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Calculation of natural periods of bridge structures by using static frame method

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ABSTRACT: Two different matrix methods for determining natural periods and mode shapes of three-dimensional vibrating continuous beam bridges are presented. One method is the consistent mass method based on the finite element approach of space frame structures. The other method is a simple calculation procedure based on the Rayleigh's method, and it is called the static frame method. Numerical examples are presented to illustrate the applicability of the static frame method and to discuss the dynamic characteristics of space bridges. The numerical results computed by two different matrix methods are given in tabular form, and the approximate quantity of natural periods obtained by the static frame method is investigated.

1. INTRODUCTION

With the progress in the construction of the traffic network these days in Japan, more and more bridges, such as multi-span continuous elevated bridges and special types of bridges, are being constructed. Bridge construction these days requires stricter and easier bridge vibration analysis. In order to calculate the inertia force needed for seismic design, the conventional method divides the entire bridge into substructure and its supporting structural member system, a unit of superstructure. However, it has recently become known that dividing the whole bridge into two structural systems is unreasonable for the calculation of inertia force. This is particularly so since the inertia force of the superstructure distributed to the substructure varies according to the stiffness and height of the piers, the characteristics of the basic ground foundation, and the types of the superstructure. Therefore, another method is needed in which the inertia force can be calculated as a whole and which can be applied to all types of bridges.

A new concept, "Seismic Design Unit," is introduced in the new Japanese Road Bridges Specification - Seismic Design Volume (February 1990). In the new concept, superstructure and substructure are considered to have the same vibration. A new method is introduced in which the inertia force can be calculated for each seismic design unit. In particular, when the seismic design unit consists of substructure and its supporting superstructure, the "seismic frame method" (referred to as the "static frame method" in this paper) developed by the public works institute (Ministry of Construction, Japan) has been introduced for the calculation of natural periods.

The static frame method is a method to calculate the approximate quantity of natural periods. The method was derived from the Rayleigh's method, which calculates natural periods based on the fact that the maximum value of strain energy equals that of kinetic energy in an SDOF system. Calculation of vibration can also be made in a higher order of the natural vibration mode by the Rayleigh's method. However, the Rayleigh's method is said to be particularly useful for calculation of fundamental natural periods. The only requirements are that it must meet the boundary conditions in the selection of displacement function which represents the shapes of the vibration mode. It is important to have a displacement curve which corresponds satisfactorily to the natural mode shape of the structures in order to obtain a good approximate quantity. One benefit of the static frame method is that it allows easy calculation of natural periods, which can be determined not by analyzing the eigenvalues problem of structures but by utilizing calculation of static displacements. However, it seems that not enough studies have been made of the application of the method for space rigid-frame structures and special types of bridges in which superstructure and substructure are regarded as an integrated system.

In this study, natural periods of space rigid-frame bridge structures are calculated by the static frame method and the consistent mass method. The effectiveness of the results is investigated by comparing these natural periods. As for basic natural *periods, the focus is on the first natural* mode of the axial direction of three-dimensional continuous girder bridge, at a right angle to and vertical with the bridge axes. Furthermore, consideration is given to the following items: 1) connecting conditions between superstructures and substructures 2) supporting conditions of abutments and pier footings and skew angles of the superstructure. Effectiveness and applicability of the static frame method are discussed in the above-mentioned consideration.

2. ANALYSIS

2.1 Natural vibration analysis

For natural vibration analysis, a suitable mathematical model and an effective analytical method should be selected according to the characteristics of geometrical structure, materials, and load, and also according to the degree of importance, and so on. According to the structural mass model, the analytical method is classified into two *systems; a continuous system and a discrete* system. The former is referred to as the continuous mass method, in which dynamic characteristics such as mass and stiffness are treated as continuous distribution quantity and strict results can be obtained. The latter is subdivided into two methods; the lumped mass method, in which the distribution mass of frame members is substituted equally at each nodal point, and the consistent mass method, in which the distribution mass is distributed by the displacement function expressed by the cubic polynomial. The results from the discrete system have approximate quantities, but the consistent mass method is better than the lumped mass method in obtaining eigenvalues as accurate as those of the strict results.

In this study, bridge structures are used as model cases of three-dimensional frames, and natural periods and natural mode shapes are calculated by using the consistent mass method (CMM). Frame member components in the model are affected only by axial force, shearing force, bending moment, and torsional moment. There are six degrees of freedom at a nodal point: in X-axis, Y-axis, *and Z-axis directions, and around X*-axis, Y-axis, and Z-axis as shown in Figure. 1. It is assumed that they are not affected by bending and torsional deformation. Natural circle frequencies are calculated as an eigenvalue problem by using the vibration equation ($\det|k-\omega^2 M| = 0$) while taking into account the above-mentioned conditions. Analysis of the eigenvalue problem is made by using the Householder Method.

2.2 Static frame method

The static fame method (SFM) is a method in which natural periods and inertia force are calculated approximately based on static displacement calculation instead of time history response analysis of the bridges affected by earthquake load or the method of dynamic analysis (model analysis) by the responding spectrum method. Generally, SFM is effective when response by the first natural vibration mode is better than that by other modes, or when static flexture shapes of the structure can closely resemble the basic natural mode shape with its dead weight statically applied to a specific direction. Specifically, the natural period (T) is determined by using the Rayleigh's method. The natural periods of the structural member system can be obtained by approximating the vibration mode with static displacement, $U_d(s)$, as in the following equation:

$$U(s) = U_d(s)\sin(\omega_d t) \qquad (1)$$

where ω_d is the natural circle frequency *obtained by static displacement, $Ud(S)$.* From Equation (1), the maximum kinetic energy of the structural member system can be expressed as the following equation:

$$T\max = \frac{1}{2}\omega_d^2 \int sM(s)\cdot U_d(s)^2\cdot ds \qquad (2)$$

where M(s) is mass at position (s). Strain energy stored in the structural member system is maximized when $\sin(\omega_d t)=1$, i.e., $U(s) = U_d(s)$, as shown in Equation (1). $U_d(s)$ is the displacement when horizontal force equivalent to the dead weight of the structural member system [M(s)g] acts toward action direction of inertia force. Therefore, the maximum strain energy equals the work done by horizontal force. That is to say, the maximum strain energy, Umax, can be given by the following equation:

$$U\max = \frac{1}{2}\int sM(s)g\, U_d(s)ds \qquad (3)$$

where g is gravity acceleration. In conservative systems with no damping force,

$$T\max = U\max \qquad (4)$$

*Therefore, the natural period (T) can be ob*tained by the following equation:

$$T = \frac{2\pi}{\omega_d} = \frac{2\pi}{\sqrt{g}}\sqrt{\frac{\int sM(s)\cdot U_d(s)^2\cdot ds}{\int sM(s)\cdot U_d(s)\cdot ds}} \qquad (5)$$

then δ is defined as follows.

$$\delta = \frac{\int sM(s)\cdot U_d(s)^2\cdot ds}{\int sM(s)\cdot U_d(s)\cdot ds} \tag{6}$$

and

$$T = 2.01\sqrt{\delta} \tag{7}$$

Finally, Equation (6) is given as the following equation by using the mass (m_i) (or dead weight, $m_i g$) and static displacement (Ui) of a rigid-frame structure in the discrete coordinate systems, shown in Figure 2.

$$\delta = \frac{\Sigma\ m_i\ u_i^2}{\Sigma\ m_i\ u_i} \tag{8}$$

Therefore, the natural periods of space rigid-frame structures in discrete coordinate systems can be calculated with Equations (7) and (8). For the convenience of formulation, the calculation method of natural periods on horizontal direction has been explained as shown in Figure 2. It is considered that axial and vertical natural periods can also be calculated because the static frame method is derived from the Rayleigh's method.

The weight of the adjoining member components is distributed equally to each nodal point of the bridge structures as an external load at each nodal point. The static displacement at each nodal point can be determined by applying the external load statically at a right angle to the axis of the bridge, in the axial direction and in the vertical direction. The natural periods in each direction can be calculated by Equations (7) and (8). Generally, it is said that natural frequencies of structural member systems obtained by using the Rayleigh's method are slightly higher. In order to improve the accuracy of the calculation of the natural periods in each direction, it is important to have a static displacement curve as close as possible to the actual natural mode shape.

3. NUMERICAL RESULTS

Three- and four-span continuous beam girder bridges are used as numerical examples. Figure 3 and 4 show a three-dimensional rigid-frame model for calculation by the static frame method and the consistent mass method. Table 1 has the physical properties needed for the calculation. Table 2 shows the connecting conditions of superstructure and substructure. Imaginary members with large section properties are assumed to be located at the connection, and rigid-body displacement is thought to occur.

3.1 Natural mode shapes by the consistent mass method

The computed natural mode shapes of the bridge with internal hinges are shown in Figure 5. The first natural mode shapes is one of horizontal bending vibrations at a right angle to the axis of the bridge. Its coupling vibrations with torsional vibrations are observed a little. T_1 (basic natural period) = 0.5817 sec. The bending vibrations at a right angle to the axis of the bridge can be observed in the 3rd, 4th, and 9th modes. The second natural vibration mode is one of bending vibrations along the axis of the bridge, and the bending vibrations in the pier dominate. Basic natural vibrations in the vertical direction are observed in the 4th mode (T_5 = 0.4664 sec). The second bending vibrations in the vertical direction is observed in the 8th mode. Torsional vibrations are prevalent in the remaining 6th, 7th, and 10th modes. When superstructure and substructure are rigidly connected, horizontal bending vibrations are observed at a right angle to the axis of the bridge (T_1 = 0.5354 sec). The natural period is slightly less than in the hinge connection. Basic bending vibrations in the axial direction of a bridge are observed in the high order of the 12th natural mode to be a rather small T_{12} = 0.1340 sec. Basic bending vibrations in the vertical direction are observed in the four th mode to be a small T_4 = 0.2538 sec. Even with the same model, when the connection conditions change from hinge to rigid connection, the stiffness of the entire structure tends to increase and the natural period tends to decrease. The conditions of connection make a difference even in the same model. In particular, the effect is remarkably apparent in the basic bending vibrations in the axial and vertical directions of a bridge.

3.2 Displacement curve by the static frame method

Figures 7 and 8 show static displacement curves of the hinge connection and the rigid connection, respectively. The dead weight of the members at each nodal point is treated as a static external load. The active direction of load application is the positive direction of the Y axis at a right angle to the bridge axis and is the positive direction of the X axis in the axial direction of the bridge. When focusing on the vertical direction, the load application direction, as indicated by the 5th mode of Figure 5 or the 4th mode of Figure 6, is the positive direction of the Z axis for the center span, and the negative direction of the Z axis for the side span. In either case, it can been seen that each static displacement curve at a right angle to the bridge axis, in the axial direction of the bridge, or in the vertical direction closely approximates each basic natural mode shape. The natural periods obtained from each curve also have satisfactory proximate values. If the dead weight at each nodal point is evenly applied in the positive direction of the Z axis, the static displacement curve in the vertical direction can be given as the displacement curve shown in Figure 9.

3.3 Effect of skew angle

A case will be considered in which the superstructure has a skew angle on abutments and bridge piers. In Table 3 and Table 4, the natural periods calculated by the SFM are compared with those calculated by the CMM for each skew angle. The comparison shows that, regardless of hinge or rigid connections, the natural periods obtained by the SFM agree closely with those obtained by the CMM. It can be concluded that the skew angle does not affect the natural periods in such bridge structures.

3.4 Effect of ground spring constant

It is discussed in this section how much effect spring stiffness has on the results of the calculation by the SFM, provided that the foundation of abutments and bridge piers is a spring support.

In Table 5, the natural periods calculated by the SFM are compared with those by the CMM when the stiffness of a spring changes from fixed support to very elastic support ($\times 10^{-2}$) on the condition that the type of straight bridge ($\theta = 90°$) with hinge connections is the standard ($\times 10^{0}$) with the above-mentioned spring support. In Table 6, the effect of a skew angle ($\theta = 60°$) is added to the above-mentioned conditions. In either case, as expected, the smaller the ground spring constants become, the larger the natural periods become. It is seen that the results calculated by the SFM agree closely with those calculated by the CMM.

4. CONCLUSIONS

In this study, the natural periods of bridge structures in the axial direction of the bridge, at a right angle to the bridge, and in the vertical direction are calculated by using the SFM. The propriety and effectiveness of the results are presented by comparing the results with those of the CMM. As discussed in numerical result, after an examination of the effect on natural periods of the conditions of superstructure and substructure connection, of the plane shape of bridges with a skew angle or the plane curve shape of bridges, of spring constant changes in abutments and pier footings with ground conditions ranging from rigid ground to soft ground, and so on, the calculation results of natural periods by the SFM agree closely with those by the CMM. For example, it is understood that a displacement curve of the SFM agrees closely with the basic natural mode shape of the CMM and the results of the calculation of natural periods have relatively good approximate quantities.

Therefore, the results computed by the SFM are highly applicable to seismic design for bridge structures. It can be concluded that the SFM is a very effective and easy calculation method of natural periods. The static frame method will be developed in the future, so that the three-dimensional dynamic analysis can be easily applicable to curved lattice girder, arch bridge, suspension bridge structures and so on in the future.

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Table 1. Structural properties of bridge

Members	$A(m^2)$	$I_y(m^4)$	$I_z(m^4)$	$J(m^4)$	$E(t/m^2)$	$W(t/m)$
Main girder (G1, G3)	0.98×10^{-1} ~1.10×10^{-1}	3.61×10^{-2} ~5.52×10^{-2}	3.96×10^{-2} ~3.99×10^{-2}	1.83×10^{-5} ~2.08×10^{-5}	2.10×10^{7}	3.048
Main girder (G2)	1.07×10^{-1} ~1.19×10^{-1}	3.42×10^{-2} ~5.71×10^{-2}	6.08×10^{-2} ~6.10×10^{-2}	2.06×10^{-5} ~2.36×10^{-5}	2.10×10^{7}	2.433
Cross beam	0.18×10^{-1}	0.56×10^{-2}	1.55×10^{-5}	5.24×10^{-7}	2.10×10^{7}	—
End sway bracing	0.10×10^{-1}	0.27×10^{-2}	—	—	2.10×10^{7}	—
Intermediate sway bracing	0.79×10^{-2}	0.20×10^{-2}	—	—	2.10×10^{7}	—
Abutment	13.120	2.799	73.516	9.820	2.35×10^{6}	32.800
Beam of pier	2.600	0.867	0.366	0.874	2.35×10^{6}	6.600
Column of pier	7.069	3.976	3.976	7.952	2.35×10^{6}	17.671

Table 2. Connecting conditions of super structure and sub structure

Nodal points	Linear displacements			Angular displacements		
	u	v	w	θ_x	θ_y	θ_z
81, 82, 83, 88, 89, 90	FIX	FIX	FIX	FIX	MOV	MOV
76, 77, 78, 95, 96, 97	MOV	FIX	FIX	FIX	MOV	MOV
The others	FIX	FIX	FIX	FIX	FIX	FIX

Table 3. Natural Periods by effect of skew angle (sec, hinge connection)

Skew angle	X-Direction		Y-Direction		Z-Direction	
	S.F.M.	C.M.M.	S.F.M.	C.M.M.	S.F.M.	C.M.M.
90°	0.5725	0.5710	0.5733	0.5817	0.4625	0.4664
75°	0.5689	0.5664	0.5742	0.5860	0.4428	0.4449
70°	0.5669	0.5676	0.5749	0.5889	0.4314	0.4353
60°	0.5627	0.5590	0.5554	0.5659	0.4108	0.4143
50°	0.5607	0.5668	0.5411	0.5513	0.3963	0.3995
45°	0.5609	0.5688	0.5336	0.5408	0.3922	0.3938

Table 4. Natural Periods by effect of skew angle (sec, rigid connection)

Skew angle	X-Direction		Y-Direction		Z-Direction	
	S.F.M.	C.M.M.	S.F.M.	C.M.M.	S.F.M.	C.M.M.
90°	0.1326	0.1340	0.5214	0.5354	0.2477	0.2538
75°	0.1328	0.1341	0.5221	0.5347	0.2478	0.2541
70°	0.1329	0.1342	0.5226	0.5349	0.2479	0.2547
60°	0.1332	0.1344	0.5176	0.5296	0.2481	0.2546
50°	0.1338	0.1348	0.5152	0.5242	0.2486	0.2556
45°	0.1343	0.1351	0.5142	0.5223	0.2490	0.2556

Table 5. Natural Periods by effect of spring stiffness (sec, θ =90°).

Spring stiffness	X-Direction		Y-Direction		Z-Direction	
	S.F.M.	C.M.M.	S.F.M.	C.M.M.	S.F.M.	C.M.M.
Fixed support	0.5725	0.5710	0.5733	0.5817	0.4625	0.4664
$\times10^{2}$	0.5727	0.5704	0.5734	0.5810	0.4625	0.4658
$\times10^{0}$	0.5844	0.5852	0.5829	0.5923	0.4630	0.4671
$\times10^{-1}$	0.6826	0.6773	0.6640	0.6773	0.4667	0.4728
$\times10^{-2}$	1.3132	1.2754	1.2001	1.2086	0.4772	0.4836

Table 6. Natural Periods by effect of spring stiffness (sec, θ =60°)

Spring stiffness	X-Direction		Y-Direction		Z-Direction	
	S.F.M.	C.M.M.	S.F.M.	C.M.M.	S.F.M.	C.M.M.
Fixed support	0.5627	0.5590	0.5554	0.5659	0.4108	0.4143
$\times10^{2}$	0.5628	0.5674	0.5555	0.5605	0.4108	0.4138
$\times10^{0}$	0.5741	0.5819	0.5654	0.5745	0.4113	0.4137
$\times10^{-1}$	0.6679	0.6764	0.6460	0.6538	0.4149	0.4187
$\times10^{-2}$	1.2309	1.2385	1.1320	1.1715	0.4278	0.4327

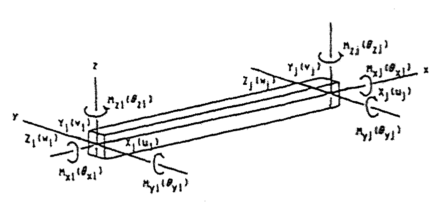

Figure 1. Force and displacement of space three-demensional beam segment

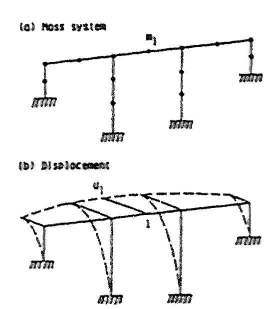

Figure 2. Mass and displacement curve

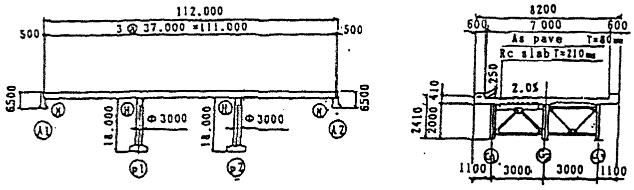

Figure 3. General view of bridge

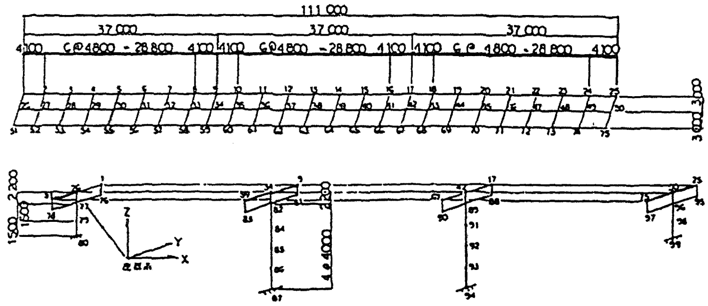

Figure 4. Model of space frame work structure

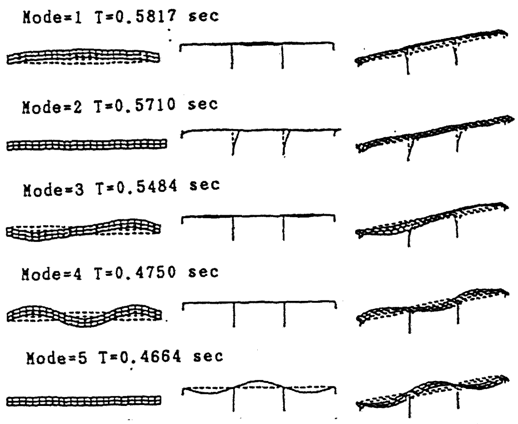

Figure 5. Natural Periods and mode shapes by C M M (hinge connection)

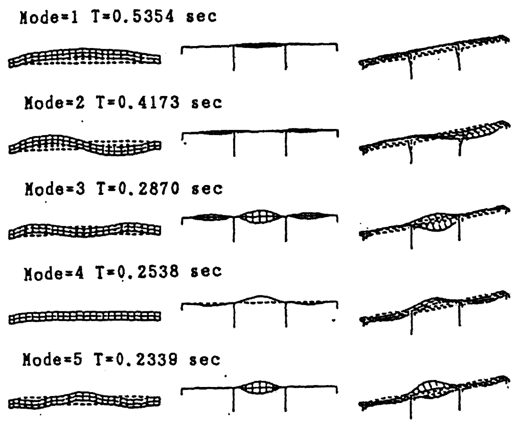

Figure 6. Natural Periods and mode shapes by C M M (rigid connection)

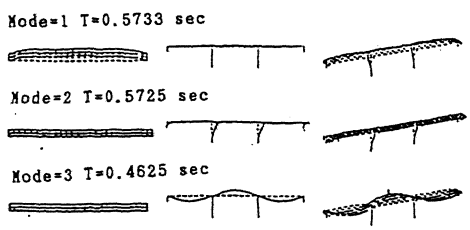

Figure 7. Static displacement curves by S F M (hinge connection)

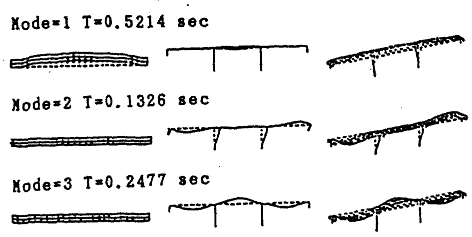

Figure 8. Static displacement curves by S F M (rigid connection)

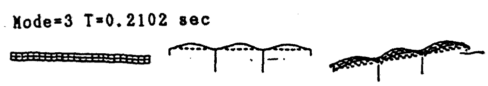

Figure 9. Static displacement curve by uniform load (rigid connection).