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Covariance analysis of multisupported linear structures

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ABSTRACT: A method basing on solving Ljapunov algebraic equation is proposed to evalate the mean square response of a multisuppeorted linear system subjected to earthquake ground motion. The earthquake acceleration at each support is modelled as an non-stationary colored noise. Spatial variations are taken into account.

1 INTRODUCTION

In the analysis of structures subjected to random excitation, a description of the input excitation in terms of a power spectral density (PSD) function is commonly used. It is well known that the PSD of the stationary response can be obtained as the product of the system transmittancy function and the PSD of the input process. However, the response quantities of engineering interest should be obtained by a Fourier transformation of the PSD into the time domain.

Covariance analysis carried out directely in the time domain is an alternative to the PSD analysis. Numerical methods for stationary cases were investigated. M.Di Paola (1983) introduced an unconditionally stable step-by-step procedure to evalate the mean square response of a linear system subjected to earthquake ground motion. A non-stationary modulated random excitation process was used.

Covariance analysis is specially efficient if the covariance matrix can be calculated by solving the algebraic Ljapunov equation without numerical integration. In general for a non-stationary case improper integrals are involved, which can only be approached by tedious numerical integration. Czerny and Popp (1984) showed that for certain commonly used simulations of non-stationary random exitation the response covariance can be obtained by solving the algebraic Ljapunov equation, if proper modification of the input simulations is undertaken. However most of the previous studies of covariance analysis take only uniform excitation into account.

In this paper, a covariance analysis method for multisupported structures is developed. The excitation at each support is simulated as a non-stationary colored process, which are obtained as the product of a determinate envelope function and stationary filtered white noise. The excitations at different supports can be so correlated, that they are assumed to be generated from the same white noise, but with different arrival time and intensities. The shape filter for each support can be chosen individually, depending on the site conditions.

With this method the covariance may always be calculated by solving Ljapunov algebraic equations. Algorithms to reduce the computing tasks by reducing the dimension of matrices and that of the corresponding matrix equations are developed.

2 FORMULATION

2.1 *State equation for multisupported systems*

The equation of a linear dynamic system can be written as

$$\mathbf{M}\ddot{\mathbf{r}}(t) + \mathbf{D}\dot{\mathbf{r}}(t) + \mathbf{K}\mathbf{r}(t) = \mathbf{p}(t), \qquad (1)$$

where $\mathbf{r}$ is the vector of nodal displacement, the dot denotes differentiation with respect to time, $\mathbf{M}$, $\mathbf{D}$ and $\mathbf{K}$ are the mass, damping and stiffness matrices respectively and $\mathbf{p}$ is the vector of dynamic load. If the system is subjected to earthquake ground motion, it is convenient to seperate the n degrees of freedom of the structure and the f degrees of freedom of the supports, which are connected to the ground:

$$\begin{bmatrix} \mathbf{M}_{ss} & \mathbf{M}_{sb} \\ \mathbf{M}_{bs} & \mathbf{M}_{bb} \end{bmatrix} \begin{bmatrix} \ddot{\mathbf{r}}_s \\ \ddot{\mathbf{r}}_b \end{bmatrix} + \begin{bmatrix} \mathbf{D}_{ss} & \mathbf{D}_{sb} \\ \mathbf{D}_{bs} & \mathbf{D}_{bb} \end{bmatrix} \begin{bmatrix} \dot{\mathbf{r}}_s \\ \dot{\mathbf{r}}_b \end{bmatrix} + \begin{bmatrix} \mathbf{K}_{ss} & \mathbf{K}_{sb} \\ \mathbf{K}_{bs} & \mathbf{K}_{bb} \end{bmatrix} \begin{bmatrix} \mathbf{r}_s \\ \mathbf{r}_b \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ \mathbf{p}_b \end{bmatrix} \qquad (2)$$

where the subscripts s and b denote the structure and the supports respectively. The total response of the structure may be seperated into quasi-static and dynamic responses

$$\mathbf{r}(t) = \begin{bmatrix} \mathbf{r}_s(t) \\ \mathbf{r}_b(t) \end{bmatrix} = \begin{bmatrix} \mathbf{r}_q(t) \\ \mathbf{r}_b(t) \end{bmatrix} + \begin{bmatrix} \mathbf{r}_d(t) \\ 0 \end{bmatrix} \quad (3)$$

The quasi-static response can be obtained with the equation

$$\mathbf{r}_q(t) = \mathbf{S}\mathbf{r}_b(t), \quad \mathbf{S} = -\mathbf{K}_{ss}^{-1}\mathbf{K}_{sb} \quad (4)$$

and the equation of dynamic response is

$$\mathbf{M}_{ss}\ddot{\mathbf{r}}_d(t) + \mathbf{D}_{ss}\dot{\mathbf{r}}_d(t) + \mathbf{K}_{ss}\mathbf{r}_d(t) = -\mathbf{M}_{sb}^d\ddot{\mathbf{r}}_b(t), \quad (5)$$

$$\mathbf{M}_{sb}^d = \mathbf{M}_{sb} + \mathbf{M}_{ss}\mathbf{S} \quad (6)$$

Equation (5) may be written as the state equation

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\ddot{\mathbf{r}}_b(t), \quad \mathbf{x}(t) = \begin{bmatrix} \mathbf{r}_d(t) \\ \dot{\mathbf{r}}_d(t) \end{bmatrix} \quad (7)$$

$$\mathbf{B} = \begin{bmatrix} \mathbf{0} \\ -\mathbf{M}_{ss}^{-1}\mathbf{M}_{sb}^d \end{bmatrix}, \mathbf{A} = \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ -\mathbf{M}_{ss}^{-1}\mathbf{K}_{ss} & -\mathbf{M}_{ss}^{-1}\mathbf{D}_{ss} \end{bmatrix} \quad (8)$$

where $\mathbf{I}$ is the identity matrix.

For a multisupported system each entry $\ddot{r}_{b_k}$ of the ground acceleration vector $\ddot{\mathbf{r}}_b$ can be represented as the product of a deteministic modulating function multiplied by a stationary random Gaussian filtered zero-mean process (model 1), namely

$$\ddot{r}_{b_k}(t) = \eta_k(t)\mathbf{H}_g\mathbf{v}_k(t), \quad \dot{\mathbf{v}}_k = \mathbf{F}_k\mathbf{v}_k + \mathbf{G}_g w_k(t) \quad (9)$$

where $w_k(t)$ denotes a white noise with constant power spectral density (PSD) S_k. Here a shape filter of first order $\mathbf{F}_k$ is used, for example a Tajimi ground filter, although other kinds of filter are also possible.

$$\mathbf{F}_k = \begin{bmatrix} 0 & 1 \\ -\omega_k^2 & -2\xi_g\omega_k \end{bmatrix} \quad (10)$$

However, for slowly varying $\eta_k(t)$, $\ddot{r}_{b_k}$ can be approched if $\eta_k(t)$ applies directly to $w_k(t)$ (Model 2)

$$\ddot{r}_{b_k}(t) = \mathbf{H}_g\mathbf{v}_k(t), \quad \dot{\mathbf{v}}_k = \mathbf{F}_k\mathbf{v}_k + \mathbf{G}_g\eta_k(t)w_k(t). \quad (11)$$

It is assumed that $w_k(t)$ for every k can be represented by the same white noise, but with different arriving time and intensity

$$w_k(t) = \chi_k w_o(t - t_k), \ k = 1, \ldots, f, \ 0 \le t_1 \ldots \le t_f, \quad (12)$$

The equations for the ground acceleration vector are

$$\ddot{\mathbf{r}}_b(t) = \mathbf{H}\mathbf{v}(t), \quad \dot{\mathbf{v}}(t) = \mathbf{F}\mathbf{v}(t) + \mathbf{G}\boldsymbol{\eta}(t)\mathbf{w}(t), \quad (13)$$

$$\mathbf{v}(t)_{(2f\times1)} = \begin{bmatrix} \mathbf{v}_1(t) & \ldots & \mathbf{v}_f(t) \end{bmatrix}^T,$$

$$\mathbf{w}(t) = \begin{bmatrix} w_1(t) & \ldots & w_f(t) \end{bmatrix}^T \quad (14)$$

$$\mathbf{H}_{(f\times2f)} = \lceil\mathbf{H}_g\rfloor, \ \mathbf{G}_{(2f\times f)} = \lceil\mathbf{G}_g\rfloor, \ \mathbf{F}_{(2f\times2f)} = \lceil\mathbf{F}_k\rfloor, \quad (15)$$

$$\boldsymbol{\eta}(t)_{(f\times f)} = \lceil\eta_k(t)\rfloor, \ \eta_k(t) = \eta_0(t - t_k), \ k = 1, \ldots, f, \quad (16)$$

where $\lceil\rfloor$ denotes diagonal matrix. Combining (5) and (11) an expanded state equation is obtained:

$$\dot{\bar{\mathbf{x}}}(t) = \bar{\mathbf{A}}\bar{\mathbf{x}}(t) + \bar{\mathbf{G}}\boldsymbol{\eta}(t)\bar{\mathbf{w}}(t), \bar{\mathbf{x}}(t) = \begin{bmatrix} \mathbf{x}(t) \\ \mathbf{v}(t) \end{bmatrix}, \quad (17)$$

$$\bar{\mathbf{A}} = \begin{bmatrix} \mathbf{A} & \mathbf{BH} \\ \mathbf{0} & \mathbf{F} \end{bmatrix}, \ \bar{\mathbf{G}} = \begin{bmatrix} \mathbf{0} \\ \mathbf{G} \end{bmatrix} \quad (18)$$

2.2 Solving for covariance matrix

The covariance matrix $\mathbf{P}_{\bar{\mathbf{x}}} = E\{\mathbf{x}(t)\}$ contains the interesting covariance matrix $\mathbf{P}_{\mathbf{x}}$, where $E\{\ .\ \}$ denotes the expectation. $\mathbf{P}_{\bar{\mathbf{x}}}$ can be calculated from the following differential equation:

$$\begin{aligned} \dot{\mathbf{P}}_{\bar{\mathbf{x}}}(t) &= \bar{\mathbf{A}}\mathbf{P}_{\bar{\mathbf{x}}}(t) + \mathbf{P}_{\bar{\mathbf{x}}}(t)\bar{\mathbf{A}}^T + E\{\bar{\mathbf{G}}\boldsymbol{\eta}(t)\mathbf{w}(t)\bar{\mathbf{x}}^T(t)\} \\ &\quad + E\{\bar{\mathbf{x}}(t)\mathbf{w}^T(t)\boldsymbol{\eta}^T(t)\bar{\mathbf{G}}^T\} \end{aligned} \quad (19)$$

Noting

$$\begin{aligned} &E\{\bar{\mathbf{x}}(t)\mathbf{w}^T(t)\boldsymbol{\eta}^T(t)\bar{\mathbf{G}}^T\} \quad (20) \\ &= \int_0^t \boldsymbol{\Phi}_{\bar{\mathbf{A}}}(t-\tau)\bar{\mathbf{G}}\boldsymbol{\eta}(\tau)E\{\mathbf{w}(\tau)\mathbf{w}^T(t)\}\boldsymbol{\eta}^T(t)\bar{\mathbf{G}}^T d\tau, \end{aligned}$$

$$\begin{aligned} &E\{\mathbf{w}(t_1)\mathbf{w}^T(t_2)\} = \delta(t_{12} - t_{kl})\mathbf{Q}_{kl} \quad (21) \\ &= \begin{cases} \mathbf{Q}_{kl} & \text{for } t_1 > t_k, t_2 > t_l, t_{12} = t_{kl} \\ \mathbf{0} & \text{otherwise} \end{cases} \end{aligned}$$

where $\delta(t)$ is the Dirac function, $t_{kl} = t_l - t_k$ and $\mathbf{Q}_{kl}$ denotes a $f \times f$ matrix, which has only one non-zero element $2\pi S_o\chi_k\chi_l$ in the k-th row and l-th column, it can be shown

$$\begin{aligned} \dot{\mathbf{P}}_{\bar{\mathbf{x}}}(t) &= \bar{\mathbf{A}}\mathbf{P}_{\bar{\mathbf{x}}}(t) + \mathbf{P}_{\bar{\mathbf{x}}}(t)\bar{\mathbf{A}}^T + \eta_0^2(t)\bar{\mathbf{G}}\mathbf{Q}_0\bar{\mathbf{G}}^T \quad (22) \\ &\quad + \sum_{l=1}^{f}\sum_{k=1}^{l-1}\eta_0^2(t-t_l)\boldsymbol{\Phi}_{\bar{\mathbf{A}}}(t_{kl})\bar{\mathbf{G}}\mathbf{Q}_{kl}\bar{\mathbf{G}}^T \\ &\quad + \sum_{k=1}^{f}\sum_{l=1}^{k-1}\eta_0^2(t-t_k)\bar{\mathbf{G}}\mathbf{Q}_{kl}\bar{\mathbf{G}}^T\boldsymbol{\Phi}_{\bar{\mathbf{A}}}^T(t_{kl}) \end{aligned}$$

where $\mathbf{Q}_0 = \sum_{k=1}^{f}\mathbf{Q}_{kk}$ and $\boldsymbol{\Phi}_{\bar{\mathbf{A}}}(t) = e^{\bar{\mathbf{A}}t}$ is the fundamental function.

Czerny and Popp (1984) demonstrated that, if eq. (10) (model 2) is applied, many commonly used modulating functions $\eta(t)$ can be integrated in the fundamental function, so that the covariance matrix can be obtained by solving algebraic Ljapunov matrix equation. For example, for $\eta_0(t) = e^{-\alpha t} - e^{-\beta t}, \quad 0 < \alpha < \beta$, the covariance matrix is

$$\mathbf{P}_{\bar{\mathbf{x}}}(t) = \mathbf{\Phi}_{\bar{\mathbf{A}}}(t-t_f)\Big\{\mathbf{P}_{\bar{\mathbf{x}}}(t_f) \tag{23}$$
$$-\sum_{l=1}^{f}\sum_{k=1}^{f}\sum_{\nu=1}^{3} c_\nu e^{-2\alpha_\nu(t_f-t_l)}\mathbf{P}_{kl\nu}\Big\}\mathbf{\Phi}_{\bar{\mathbf{A}}}^T(t-t_f)$$
$$-\sum_{l=1}^{f}\sum_{k=1}^{f}\sum_{\nu=1}^{3} c_\nu e^{-2\alpha_\nu(t_f-t_l)}\mathbf{P}_{kl\nu}.$$

where $\alpha_1 = \alpha, \alpha_2 = \beta, \alpha_3 = \frac{\alpha+\beta}{2}$; $c_1 = c_2 = 1, c_3 = -2$.

The constant matrices $\mathbf{P}_{kl\nu}$ are the solutions of the Ljapunov equations

$$\mathbf{\Phi}_{\bar{\mathbf{A}}}^T(t_{kl})\bar{\mathbf{G}}\mathbf{Q}_{kl}\bar{\mathbf{G}}^T = -\bar{\mathbf{A}}_\nu P_{kl\nu} - P_{kl\nu}\bar{\mathbf{A}}_\nu^T, k \leq l,$$
$$\bar{\mathbf{G}}\mathbf{Q}_{kl}\bar{\mathbf{G}}^T\mathbf{\Phi}_{\bar{\mathbf{A}}}^T(t_{kl}) = -\bar{\mathbf{A}}_\nu P_{kl\nu} - P_{kl\nu}\bar{\mathbf{A}}_\nu^T, k \geq l, \tag{24}$$

where $\bar{\mathbf{A}}_\nu = \bar{\mathbf{A}} + \alpha_\nu\mathbf{I}, \quad \nu = 1,2,3$. For the solution of eq.(24) very efficient algorithms are available, for example that of Smith (1968).

2.3 *Reduction of matrix dimension*

The computing efforts depend on the dimension of the matrices involved. Dividing the matrices into submatrices is advantageous in solving the Ljapunov equtions as well as in calculating the fundamental matrix. In fact only the submatrix $\mathbf{P}_{\mathbf{x}}$ of $\mathbf{P}_{\bar{\mathbf{x}}}$ is of interest.

Expressing $\mathbf{P}_{\bar{\mathbf{x}}} = \begin{bmatrix} \mathbf{P}^{\mathbf{x}}(t) & \mathbf{P}^{\mathbf{xv}}(t) \\ \mathbf{P}^{\mathbf{vx}}(t) & \mathbf{P}^{\mathbf{v}}(t) \end{bmatrix}$, eq.(24) can be replaced by following equations:

$$\begin{aligned}
&\mathbf{F}_\nu P_{kl\nu}^{\mathbf{v}} + P_{kl\nu}^{\mathbf{v}}\mathbf{F}_\nu^T + \mathbf{\Phi}_{\mathbf{F}}(t_{kl})\mathbf{G}\mathbf{Q}_{\mathbf{kl}}\mathbf{G}^T = 0, \ k \leq l,\\
&\mathbf{F}_\nu P_{kl\nu}^{\mathbf{v}} \ + \ P_{kl\nu}^{\mathbf{v}}\mathbf{F}_\nu^T + \mathbf{G}\mathbf{Q}_{\mathbf{kl}}\mathbf{G}^T\mathbf{\Phi}_{\mathbf{F}}^T(t_{kl}) = 0; \ k \geq l,\\
&\mathbf{A}_\nu P_{kl\nu}^{\mathbf{xv}} \ + \ P_{kl\nu}^{\mathbf{xv}}\mathbf{F}_\nu^T + \mathbf{BH}P_{kl\nu}^{\mathbf{v}}\\
&\qquad + \ \mathbf{\Phi}_{\mathbf{K}}(t_{kl})\mathbf{G}\mathbf{Q}_{\mathbf{kl}}\mathbf{G}^T = 0, \ k \leq l, \qquad (25)\\
&\mathbf{A}_\nu P_{kl\nu}^{\mathbf{xv}} \ + \ P_{kl\nu}^{\mathbf{xv}}\mathbf{F}_\nu^T + \mathbf{BH}P_{kl\nu}^{\mathbf{v}} = 0; \ k \geq l,\\
&\mathbf{F}_\nu P_{kl\nu}^{\mathbf{vx}} \ + \ P_{kl\nu}^{\mathbf{vx}}\mathbf{A}_\nu^T + \mathbf{P}_{kl\nu}^{\mathbf{v}}\mathbf{H}^T\mathbf{B}^T = 0, \ k \leq l,\\
&\mathbf{F}_\nu P_{kl\nu}^{\mathbf{vx}} \ + \ P_{kl\nu}^{\mathbf{vx}}\mathbf{A}_\nu^T + P_{kl\nu}^{\mathbf{v}}\mathbf{H}^T\mathbf{B}^T\\
&\qquad + \ \mathbf{G}\mathbf{Q}_{\mathbf{kl}}\mathbf{G}^T\mathbf{\Phi}_{\mathbf{K}}^T(t_{kl}) = 0; \ k \geq l,\\
&\mathbf{A}_\nu P_{kl\nu}^{\mathbf{x}} \ + \ P_{kl\nu}^{\mathbf{x}}\mathbf{A}_\nu^T + \mathbf{BH}P_{kl\nu}^{\mathbf{vx}} + P_{kl\nu}^{\mathbf{xv}}\mathbf{H}^T\mathbf{B}^T = 0,
\end{aligned}$$

where $\mathbf{F}_\nu = \mathbf{F} + \alpha_\nu\mathbf{I}$.

Expressing the fundamental matrix $\mathbf{\Phi}_{\bar{\mathbf{A}}}(t)$ as

$$\mathbf{\Phi}_{\bar{\mathbf{A}}}(t) = \begin{bmatrix} \mathbf{\Phi}_{\mathbf{A}}(t) & \mathbf{\Phi}(t) \\ 0 & \mathbf{\Phi}_{\mathbf{F}}(t) \end{bmatrix} \tag{26}$$

where $\mathbf{\Phi}_{\mathbf{A}}(t) = e^{\mathbf{A}t}$ and $\mathbf{\Phi}_{\mathbf{F}}(t) = e^{\mathbf{F}t}$, it can be shown

$$\mathbf{\Phi}(t) = \mathbf{\Phi}_{\mathbf{A}}(t)\mathbf{C} - \mathbf{C}\mathbf{\Phi}_{\mathbf{F}}(t), \tag{27}$$

where the constant matrix $\mathbf{C}$ can be obtained easily by solving the Ljapunov equation

$$\mathbf{AC} - \mathbf{CF} = \mathbf{BH} \tag{28}$$

In this way the fundamental matrix can be calculated very efficiently. Especially when it is noted, that $\mathbf{\Phi}_{\mathbf{F}}(t)$ is consisted of only f 2×2 submatrices $\mathbf{\Phi}_{\mathbf{F}_k}(t)$, namely $\mathbf{\Phi}_{\mathbf{F}}(t) = \lceil\mathbf{\Phi}_{\mathbf{F}_k}(t)\rfloor$. If modal coordinates are used, $\mathbf{\Phi}_{\mathbf{A}}(t)$ for each mode is a 2×2 matrix, too.

2.4 *Covariance matrix of total response*

The total response is the sum of the dynamic and the quasi-static responses. The covariance matrix of the total response is

$$\mathbf{P}_{\bar{\mathbf{x}}_t} = \mathbf{P}_{\bar{\mathbf{x}}} + \mathbf{P}_{\bar{\mathbf{x}}_q} + \mathbf{P}_{\bar{\mathbf{x}}_{dq}} + \mathbf{P}_{\bar{\mathbf{x}}_{qd}}. \tag{29}$$

where $\mathbf{P}_{\bar{\mathbf{x}}_q}$ is the expanded covariance matrix of quasi-static response and $\mathbf{P}_{\bar{\mathbf{x}}_{dq}} = \mathbf{P}_{\bar{\mathbf{x}}_{qd}}$ is the expanded cross covariance matrix of dynamic and quasi-static response.

$\mathbf{P}_{\bar{\mathbf{x}}_q}$ and $\mathbf{P}_{\bar{\mathbf{x}}_{dq}}$ can be calculated in the same way as $\mathbf{P}_{\bar{\mathbf{x}}}$ using the method described above. Only at the corresponding places in the equations $\mathbf{A}$ and $\mathbf{B}$ should be replaced by $\mathbf{A}_q$ and $\mathbf{B}_q$, where

$$\mathbf{B}_q = \begin{bmatrix} 0 \\ \mathbf{S} \end{bmatrix}, \quad \mathbf{A}_q = \begin{bmatrix} 0 & \mathbf{I} \\ 0 & 0 \end{bmatrix}. \tag{30}$$

3 EXAMPLE

As examples for the application of the proposed method, six three-span beams subjected to excitation at supports are investigated. Each beam is equal-spanned. They have the same moment of inertia about the vertical axis but different length and mass:

System	1	2	3	4	5	6
spanlength (m)	160	160	80	80	40	40
natural freq.(Hz)	1/8	1/4	1/2	1	2	4

It is assumed that an SH-wave propagates along the beam axis. Three cases, namely for wave velocities $V = 1600m/s, 800m/s, 400m/s$, are studied. The same ground filters are used at all supports. That means only the wave propagation effect is considered in this example. However, if a proper correlation function for the spatial variation of the support movement processes is available, the proposed method can also take this into account. Following parameters describe the exciting process:

for the white noise: $R_w(t-\tau) = E\{w(t)w^T(t)\} = q_w^2\delta(t-\tau), q_w = 20m/s^2$;

for the ground filter: $w_g = 15.7/s, \quad \xi_g = 0.6$;

for the modulating function: $\eta(t) = e^{-\alpha t} - e^{-\beta t}, \ \alpha = 0.25/s, \beta = 0.5/s$.

Table 1 shows the comparisions between responses with and without the wave propagation effect. As

Table 1: Variance of the max. curvature compared to simutaneous excitation

a) Point 7

	system					
	1	2	3	4	5	6
V_1	86%	71%	92%	80%	81%	42%
V_2	61%	47%	77%	40%	41%	10%
V_3	35%	62%	42%	3%	42%	52%

b) Point 3

	system					
	1	2	3	4	5	6
V_1	186%	158%	116%	117%	127%	141%
V_2	205%	322%	115%	134%	191%	213%
V_3	126%	127%	130%	137%	270%	132%

V: Wave velocity; V_1=1600m/s, V_2=800m/s, V_3=400m/s;

expected, with the assumption of simultaneous excitation the response at point 7 (the middle point of second span) is always overestimated, while it is in most cases underestimated at point 3 (the middle point of first span). The effect of wave propagation can be significant, even for a moderate span length of 40 m and a wave velocity of 800 m/s.

Figure 1 shows the typical time history of response. For stiff systems, here sytem 5 and 6, the quasi-static response is not negligible, see Table 2. In such cases the cross response has to be taken into account. This is sometimes negative, indicating that the dynamic and quasi-static responses are moving opposed each other.

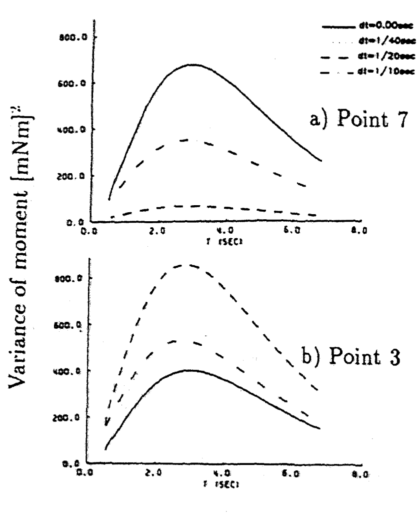

Figure 1. Variance of bending moment , system 6, l=40m, f=4/sec

Table 2: Percentage of dynamic, quasi-static variance and covariance in the total variance of curvature

a) Point 7

System		3	4	5	6
V_1	D	100.0%	100.0%	99.8%	97.9%
	S	0.0%	0.0%	0.0%	0.0%
	C	0.0%	0.0%	0.2%	2.0%
V_2	D	100.3%	100.2%	98.7%	73.5%
	S	0.0%	0.0%	0.0%	4.2%
	C	-0.3%	-0.2%	1.3%	22.3%
V_3	D	100.2%	92.7%	84.6%	77.7%
	S	0.0%	1.5%	2.5%	5.2%
	C	-0.2%	5.8%	13.0%	17.1%

b) Point 3

System		3	4	5	6
V_1	D	100.0%	100.2%	99.7%	98.9%
	Q	0.0%	0.0%	0.0%	0.0%
	C	0.0%	0.2%	0.3%	1.1%
V_2	D	100.9%	100.4%	99.9%	96.0%
	Q	0.0%	0.0%	0.0%	0.2%
	C	-1.0%	0.5%	0.1%	3.8%
V_3	D	100.8%	99.3%	98.0%	80.1%
	Q	0.1%	0.2%	0.1%	3.2%
	C	-0.9%	0.5%	1.9%	16.7%

D, S: Dynamic and quasi-statc responses; C: covariance of dynamic and quasi-static response

4 CONCLUSIONS

The proposed covariance analysis method can be applied to linear structures subjected to non-white nonstationary spatially variating excitation processes. It is showed that to calculate the covariance matrix by solving Ljapunov algebraic equation no numerical integrations are needed, so that the method is very efficient and accurate.

5 REFFERENCES

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