Earthquake Engineering, Tenth World Conference © 1992 Balkema, Rotterdam. ISBN 90 5410 060 5

Stability of arch bridges on elastic soil

N.M.Auciello
Department of Structural Engineering and Soil Mechanics, Potenza, Italy

ABSTRACT: The aim of the present paper is to study the critical load multipliers of arch structures on elastic foundation. The flexibility is defined by means of a full 3 x 3 matrix, which allows us to define the most general scheme of arch-soil interaction.
The structure is discretized according to the so-called "cell discretization method" (Raithel and Franciosi 1985), which seems to be particularly well suited to investigate the static and dynamic behaviour of arch structures. A numerical example allows us to verify the precision of the proposed approach

1. INTRODUCTION

In the last decades the arch structures have been rediscovered, mainly because of a noticeable technological progress, and now it is quite frequent to come across an arch bridge with large span. More particularly, some economical technical procedures allowed the practical building of many arch bridges, in such a way that this structural scheme seems to flourish again. The segment by segment cantilever method, and the so-called 'Bung' system are two classical examples of such technological progress, which make again competitive the arch bridge structure. The Bloukrans bridge (R.S.A. 272m.), the Krummback bridge (Suisse 134m) were both built by means of the segment by segment cantilever method, whereas the Argentobel bridge, (R.F.D), was built by adopting the 'Bung' method. Basically, the bridge is built near the abutments, and then the two halves are rotated till the crown can be joined. In both these cases, the abutments are totally responsible of the static behaviour of the whole bridge. Therefore, it seems useful to propose a structural model which take into account the soil-interaction problem.

The present paper aims to study the critical load multipliers of arches in the presence of flexible abutments, where the flexibilities are defined in the most general way. The structure is discretized according the so-called 'cell discretization method', which reduce the structure to a set of rigid bars linked by means of elastic cells, in which the elastic strain energy is supposed to be concentrated. This method has already been adopted to study the static and dynamic behaviour of arch bridges (Raithel and Franciosi, 1985).

If the structure is divided into t bars, then the t-2 rotations of the firts t-2 bars can be assumed as Lagrangian coordinates, in order to fully describe all the geometric configurations of the structure. In addition, it is necessary to add the six possible displacements of the abutments, so that the system has t+4 Lagrangian coordinates.

2. THE STRUCTURAL MODEL

Let us consider the parabolic arch in figure 1, and let c be the array of the t+4 Lagrangian coordinates. Let us order the c array in such a way that (n = t-2):

$$c^T = \{r_1, r_2, \ldots, r_n, r_A, r_B, v_A, v_B, w_A, w_B\} \quad (1)$$

where r_i is the rotation of the i-th bar, whereas r_A, r_B, v_A, v_B, w_A, w_B are the abutment displacements.

At the i-th cell the strain energy of the two adjacent bars is supposed to be concentrated, in such a way that at each cell it will be (Raithel and Franciosi 1985):

$$\Delta r = \frac{M}{k_i} \tag{2}$$

with:

$$k_i = 2E\left(\frac{s_i}{I_i} + \frac{s_{i-1}}{I_{i-1}}\right) \qquad i = 1,2,..n+3 \tag{3}$$

where I_i is the second moment of area of the cross section, s_i is the length of the i-th bra, and E is the Young modulus. The potential energy of the system is a function of the Lagrangian coordinates, and it is given by the sum of the strain energy and of the quadratic potential energy of the applied forces. In fact, the arch axis is supposed to be funicular of the applied vertical forces.

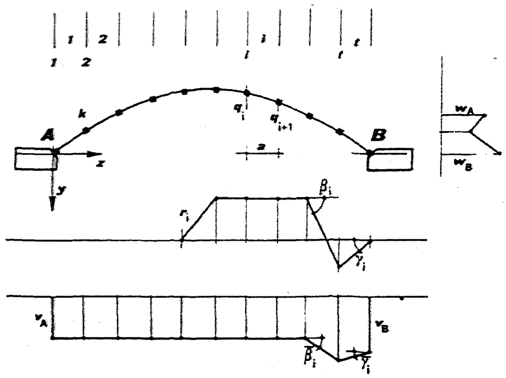

Figure 1.Lagrangian coordinates.

The relative rotations at the elastic cells abscissae are given by:

$$\Delta r_1 = r_1 - r_A$$

$$\Delta r_i = r_i - r_{i-1}, \qquad i = 2,\dots n$$

$$\Delta r_{n-1} = \sum_{i=1}^{n} \beta_i r_i + \sum_{h=1}^{6} \bar{\beta}_h c_h - r_n \tag{4}$$

$$\Delta r_{n-2} = \sum_{i=1}^{n} (\gamma_i - \beta_i) r_i + \sum_{h=1}^{6} (\bar{\gamma}_h - \bar{\beta}_h) c_h$$

$$\Delta r_{n-3} = r_B - \sum_{i=1}^{n} \gamma_i r_i - \sum_{h=1}^{6} \bar{\gamma}_h c_h$$

where:

$$\beta_i = \frac{\Delta q_{n-2} - \Delta q_i}{\Delta q_{n-1} - \Delta q_{n-2}} \tag{5}$$

$$\gamma_i = \frac{\Delta q_{n-1} - \Delta q_i}{\Delta q_{n-1} - \Delta q_{n-2}}$$

and:

$$c_1 = r_A \qquad \bar{\beta}_1 = 0 \qquad \bar{\gamma}_1 = 0$$

$$c_2 = r_B \qquad \bar{\beta}_2 = 0 \qquad \bar{\gamma}_2 = 0$$

$$c_3 = v_A \qquad \bar{\beta}_3 = -\Delta\frac{q_{n-2}}{\alpha} \qquad \bar{\gamma}_3 = \Delta\frac{q_{n-1}}{\alpha} \tag{6}$$

$$c_4 = v_B \qquad \bar{\beta}_4 = -\bar{\beta}_3 \qquad \bar{\gamma}_4 = -\bar{\gamma}_3$$

$$c_5 = w_A \qquad \bar{\beta}_5 = \frac{a}{\alpha} \qquad \bar{\gamma}_5 = -\frac{a}{\alpha}$$

$$c_6 = w_B \qquad \bar{\beta}_6 = -\bar{\beta}_5 \qquad \bar{\gamma}_6 = -\bar{\gamma}_5$$

The quantities q_i are calculated at the t+1 cells abscissae, and they are assumed to be positive if the cells is above the horizontal line passing through A (fig.1). α is given by:

$$\alpha = a(2q_{n-2} - q_{n-1} - q_{n-3}) \tag{7}$$

If Δr is the array of the relative rotations at the cells abscissae, then eqn.(4) can be written as:

$$\Delta r = Ac \tag{8}$$

where A is a rectangular (n+3,n+6) matrix.

3. STRAIN ENERGY

The strain energy of the structure is given by the strain energy of the cells, and by the strain energy of the soil:

$$L = L_c + L_t \tag{9}$$

The strain energy of the cells is given by:

$$L_c = \frac{1}{2}\sum_{i=1}^{n+3} k_i \Delta r_i^2 = \frac{1}{2}\Delta r^T D \Delta r \tag{10}$$

where D is the diagonal (n+3,n+3) matrix with the k_i terms along the diagonal. From eqn.(8) we can deduce:

$$L_c = \frac{1}{2}c^T A^T D A c = \frac{1}{2}c^T K_c c \tag{11}$$

where K_c is the (n+6,n+6) symmetric stiffness matrix of the arch.

At the left abutment it is:

$$f_A = K_A s_A \qquad (12)$$

where:

$$s_A^T = \langle r_A, v_A, w_A \rangle \qquad f_A^T = \langle M_A, V_A, H_A \rangle,$$

and K_A is the stiffness matrix of the left abutment.

The abutment is supposed to be a rigid block totally embedded into a Winkler soil, and its flexibility is defined by a 3 x 3 matrix.

In the same way, at the right abutment it is possible to write:

$$f_B = K_B s_B \qquad (13)$$

and finally the strain energy of the soil can be written as:

$$L_t = \frac{1}{2} s_A^T K_A s_A + \frac{1}{2} s_B^T K_B s_B. \qquad (14)$$

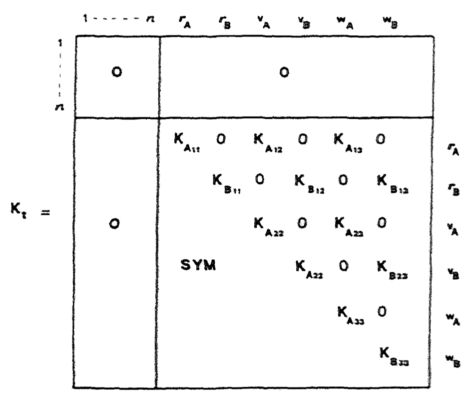

Figure 2.Stiffness matrix of the soil

The strain energy of the whole system is therefore given by:

$$L = \frac{1}{2} c^T K c \qquad (15)$$

where:

$$K = K_c + K_t \qquad (16)$$

and K_t is the (n+6,n+6) matrix illustrated in figure 2. It is worth noting that symmetry considerations lead to deduce:

$$K_{B11} = K_{A11} \quad ; \quad K_{B22} = K_{A22} \quad ; \quad K_{B33} = K_{A33}$$

$$K_{B12} = -K_{A12} \quad ; \quad K_{B13} = K_{A13} \quad ; \quad K_{B23} = -K_{A23}$$

K is the global stiffness matrix of the system and it is therefore symmetric and positive definite.

4. POTENTIAL ENERGY OF THE APPLIED LOADS

The arch axis is supposed to be funicular curve of the deda load, and therefore the first order potential energy is zero whereas the second order potential energy is given by (figure 3) (Franciosi and Auciello 1988):

$$P = -\sum F_i v_i^{(2)} - V_B v_B^{(2)} - H_B w_B^{(2)} \qquad (18)$$

The forces F_i are due to the dead load and are supposed to be concentrated at the cells abscissae.

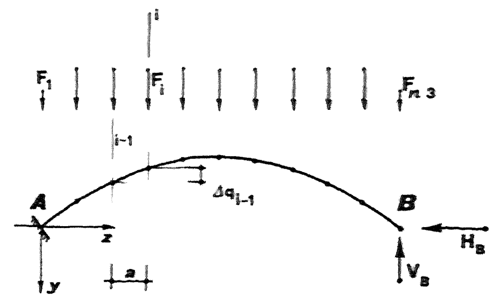

Figure 3. Applied forces

Eqn (18) can be also expressed as:

$$P = -\frac{1}{2} c^T B c, \qquad (19)$$

where B is given by:

$$B_{ii} = R_i + \beta_i^2 R_{n-1} + \gamma_i^2 R_{n-2} \qquad i = 1,2,\ldots n$$

$$B_{n-h,n-h} = \bar{\beta}_h^2 R_{n-1} + \bar{\gamma}_h^2 R_{n-2} \qquad h = 1,2,\ldots 6$$

$$B_{ij} = \beta_i \beta_j R_{ij} + \gamma_i \gamma_j R_{n-2} \qquad i = 1,\ldots n; j = i+1,\ldots n$$

$$B_{i,n-h} = \beta_i \bar{\beta}_h R_{n-1} + \gamma_i \bar{\gamma}_h R_{n-2} \qquad i = 1,\ldots n; h = 1,\ldots 6$$

$$B_{n-h,n-k} = \bar{\beta}_h \bar{\beta}_k R_{n-1} + \bar{\gamma}_h \bar{\gamma}_k R_{n-2} \qquad h,k = 1,\ldots 6$$

with:

$$R_i = -H_B a + \Delta q_i \left(\sum_{j=i-1}^{n-3} F_i + V_B \right) \qquad (21)$$

The total potential energy is given by:

$$E_t = \frac{1}{2} c^T K c - \frac{1}{2} \lambda c^T B c \qquad (22)$$

where λ is the load multiplier. The equilibrium conditions:

$$\frac{\partial E_t}{\partial c_i} = 0 \qquad i = 1,2,\ldots n+4 \qquad (23)$$

lead to the homogeneous system:

$$Kc - \lambda Bc = 0 \qquad (24)$$

The eigenvalues λ_i are real and positive, because of the properties of the matrices.

5. NUMERICAL EXAMPLES

As a numerical example, let us consider a parabolic arch with span equal to 200 meters, and rectangular foundation blocks with dimension 26 x 16 x 5 meters. The Winkler soil is assumed to be defined by the coefficients:

$$\beta_v = 2000t/mc \qquad \beta_H = 200t/mc$$

The stiffness matrix of the left abutment is given by:

$$K_A = \begin{pmatrix} 1.21\times10^7 tm & 6.79\times10^6 t & 2.05\times10^5 t \\ 6.79\times10^6 t & 5.21\times10^5 t/m & 61.32t/m \\ 2.05\times10^5 t & 61.32t/m & 12778t/m \end{pmatrix}$$

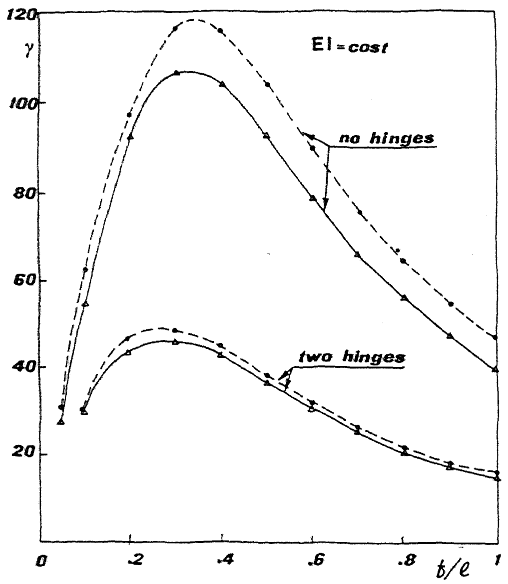

Figure 4. Nondimensional coefficient for EI=const.

If the abutments were perfect, then the critical loads can be written as:

$$q_c = \gamma \frac{EI}{l^3}$$

where E is the Young modulus and I is the cross sectional second moment of area. The γ parameter has been given by many authors as a function of the ratio f/l, both for arches with constant cross section and for arches with varying cross sections (Timoshenko and Gere 1961), (Pfluger 1950). It is also possible to give the parameter γ in the presence of elastic abutments, as a function of the ratio f/l. The results are given in the figure 4-5, where the γ values for rigid abutments are also reported (dashed line). It is worth noting that the influence of the foundation blocks is noticeable, expecially for the clamped arch.

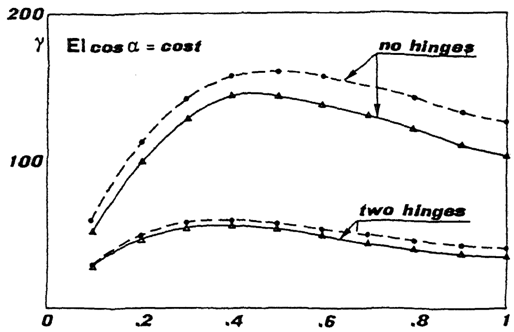

Figure 5. Nondimensional coefficient for EI cosα =const.

Finally, all the results have been obtained by dividing the structure into 15 rigid bars, so obtaining a 21-degrees-of-freedom system.

6. CONCLUSIONS

In a large arch bridge, the analysis of the abutment flexibilities cannot be limited to the sole rotational flexibility, but it is mandatory to introduce the global flexibility of the abutment. The soil-structure interaction problem has been studied for a simplified model, and the critical load multipliers have been obtained. The effect of the soil behaviour on the critical load multiplier seems to be noticeable.

REFERENCES

Timoshenko, S.P. & Gere J.M. 1961 *Theory of elastic stability*. McGraw-Hill, New York

Pfluger, A. 1950. *Stabilitata Probleme der Elastostatik*. Springer, Berlin

Raithel, A & Franciosi,C. 1985, The Stability of Arches in the Lagrangian Approach. *Int.J.Solids Struct*. 21(5), 427-446

Franciosi,V. & Auciello, N.M., 1988, Analisi modale degli archi su imposte elastiche comunque cedevoli. *Industria Italiana del Cemento* 627, 706-711