	$I_y(m^4)$	$I_z(m^4)$	w(t/m)
Main girder (G1, G2)	0.0325 ~0.0704	0.209×10^{-5} ~1.660×10^{-5}	0.0193 ~0.0506	0.147×10^{-3} ~1.228×10^{-3}	4.906 ~5.528
Floor beam	0.0206	0.626×10^{-6}	0.0076	0.265×10^{-4}	0.217
Leg strut	0.0155	0.764×10^{-6}	0.0011	0.852×10^{-4}	0.840
Lateral bracing	0.0038	0.200×10^{-6}	0.716×10^{-5}	0.710×10^{-5}	0.056
Sway bracing	0.0065	0.200×10^{-6}	0.545×10^{-4}	0.178×10^{-4}	0.147

consistent mass methods. Figures 5 and 6 show the first 10 natural mode shapes of the space strutted rigid-frame bridge without and with lateral bracing members, respectively. In the case of the vibrating bridge

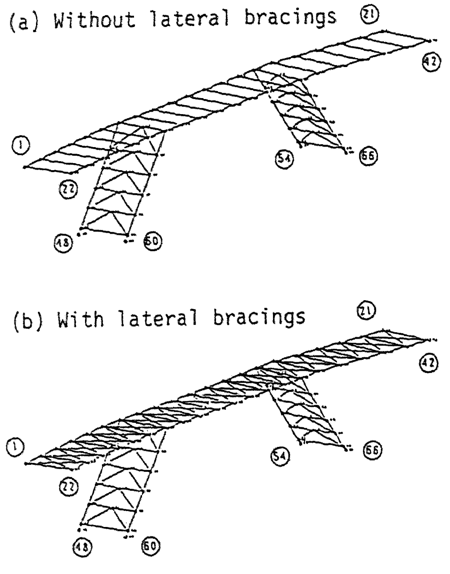

Figure 4. Three-dimensional framework models

without lateral bracings, there are unnatural mode shapes with the exception of the 3rd, 9th and 10th modes. This is dependent on the mismatching mathematical model which neglects the stiffness effect of lateral bracings in dynamic analysis. The 3rd, 9th and 10th mode shapes in Figure 5 correspond to the 1st, 3rd and 4th mode shapes presented in Figure 6, respectively. The other mode shapes as shown in Figure 5 are indistinct computed results. The relationship between the natural circular frequency and the mode order is presented in Figure 7.

4.3 Idealization of mass

Figure 8 shows the computed natural circular frequencies due to the different treatments of beam mass. According to a large number of nodal points, there is no considerable difference between the two computed natural circular frequencies by using the lumped and consistent mass methods. In general, the values of natural circular frequencies calculated by the consistent mass method are relatively large in comparison with those of the lumped mass method. Also, the computed values of natural circular frequencies in consideration of mass of each beam segment are in agreement with those obtained by

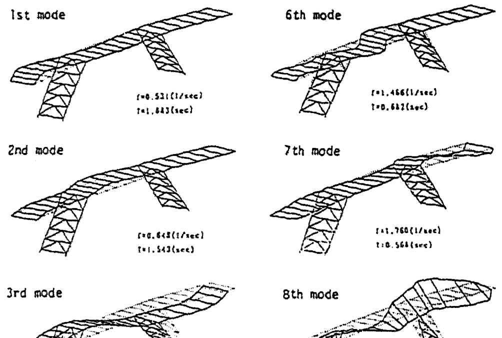

Figure 5. Computed natural mode shapes (without lateral bracing members)

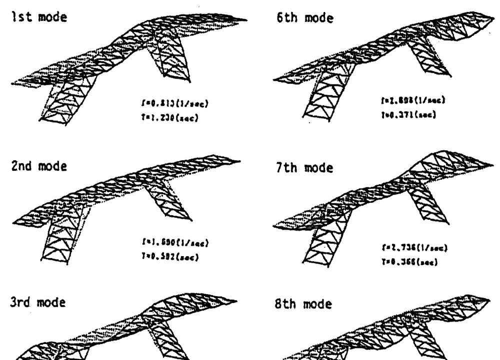

Figure 6. Computed natural mode shapes (with lateral bracing members)

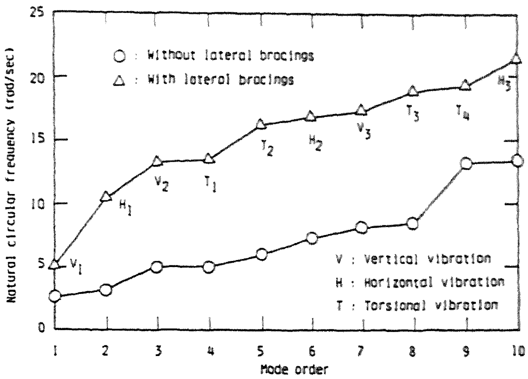

Figure 7. Effect of lateral bracing members

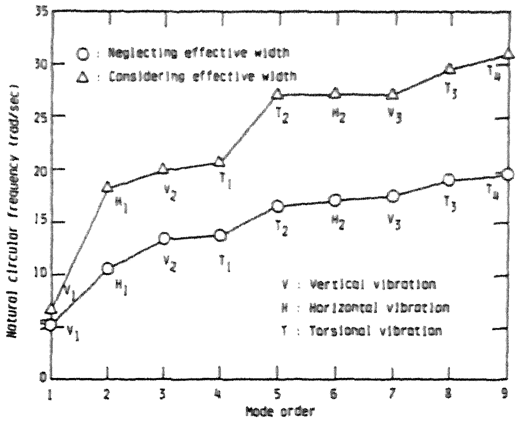

Figure 9. Effect of effective width

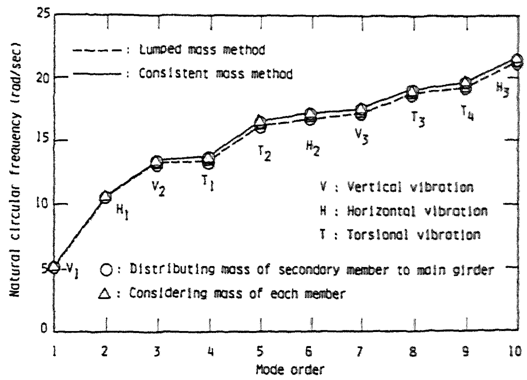

Figure 8. Effect of mass of structural beam members

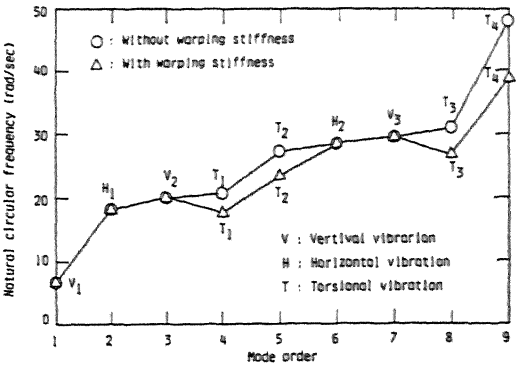

Figure 10. Effect of warping stiffness

distributing mass of secondary members of the bridge to main girders.

4.4 Effective width of floor slab

This space strutted rigid-frame bridge has the concrete floor slab with the radius of curvature R=250m. Then, the composite effect between the main girder of steel and the concrete slab is expected. By evaluating the flexural rigidity of main girders as the composite girders,the computed results in consideration of effective widths are shown in Figure 9, in comparison with those by neglecting effective widths. The adoption of the effective widths of slab offers the advantage of increased stiffness. This increased stiffness of the bridge has the tendency to increase the values of natural circular frequencies, in other words, to decrease the values of natural periods. In general, the values of natural circular frequencies calculated by considering the effective widths are larger than the computed results by neglecting the effective widths.

4.5 Warping deformation of main girders

The main girders of the bridge presented in this study consist of a thin-walled plate of I-type cross section. Also, these main girders have horizontally bent-angles at the strut legs. Natural vibration analysis in consideration of the bending-torsional stiffness of main girders is carried out. The numerical results obtained by considering warping stiffness are compared with those by neglecting warping stiffness as shown in Figure 10. In the torsional modes, there is a slight difference between the calculated results without and with warping stiffness. While in the vertical and horizontal vibration modes, it can be considered that there is no effect on warping deformation of main girders.

Table 2. Computed modal cross-correlation coefficients of strutted rigid-frame bridge

Mode order	Natural frequency (Hz)	Modal cross-correlation coefficients ρ_{ij}									
		1	2	3	4	5	6	7	8	9	10
1	0.813	1.0000	0.0027	0.0014	0.0013	0.0009	0.0008	0.0008	0.0007	0.0007	0.0005
2	1.690	0.0027	1.0000	0.0298	010236	0.0083	0.0069	0.0065	0.0046	0.0044	0.0027
3	2.121	0.0014	0.0298	1.0000	0.6624	0.0376	0.0266	0.0238	0.0133	0.0125	0.0061
4	2.183	0.0013	0.0236	0.6624	1.0000	0.0506	0.0340	0.0300	0.0159	0.0148	0.0069
5	2.595	0.0009	0.0083	0.0376	0.0504	1.0000	0.5117	0.3642	0.0768	0.0662	0.0176
6	2.698	0.0008	0.0069	0.0266	0.0340	0.5117	1.0000	0.8941	0.1393	0.1551	0.0233
7	2.736	0.0008	0.0065	0.0238	0.0300	0.3642	0.8941	1.0000	0.1791	0.1451	0.0259
8	2.980	0.0007	0.0046	0.0133	0.0159	0.0768	0.1393	0.1791	1.0000	0.9243	0.0600
9	3.014	0.0007	0.0044	0.0125	0.0148	0.0662	0.1551	0.1451	0.9243	1.0000	0.0691
10	3.490	0.0005	0.0027	0.0061	0.0069	0.0176	0.0233	0.0259	0.0600	0.0691	1.0000

4.6 Modal cross-correlation coefficients

By assuming 2 percent damping and the natural circular frequencies obtained by the consistent mass method, the modal cross-correlation coefficients ρ_{ij} of equation (7) can be calculated as shown in Table 2. As seen in the example structure, the closeness of the natural frequencies between the 3rd and 4th modes, between the 6th and 7th modes and between the 8th and 9th modes is recognized. The modal cross-correlation coefficients corresponding to these modes are considerably large in comparison with the others in Table 2. Therefore, it can be considered for this case that the higher modes significantly influence seismic response analysis for combining modal maxima.

5 CONCLUSIONS

Natural vibration analysis of the three-dimensional strutted rigid-frame bridge with horizontally bent-angles is presented by the lumped and consistent mass methods. The mathematical models of bridge structures for three-dimensional free vibration are investigated in this presentation. It is impossible to disregard the effects of lateral bracings as the secondary members and of effective widths of concrete floor slabs in discussing the eigenvalue problem of space framework structures. Although there is a slighting effect of warping stiffness of main girders in the torsional modes, it can be concluded that there is no effect of mass from mathematical treatment in the calculated results of natural circular frequencies. The values of modal cross-correlation coefficients estimated by some higher modes are larger than those calculated by the first few lower modes. It is considered that the higher modes take an important role in the dynamic response analysis of these kinds of three-dimensional bridge structures.

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