

# Dynamic analysis of cable stayed bridges by means of 3D analytical and physical modelling

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**ABSTRACT:** A small-scale elastic model of a existing typical cable stayed bridge has been designed and built. Hammer, sinusoidal and random signal tests have been made to determine the dynamic characteristics and damping of the model. By using the shaking table, two types of seismic excitation were applied and corresponding response quantities in the critical sections were obtained. A comparison of the test results with those obtained from the mathematical model of the prototype has been made.

## 1 INTRODUCTION

The most common practice for definition of the dynamic characteristics and the earthquake response of cable stayed bridges is through analytical solutions. No matter how perfect the analytical model may be, idealization of the structure is always a starting point. Therefore, in order to improve the mathematical model of the cable stayed bridges more data can be obtained from different experimental tests.

The main objective of the investigations presented in these paper is correlation between the experimental data obtained from dynamic testing of a small - scale model of a typical cable - stayed bridge and the analytical results obtained from the finite element analysis of the prototype.

Measurement of damping of these structures was another task of the research.

## 2 CHARACTERISTICS OF THE PROTOTYPE

The Jindo bridge [1], Fig. 1 has three span and the stiffening girder is continuous from one end to the other. The main span is 344m and the side spans are each 70m. The stays are arranged in a fan and converge at the top of each A-frame tower; each tower carries 24 stays. These are of four different diameters: 87mm, 76mm, 67mm and 56mm. The towers are cantilevers and top out 69m above the pier. The cross section of the stiffening steel girder has a hexagonal shape with the following average properties:  $A = 0.464\text{m}^2$   $I_x = 0.54\text{m}^4$   $I_y = 5.4\text{m}^4$ . The steel legs are rectangular box sections with stiffness characteristics of  $A = 0.237\text{m}^2$   $I_x = 0.276\text{m}^4$   $I_y = 0.230\text{m}^4$

## 3 MATHEMATICAL MODEL FORMULATION OF THE JINDO BRIDGE

The mathematical model has been formulated for the dynamic analysis of the Jindo Bridge. 248 3D beam elements were introduced for discretisation of the stiffening girder and the towers. The box girder deck was represented by an equivalent beam element passing through the central line of the deck. In order to attain equivalent torsional properties, infinitely rigid cross beam elements were connected to the central beams with an equivalent mass and mass moment of inertia added to the outer ends. The cable stays were represented by 52 flexible elements, with equivalent modulus of elasticity. Since the structural elements are well defined by the geometry, a lumped mass matrix was employed.

A computer plot of the mathematical model is shown in Fig. 2

## 4 PHYSICAL MODELING OF THE JINDO BRIDGE

### 4.1 Scaling Factors

Long structures (suspension bridges, cable-stayed bridges) require models of very small scales since the scaling factor in this case is very high.

The present case is even more complicated as it deals with a particular structure, i. e., Jindo Bridge dictating the proportions of the model. Practically, it is very difficult to scale correctly all the proportions of the bridge. Most frequently, there are some slight differences between the scaled proportions.

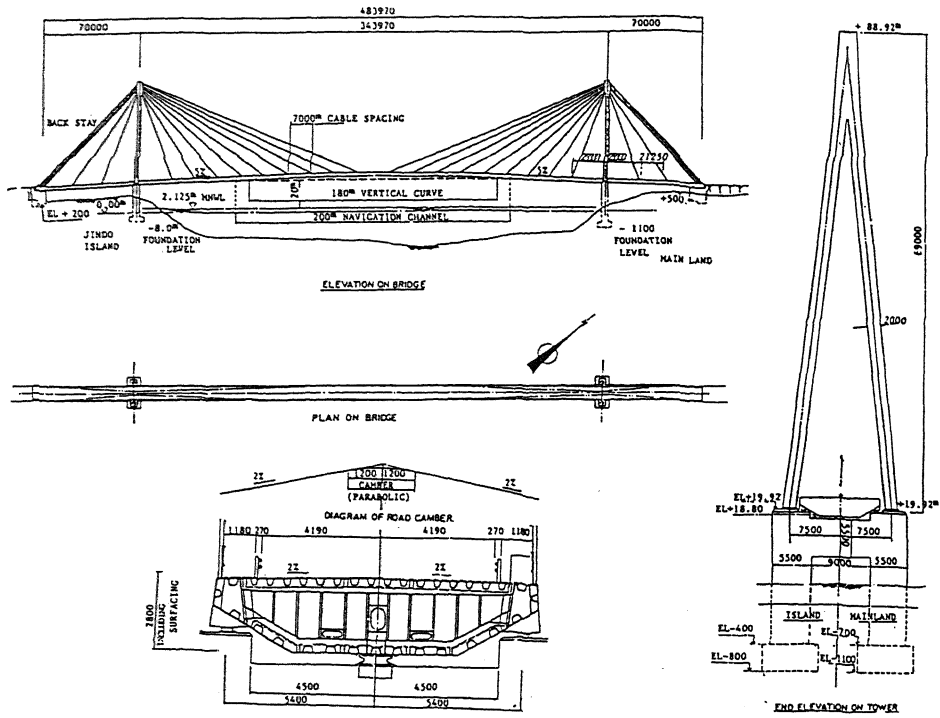


Fig. 1 General arrangement of the Jindo Bridge

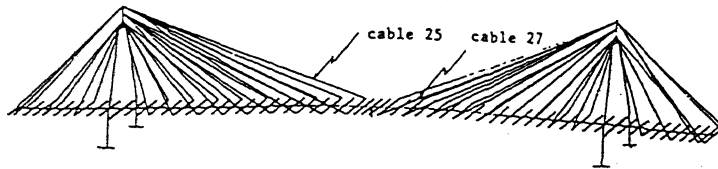


Fig. 2 3D mathematical model of the Jindo Bridge

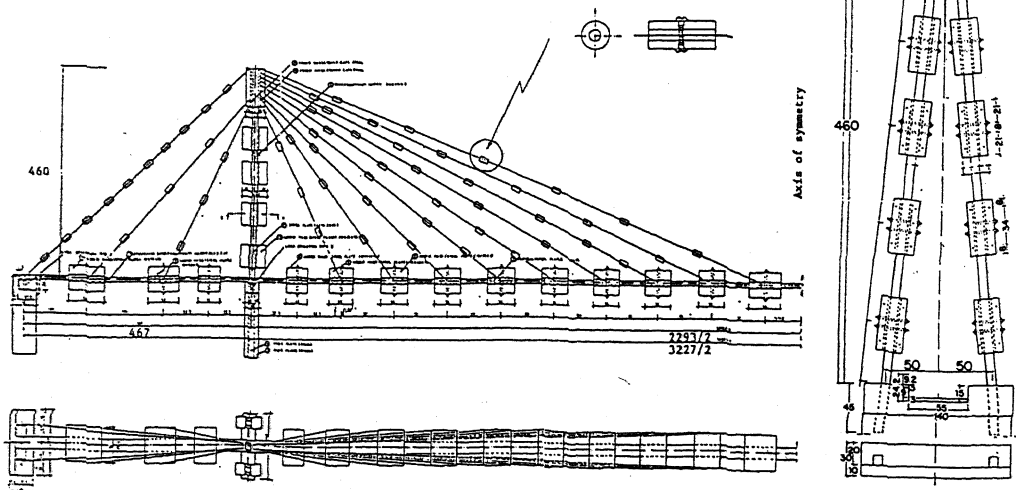


Fig. 3 Structural dimensions and elevations of the small-scale model of the Jindo Bridge

The selection of the scaling factors for the physical model of the Jindo Bridge depended on the characteristics of the shaking table at the University of Bristol [2], since not only the dynamic characteristics of the bridge but also its behaviour under different types of seismic excitation had to be defined.

The total span of the prototype  $l_p$  is 483.47m and the lowest linear scale factor  $l_r$  to fit the shaking table was 1/150. This gives a value for the total length of the model,  $l_m$ , of 3.22m. Although this is longer than the width of the table, testing is possible by overhanging the platform by 0.11m on each side. Such a selection of a linear scale does not give the possibility to construct a true replica physical model of the Jindo Bridge. Therefore, considering the selection of the linear scale, the so called model with artificial mass simulation is used.

The relationship between frequency scale,  $f_r$ , and linear scale,  $l_r$ , is given by  $f_r = l_r^{-1/2}$ , so that if  $l_r = 1/150$  then  $f_r = 12.25$ . Using this scale all the significant modes of vibrations of the model will have frequencies within the working range of the platform. In fact, the frequency characteristics of the shaking table would have allowed the use of a higher linear scale factor if there had not been the problem of finding an appropriate wire to reproduce correctly the axial stiffness characteristics of the stays.

As a very small model is considered, its weight does not exceed the bearing characteristics of the platform. Furthermore, in order to decrease the cost and the total weight of the model, it was decided that steel is not an appropriate material to be used for the construction of the towers and the superstructure of the model as it is the case with the prototype. Therefore, an aluminium alloy has been used instead. The masses and forces were therefore scaled by the ratio  $M_r = F_r = E_r l_r$  (where  $E_r$  is the scaling factor for Young's modulus). Thus  $E_r$  equalled 0.333 which reduced the additional mass required.

## 4.2 Design and construction

A distorted model with similarity for the bending stiffness using simplified sections of the girders and the towers can be taken into account, since the material non-linearity is not the objective of this study. So, rectangular sections have been used for the cross sections of the girders and the tower legs instead of box sections. As a result of this, the bending stiffnesses were correctly scaled which was not the case with the axial stiffnesses for the girder and the towers.

The designed axial stiffnesses were larger, but they did not significantly affect the global behaviour of the bridge. As to the axial stiffness of the stays, it was correctly scaled since the cable elements are characterized by axial stiffness only so that any inappropriate scaling of the cable cross-section would

exceed to a great extent the model characteristics. The following materials have been used for the construction of the model: (1) Aluminium alloy solid beams (for the main girders and the tower legs); (2) Piano wires (for the stays); (3) Steel plates (for the masses added to the superstructure and the towers); (4) Steel cylinders (for the masses added to the cables); (5) Lead cylinders (for the masses added to the back stays). The steel plates are attached to the main girder by means of two bolts with a diameter of 2.5 mm. In this way, the beam stiffness changes to a minimum extent at the cross-sections where masses are added. To avoid any additional damping, there is a 1.5 mm gap between the steel plates and the main girder. The additional masses for the towers have been attached in the same way. The only difference is that they are attached by 2mm bolts.

As mentioned before, special care was taken when choosing the correct diameters for the cables and placing them appropriately. The only difference between the prototype and the model were the back stays. The back stays of the prototype were formed by 6 x 87 mm separate cables, while those of the physical model were simulated by a No. 11 piano wire. No. 00, 0 and 2 piano wires were used to represent the other cables.

Another problem which occurred in designing the model was the placement of the cylinders simulating the dead weight of the cables. The attachment of the cylinders to the wires having  $d = 0.23\text{mm}$  was performed with a considerable difficulty.

Since the same rule (the additional masses must not affect stiffness and damping) applies here, the cylinders and the stays were connected by means of two bolts (Fig. 3) that only touched the wire slightly along a length of 2mm.

The model was constructed in the workshop of the Civil Engineering Department at the University of Bristol.

Structural dimensions and elevations of the small-scaled model of the Jindo Bridge are presented at Fig. 3

## 5 DYNAMIC TESTING

### 5.1 Dynamic characteristic measurement of model

Good agreement between the dynamic characteristics of the mathematical model and the experimental model does not necessarily imply a good reproduction of a prototype behaviour, and so static tests were also performed, to ensure that the stiffness characteristics and boundary conditions were properly simulated.

As good agreement had been obtained for static behaviour [4], it was possible to proceed with dynamic tests. Three types of excitation were used: hammer

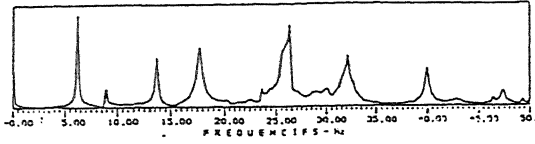


Fig. 4 Typical spectral response from shaking table random test of the vertical movement of the model

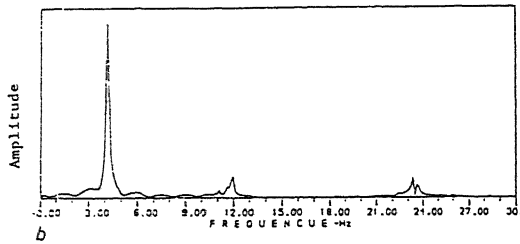
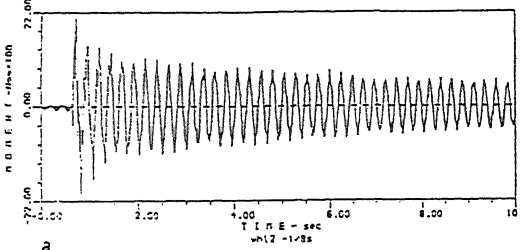


Fig. 5 Typical response from hammer test of the lateral movement of the model; a, movement at  $1/8$  c.s.; b, spectral response at  $1/8$  c.s.

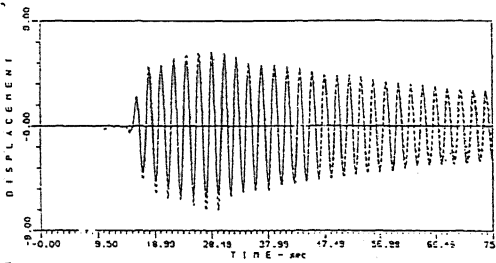


Fig. 7 Analytical and experimental time history of vertical displacement, at centre of the central span for the 3DSROM input

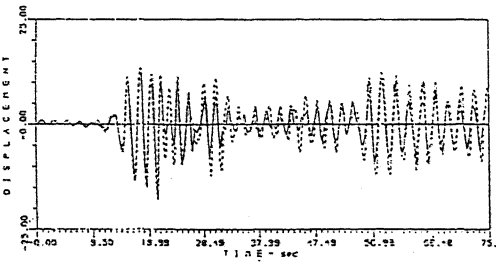


Fig. 8 Analytical and experimental displacement for the same cross-section and same direction under the 3DEL45 input

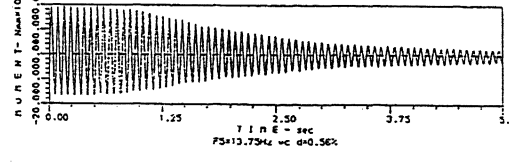
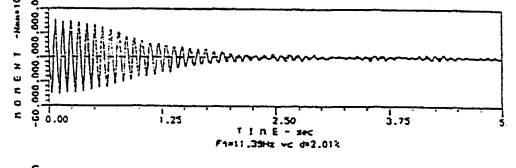
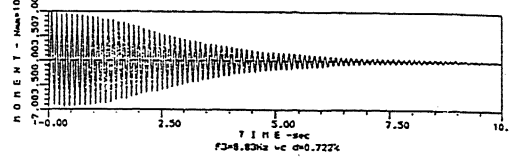
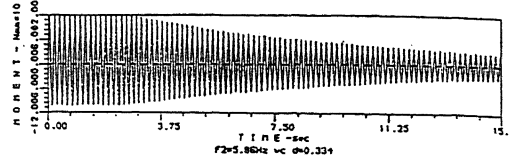
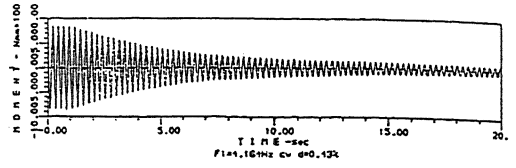


Fig. 6 Typical time-decay records for the first 5 modes of vibration (excitation with shaking table)

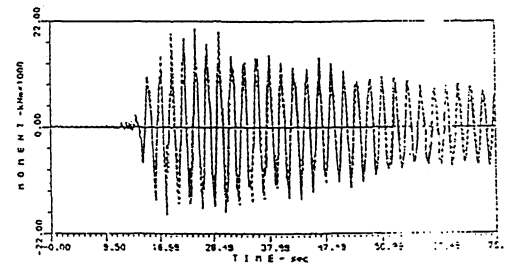


Fig. 9 Time history for the moment at the same cross-section and direction under the 3DEL45 seismic input

blows, random force and steady state sinusoidal force tests.

An impulsive force was applied by an instrumented hammer at the centre of the span, at the quarter span and the top of the tower.

The random and sinusoidal signals were applied to the model by means of portable electro-magnetic shakers and by placing the model on the shaking table. The response analysis from the random signal was carried out in real time using two channel spectrum analyzer. The responses from the hammer tests and the sinusoidal input were collected and saved on a micro computer. FFT analysis was applied to calculate spectral response curves from the hammer and random tests records. The most useful spectral response data were obtained from the strain gauges located at the 1/8 point on the central span because all significant modes were present at this location. Fig 4 shows the spectral response curve for the vertical vibration obtained from the strain gauge. The same quantities for the lateral vibration are shown in Fig 5.

From these data, the most significant natural frequencies of the model were quickly found. As well as random excitation, a sinusoidal excitation force from the small shakers was used to excite the bridge to determine the mode shape. Normalized mode shapes were obtained by comparing the amplitude and phase of each instrumented location with those of the reference location. In table 1 are presented analytically obtained frequencies of the prototype and those measured from the model.

Table 1 Frequencies of the Jindo Bridge obtained from the analytical and experimental model.

Mode no.	Type of mode	Frequency /Hz	
		Analytical	Experimental
1	LS	0.344	0.339
2	VS	0.468	0.478
3	VA	0.654	0.720
4	LA	0.971	0.928
5	VS	0.964	1.121
6	VA	1.287	1.456
7	VS	1.616	1.851
8	LS	1.927	1.899
9	VA	2.404	2.689
10	VA	2.975	3.330

### 5.2 Damping Measurement

As it was mentioned in the introduction, the objective of the investigation was to determine the damping values of the model, too

The knowledge on damping of the prototype is of the greatest importance for dynamic analysis of cable-stayed bridges. As to the dynamic response, the error made as a result of wrong estimates of damping

is much higher than that made when large displacements are not taken into account [4]

The importance of the model damping involves several aspects: (1) the comparison between the dynamic response of the physical model and the dynamic response of the mathematical model of the prototype is only made possible if the exact value of model damping is obtained and adopted in the analytical procedure, (2) measurement of the model damping for the purpose of adopting a damping value for the prototype.

There are, in general to ways of estimating damping from experimental response data; by analyzing the signal either in the frequency domain or in the time domain. Here, because the level of damping in the model was low, the results obtained from the logarithmic decrement method were more accurate and they were taken as being representative. Damped harmonic oscillations were created by exciting the models in two ways, first with the shaking table and second with portable shakers. When steady state motion had been achieved in each mode, the excitation was switched off. The damping values of each excited mode were calculated from the decay curves Fig.6 by using exponential curve fitting. The damping values for the first five modes were estimated using this method and are shown in table 2

Table 2 Experimentally measured damping

Mode no.	Type	Physical model			
		Shaking table		Portable shakers	
		Frequency /Hz	Damping %	Frequency /Hz	Damping %
1	Lateral	4.164	0.43	4.168	0.344
2	Vertical	5.86	0.334	5.81	0.559
3	Vertical	8.83	0.722	8.66	0.834
4	Lateral	11.39	2.01	11.58	1.022
5	Vertical	13.75	0.56	13.81	1.23

### 5.3 Earthquake tests using shaking table simulator

The earthquake tests have been the most interesting from the viewpoint of comparison with the analytical ones. This type of experiment is of a special importance although it has some disadvantages, because the shaking table is unable to reproduce thoroughly the given earthquake input. The most common way of overcoming this disadvantage is to use the modified input recorded by the accelerometers of the shaking table in the analytical models in order to avoid errors in the comparison of the results. In order to use this record in the dynamic analytical analysis of the Jindo bridge, a reversible procedure has to take place, i.e., the record is divided by factor  $Tr = Ir^{1/2}$

The seismic input was selected with no consideration of the prototype site but with a

tendency to choose records with a frequency content similar to the fundamental circular frequencies of the model, i.e., the prototype. Beside the Romanian record (3DSROM), the second motion used in this study was derived from the El Centro 1940 (3DELC45) accelerogram. As this type of accelerogram accelerated by time scaling factor did not have any large effect on the model, i.e., was not capable of inducing larger deformations, it was decided that 4.5 sec of this record be used without any modification. In this way, the maximum peak accelerations from the first 4.5 seconds were involved, the frequency content of the record being similar to the fundamental frequencies of the model. Both types of records have been normalised regarding the same maximum peak acceleration of 0.1g.

As to the seismic test itself, the seismic shaking table at Bristol University has been used, i.e., a simultaneous three-dimensional motion of the shaking table has been simulated.

Fig 7 displays analytical (solid line) versus experimental one (dashed line) vertical displacement at the centre of the main span, for the case of the 3DSROM input. Presented in Fig. 8 are the analytical and experimental dynamic displacements for the same cross-section and same direction under the 3DELC45 seismic input. Fig. 9, show the time histories for the moment at the same characteristic cross-sections of the mathematical and the physical model exposed to the 3DSROM seismic input.

## 6 CONCLUSIONS

Based on the analytical and experimental dynamic response of the Jindo bridge, it can be concluded that they vibrate in the first fundamental modes of vibration. Therefore, the mode superposition method is much more efficient in the linear analysis of these structures than is the direct integration method. The experimental results obtained for the physical model of the Jindo Bridge give the right to the author to conclude that very small models can provide useful data despite the doubtful attitude of a large number of experts. This is proved by the fact that very close analytical and experimental results have been obtained.

As to the model measurements of damping, the opinion of the authors is that measured damping of models which are not true replica models should not be considered representative for the prototype structure. This is, in fact, proved by the different results of measurements performed for the Parana bridges [5] giving very high damping values and those performed for the Ruck-a-Chucky bridge [6] showing very low values of damping.

The differences in measured damping in model studies arise probably because of the inadequate similarity in materials and the fabrication techniques used for the model and the prototype.

On the basis of the good analytical and experimental correlation of the static values, the frequencies (Table 1) and time histories for the simulated two types of a 3D input (Fig. 7 - Fig. 9), the selected mathematical model can be considered as representative for the global linear analysis of the prototype.

The investigations performed within the framework of this study might contribute a lot to the definition of the dynamic behaviour of these bridges. However, additional research will be quite useful for the verification of the mathematical model.

Verification of the mathematical model of the Jindo Bridge has been made in this study by experimental tests of its physical model. Finding a way of performing full-scale testing of Jindo Bridge would be a possibility for another verification of the mathematical model as well as estimation of the extent to which damping of the physical model is representative for the prototype.

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