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Analysis of longitudinal vibration of earth dam in triangular canyons

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ABSTRACT In this paper, firstly, a partial differential equation of longitudinal motion for a earth dam with symmetrical cross-section in a triangular canyon by means of shear wedge analysis is presented. Then by using the method of separation of variables and Bubnov-Galerkin approach, an approximate eigenvalue solution of this equation for the fundamental natural frequency of vibration of the dam and some calculation formulas for longitudinal earthquake response of dam are given. These formulas are very simple and dynamic analyses may readily be made by hand calculation. At the end, a sample calculation has been presented.

1 INTRODUCTION

In the majority of earth dams shaken by severe earthquakes, two primary types of damage have occurred (Ambraseys 1960): longitudinal cracks at the top of the dam and transverse cracks sometimes accompanied by crest settlement. The longitudinal cracks appear to have been caused primarily by the horizontal component of the earthquake motion in the upstream-downstream direction, that is, the direction perpendicular to the longitudinal axis of the dam. In contrast, transverse cracking of an earth dam can result from longitudinal dynamic strains induced by earthquake motion in the longitudinal direction (Seed et al 1978). This paper develops an approximate analytical method for evaluating the dynamic characteristics of earth dams in the direction parallel to the dam axis.

2 DIFFERENTIAL EQUATION OF LONGITUDINAL VIBRATION OF EARTH DAM

Fig. 1 shows a max. longitudinal section, a max. transversal section and a longitudinal slice of dam. Assume longitudinal section and transversal section are symmetrical triangle. The assumptions inherent to a shear wedge analysis of a symmetrical earth dam are as follows: (1) The canyon wall are perfectly rigid; (2) The direction of ground motion is horizontal and parallel to dam axis and there are no displacements in other directions; (3) The dam is homogeneous and the dam materials are linearly elastic; (4) Interaction between water in the reservoir and the dam is negligible; (5) Only shear deformation is taken into account.

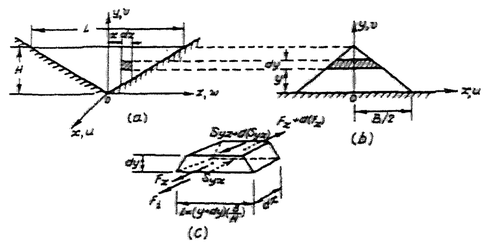

Fig. 1 Analytical model of dam in triangular canyon for shear wedge analysis

Forces acting on an element in the longitudinal direction, as shown in Fig. 1(c), are:

1. Inertial force:

$$F_i = \rho\left(l + \frac{1}{2}\frac{\partial l}{\partial y}dy\right)\frac{\partial^2 w}{\partial t^2}$$

2. Shear force on bottom:

$$S_{yz} = l\,G\frac{\partial w}{\partial y}dz$$

3. Shear force on top:

$$S_{yz} + d(S_{yz}) = G\left(\frac{\partial w}{\partial y} + \frac{\partial^2 w}{\partial y^2}dy\right)\left(l + \frac{\partial l}{\partial y}dy\right)dz$$

4. Axial (normal) force on front:

$$F_z = \sigma_z\left(l + \frac{1}{2}\frac{\partial l}{\partial y}dy\right)dy =$$

$$= E\frac{\partial w}{\partial z}\left(l + \frac{1}{2}\frac{\partial l}{\partial y}dy\right)dy$$

5. Axial (normal) force on back face

$$F_z + dF_z = E\left(\frac{\partial w}{\partial z} + \frac{\partial^2 w}{\partial z^2} dz\right)\left(l + \frac{1}{2}\frac{\partial l}{\partial y} dy\right) dy$$

where ρ is density of the material, l the width of element in x direction, E Young's modulus of the material, G shear modulus of the material, σ_z longitudinal (axial) normal stress and t time. $E=2(1+\mu)G=\xi G$ in which μ is the Poisson's ratio of the material.

For the equilibrium of an element (Fig. 1 (c)), the equation of motion governing free longitudinal vibration of dam is obtained:

$$v_s^2 \frac{\partial^2 w}{\partial y^2} + \xi v_s^2 \frac{\partial^2 w}{\partial z^2} + \frac{v_s^2}{y-H}\frac{\partial w}{\partial y} - \frac{\partial^2 w}{\partial t^2} = 0 \qquad (1)$$

where $v_s=\sqrt{G/\rho}$ is shear wave velocity of the material, H the height of the dam.

The following boundary conditions are applicable to the case of a symmetrical dam in a triangular canyon:

$$\left.\begin{aligned} \frac{\partial w}{\partial y} &= 0 \quad at \quad y = H \\ w &= 0 \quad at \quad y = \frac{2H}{L} z \end{aligned}\right\} \qquad (2)$$

where L is length of dam crest.

3 SOLUTION FOR FIRST NATURAL FREQUENCY

By the method of separation of variable $[W=\phi(y,z)T(t)]$, the following equations are obtained:

$$\frac{\partial^2 T}{\partial t^2} + \omega^2 T = 0 \qquad (3)$$

$$\frac{\partial^2 \phi}{\partial y^2} + \xi \frac{\partial^2 \phi}{\partial z^2} + \frac{1}{y-H}\frac{\partial \phi}{\partial y} + \frac{\omega^2}{v_s^2}\phi = 0 \qquad (4)$$

where ω is the natural frequency. Therefore:

$$T = A_1 \cos \omega t + A_2 \sin \omega t \qquad (5)$$

where A_1 and A_2 are arbitrary constant.

Since the boundary conditions given by equation (2) have to be satisfied at all times, the following boundary conditions can be imposed on the function ϕ :

$$\left.\begin{aligned} \frac{\partial \phi}{\partial y} &= 0 \quad at \quad y = H \\ \phi &= 0 \quad at \quad y = \frac{2H}{L} z = Kz \end{aligned}\right\} \qquad (6)$$

Solution in closed form of equation (4) is difficult to obtain. However, an approximate eigenvalue solution of equation (4) can easily be used to obtain a rather accurate value for the first natural frequency of vibration of the system.

According to the Bubnov-Galerkin method (Zienkiewicz 1971), if a function ϕ which satisfies the boundary conditions given by equations (6) can be found, the following integral:

$$\int_0^H\int_0^{y/K}\left(\frac{\partial^2 \phi}{\partial y^2} + \xi\frac{\partial^2 \phi}{\partial z^2} + \frac{1}{y-H}\frac{\partial \phi}{\partial y} + \frac{\omega^2}{v_s^2}\phi\right)\phi \, dz\, dy = 0 \qquad (7)$$

yields an algebraic equation from which the frequency of the system can be determined.

It can easily be shown that the function:

$$\phi = \frac{1}{H^4}(y+Kz)(y-Kz)(y-2H+Kz)(y-2H-Kz) \qquad (8)$$

satisfies the equation (6). After substituting equation (8) into equation (7) and performing the integration the following algebraic equation is obtained:

$$\frac{32}{225}\frac{\omega^2}{K}\frac{H^2}{v_s^2} - \frac{8}{5K} - \frac{32}{45}\xi K = 0 \qquad (9)$$

Solving equation (9) for ω, we get:

$$\omega = \frac{v_s}{H}\sqrt{\frac{45}{4} + 40(1+\mu)\frac{H^2}{L^2}} \qquad (10)$$

This expression gives the first natural frequency i.e. ω_1 of a symmetrical dam in a triangular canyon under longitudinal vibration, the function ϕ in equation (8) is corresponded to the first mode shape of vibration, i.e. ϕ_1 .

4 LONGITUDINAL EARTHQUAKE RESPONSE OF DAM

It is easily proved that the equation governing longitudinal vibration of the dam with damping under earthquake can be written as:

$$\frac{\partial^2 w}{\partial t^2} + \frac{c}{\rho}\frac{\partial w}{\partial t} - \frac{G}{\rho}\left(\frac{\partial^2 w}{\partial y^2} + \xi\frac{\partial^2 w}{\partial z^2} + \frac{1}{y-H}\frac{\partial w}{\partial y}\right) = -\ddot{w}_g(t) \qquad (11)$$

where $\ddot{w}_g(t)$ is acceleration of rigid canyon in the z direction and c is coefficient of damping.

By the method of separation of variables $[w=\sum_1^\infty \phi_n(y,z)T_n(t)]$ and based upon the orthogonality of mode shape, the following two equations for the first mode shape are obtained:

$$\frac{\partial^2 \phi_1}{\partial y^2} + \xi\frac{\partial^2 \phi_1}{\partial z^2} + \frac{1}{y-H}\frac{\partial \phi_1}{\partial y} + \frac{\omega_1^2}{v_s^2}\phi = 0 \qquad (12)$$

$$\frac{\partial^2 T_1}{\partial t^2} + 2\lambda_1\omega_1\frac{\partial T}{\partial t} + \omega_1^2 T_1 = -\eta_1 \ddot{w}_g(t) \qquad (13)$$

where ω_1 is first natural frequency given by equation (10), λ_1 is damping ratio of first mode, it is equal to $c/2\rho\omega_1$, η_1 is mode participate coefficient

$$\eta_1 = \frac{\int_0^H \int_0^{y/K} \phi_1 (H-y)\, dy\, dz}{\int_0^H \int_0^{y/K} \phi_1^2 (H-y)\, dy\, dz} \qquad (14)$$

After substituting the ϕ_1 from equation (8) into equation (14) and performing the integration, then obtains $\eta = 1.839$.

The solution of equation (14) is:

$$T_1 = \frac{-1.839}{\omega_1'} \int_0^t \ddot{w}_g(\tau) e^{-\lambda_1 \omega_1 (t-\tau)} \sin \omega_1' (t-\tau) d\tau \qquad (15)$$

where $\omega_1' = \omega_1\sqrt{1-\lambda_1^2}$, the Duhamal integral may be calculated by numerical integration method.

Because the higher modes have little effect on earthquake response of dam, only a few lower modes (1~3 order) are adopted for practical requirement. Then the longitudinal earthquake responses of dam in triangular canyons can be approximately written as follows: $w \approx \phi_1 T_1$; $\dot{w} \approx \phi_1 \dot{T}_1$; $\ddot{w} \approx \phi_1 \ddot{T}$; $\tau_{yz} \approx G\phi_{1y}' T_1$; $\sigma_z \approx E\phi_{1z}' T_1$, where ϕ_1 , ϕ_{1y}' and ϕ_{1z}' can be determined by equation (8) and its derivative, T_1 , $\dot{T}_1$ and $\ddot{T}_1$ can be obtained by equation (15) and its derivative.

In engineering it is most interesting in the max. response of dam and so the following formulas of max. response are useful for earthquake-resistant design of dam:

$$\left.\begin{aligned}
w_{max} &\approx |\eta_1 \phi_1| \, S_d \approx |1.839\, \phi_1| \, S_d \\
\dot{w}_{max} &\approx |\eta_1 \phi_1| \, S_v \approx |1.839\, \phi_1| \, S_v \\
\ddot{w}_{max} &\approx |\eta_1 \phi_1| \, S_a \approx |1.839\, \phi_1| \, S_a \\
\tau_{yz,max} &\approx |\eta_1 \phi_{1y}'| \approx |1.839\, \phi_{1y}'| \, S_d \\
\sigma_{z,max} &\approx |\eta_1 \phi_{1z}'| \approx |1.839\, \phi_{1z}'| \, S_d
\end{aligned}\right\} \qquad (16)$$

where S_d, S_v and S_a are displacement response spectrum, velocity response spectrum and acceleration response spectrum respectively.

5 EXAMPLE

Suppose the symmetrical earth dam in the triangular canyon is subjected a longitudinal earthquake (EL Centro record in 1940), the max. length of dam L=200 m, max. height H=50 m, the property of dam material are: $G=80$ MP_a, $V_s=200$ m/s, $\mu=0.3$, $\lambda_1=0.1$. Determine the various max. responses in the central and L/4 sections of dam.

Solution: $K = \frac{2H}{L} = \frac{2\times 50}{200} = \frac{1}{2}$

$$E = 2G(1+\mu) = 2\times 80\,(1+0.3) = 208\ MPa$$

Substituting the values of V_s, H, μ, K, L, into equation (10), we get the first natural frequency ω_1 and first natural period T_{D1} as follows:

$$\omega_1 = \frac{200}{50}\sqrt{\frac{4.5}{4} + 4.0\times 1.3\left(\frac{1}{4}\right)^2} = 15.2/s$$

$$T_{D1} = \frac{2\pi}{\omega_1} = \frac{2\times 3.14}{15.2} = 0.41s$$

According to T_{D1} and from the charts of response spectrum (Wiegel (1970)) we get:

$S_v=24.4$ cm/s, $S_d=1.52$ cm, $S_a=400$ cm/s

Substituting the values of S_v, S_d, S_a, G and E into equations (16), the computed results are summarized as shown in Fig. 2.

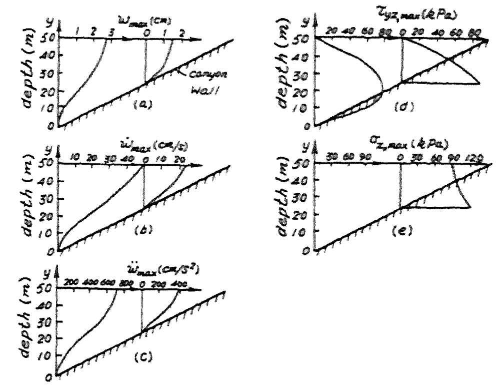

Fig. 2 The various max. responses in central and L/4 section

6 CONCLUSION

The approximately analytical formulas developed in this paper are very simple and they can be used for analysis of longitudinal vibration of earth dam in triangular canyons under earthquake motion, by corresponding simplified method, such as response spectrum technique. Generally, analyses may readily be made by hand calculation even without computer. A detailed example computation has shown that the analytical model presented here will provide information of practical as well as academic significance.

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