Earthquake Engineering, Tenth World Conference © 1992 Balkema, Rotterdam. ISBN 90 5410 060 5

Efficient solution schemes for fluid-structure-soil interaction problems

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ABSTRACT: The equations governing the coupled fluid–structure–soil interaction problem are presented. The resulting system is solved via a partitioned solution procedure which allows parallel processing. A simple isotropic continuum damage model is used for the structural material (concrete). "Transparent" boundaries are introduced to allow outgoing waves to leave the domain and incoming pressure and velocity waves to provide the seismic input. A test example and the analysis of a dam–reservoir system are presented.

1 INTRODUCTION

Fluid–structure interaction problems are of great importance in many branches of engineering. In earthquake engineering the problem is usually augmented because in most cases the foundation must be taken into consideration, yielding a fluid–structure–soil model to be solved. This is a typical coupled problem. In fact, it is the archetypical example of a class of coupled problems where interaction occurs at the interface between different physical domains. Here, neither the structure nor the fluid can be solved independently of the other: motion of the structure depends on the hydrodynamic pressures at the interface, and pressures in the fluid depend on the normal acceleration at the wet wall. It must also be considered that earthquake excitation reaches the structure travelling through the soil region around the structure. This means that soil–structure interaction has to be modelled in some way.

Both the fluid and the soil are semi–infinite unbounded domains. For static or quasi-static loading, a fictitious boundary at a sufficient large distance from the structure, can be introduced. The physical model is hence "cut off", expecting that from the practical point of view, the structural response will not be affected. However, for dynamic loading this procedure cannot be used. The fictitious boundary would reflect outgoing waves back into the domain of interest, spoiling completely the computed solution for any practical purpose. Thus, it is neccesary to model appropriately the fictitious boundaries to allow the incoming seismic waves to enter into the domain, as well as to ensure that the outgoing waves are not reflected.

Figure 1 shows the elements of the computational model necessary to solve the fluid–structure–soil interaction problem in earthquake engineering. All of these are discussed in this paper:

- A discretized structural model that can include nonlinear behaviour of the material.
- A discretized model for the soil. This model can be the same used for the structure, but generally a greater degree of simplicity can be acceptable.
- A discretized model for the fluid that must include compressibility effects.
- A model for the real, interaction and fictitious boundaries: Γ_{ff} free surface in the fluid domain, Γ_{fs} fluid-structure and fluid-soil interaction boundaries, Γ_{sf} structure-fluid and soil-fluid interaction boundaries, Γ_{ss} structure-soil interaction boundary, Γ_{fr} radiating boundary for the fluid, Γ_{fi} radiating boundary for the fluid with incoming seismic waves, Γ_{sr} radiating boundary for the soil domain, and Γ_{si} radiating boundary for the soil with incoming seismic waves.
- A model for discretization in time.
- A model for interaction.

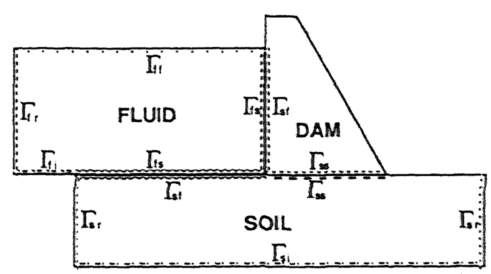

Figure 1. Elements of the discretized model

2 STRUCTURAL MODEL

The semi-discrete dynamic equation for a structure including nonlinear behaviour of the material can be written as

$$M_s\ddot{a} + C_s\dot{a} + p_s(a) = f_s \tag{1}$$

where M_s and C_s are the mass and damping matrices of the structure; $a, \dot{a}, \ddot{a}$ are the structural nodal displacements, velocities and accelerations, respectively; $p_s(a)$ is the internal force vector and f_s is the external force vector. If the displacements u inside a finite element are interpolated in the standard way given by the shape function N_s and the nodal displacements a, the expressions for matrix M_s and vectors $p_s(a)$ and f_s are easily obtained (Zienkiewicz and Taylor (1991)). Rayleigh damping is usually assumed to define the matrix C_s.

The model is completed with an appropriate constitutive equation $\sigma = \sigma(\varepsilon(a))$. Actually, there are a lot of constitutive models in the literature. However, it is a still a challenge to develop a model that can be used in large scale 3D problems under dynamic loading, with fluid–structure–soil interaction. Such a model should represent the most important features of structural material such as concrete but also must be as simple and computationally efficient as possible. In recent years, the so-called continuum damage models have been widely accepted as an alternative to deal with complex constitutive behaviour. The approach has proved simple and versatile, as well as rigorously based on fundamental constitutive theory. The authors have presented an isotropic continuum damage model that exhibits attractive features for numerical analysis of concrete dams (Cervera et al (1991)). The constitutive equation has the simple form:

$$\sigma^t = (1 - d^t)D : \varepsilon^t \tag{4}$$

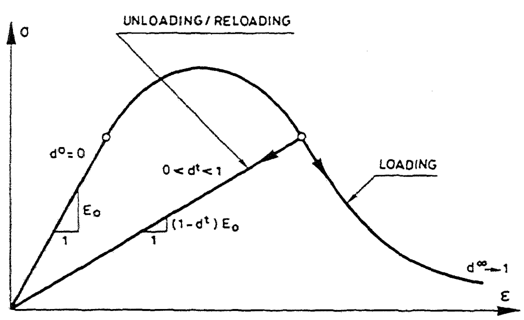

Figure 2. Uniaxial stress-strain curve for the damage model.

where σ^t and ε^t are the current stress and strain tensors, respectively; D is the elastic constitutive tensor and $d^t(\varepsilon^t)$ is a scalar damage variable. The damage or degradation variable d^t is a measure of the loss of secant stiffness of the material, and it ranges from 0 for the undamaged material to 1 for the fully degradated one ($0 \leq d \leq 1$). Figure 2 shows a one dimensional representation of the stiffness evolution of the material. The model defined by equation (4) is fully determined if the value of d^t can be evaluated at any time during the deformation process. In the previous reference the authors present the explicit form of d^t in terms of the strain tensor ε^t at time t.

3 SOIL MODEL

Soils behave nonlinearly when excited by the strong earthquake that are of interest to structural engineers. A possibility is to use a similar constitutive model to that used for the structure. However, when determining the input seismic excitation starting from the motion of one free-field surface control point, superposition is used. This means that a linear system is actually assumed. The fact that only linear analysis are feasible shows how primitive today's state of the art of soil–structure interaction for seismic excitation really is. The simplification of using linear model soil medium should be considered in connection with the unsatisfactory way of determining and specifying the seismic input.

4 FLUID MODEL

The governing equations for acoustic waves written in terms of the pressure for an inviscid, compressible fluid with small amplitudes is:

$$\nabla^2 P = \frac{1}{c^2}\ddot{P} \qquad in \quad \Omega_f \tag{5}$$

where P is the pressure and c is the acoustic speed. After discretization in space, equation (5) reads

$$M_f\ddot{p} + K_f p = f_f \tag{6}$$

where M_f and K_f are the 'mass' and 'stiffness' matrices for the fluid, and f_f is a 'force' vector. If the pressure P inside a finite element is interpolated in the standard way given by the shape function N_f and the nodal pressures p, then

$$\begin{aligned} M_f &= \frac{1}{c^2}\mathbf{A}_e \int_{\Omega_f} N_f^T N_f \, d\Omega \\ K_f &= \mathbf{A}_e \int_{\Omega_f} \nabla N_f^T \nabla N_f \, d\Omega \\ f_f &= \mathbf{A}_e \int_{\Omega_f} \rho_f \nabla N_f^T b_f \, d\Omega \end{aligned} \tag{7}$$

where $\mathbf{A}_e$ is the assembly operator, ρ_f is the fluid density, and b_f are the forces per unit mass. Note that only natural boundary conditions have been considered when deriving equation (6).

5 BOUNDARIES MODEL

5.1 Solid Domain

In the solid domain there are four types of boundaries to be considered (n is the outward normal):

(a) Solid-fluid interaction boundary: Γ_{sf}

$$f_{cs} = -Pn \tag{8}$$

where f_{cs} is the coupling force on the solid.

(b) Radiating boundary: Γ_{sr}

It is possible to define a radiating boundary for elastic waves (Oliver at al, 1992) imposing that

$$f_{rs} = -D_{rs}\dot{a} \tag{9}$$

where f_{rs} is the force acting on the boundary and D_{rs} is a matrix that depends on the elastic properties of the medium and it is given by

$$D_{rs} = \begin{bmatrix} \frac{\lambda+2\mu}{c_u} & 0 & 0 \\ 0 & \frac{\mu}{c_v} & 0 \\ 0 & 0 & \frac{\mu}{c_v} \end{bmatrix} \qquad \begin{array}{l} c_u = \sqrt{\frac{\lambda+2\mu}{\rho_s}} \\ c_v = \sqrt{\frac{\mu}{\rho_s}} \end{array}$$

This radiating boundary is derived for outgoing plane waves propagating in a direction normal to the boundary. In the general case when the wave is not plane, the given expression is only an approximation. Numerical experiments show that the error involved in this approximation is acceptable from an engineering point of view.

(c) Radiating boundary with incoming wave: Γ_{si}

$$f_{rs} = -D_{rs}(\dot{a} - 2\dot{U}_i) \tag{10}$$

where $\dot{U}_i$ is the prescribed incoming velocity wave.

(d) Solid–solid interaction boundary: Γ_{ss}

This is a sub–structuring problem. Figure 3 shows a solution scheme based on a 'splitting zone'. The problem is solved equating the displacements U_1 y U_2 corresponding to SOLID1 and SOLID2, respectively.

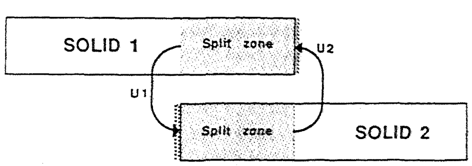

Figure 3. Solid–solid interaction

5.2 Fluid Domain

In the fluid domain there are four types of possible boundaries to be considered (n is the outward normal):

(a) Fluid-solid interaction boundary: Γ_{fs}

$$\nabla P \cdot n = -\rho_f \ddot{a} \cdot n \tag{11}$$

where $\ddot{a}$ is the acceleration at the interface.

(b) Free surface boundary: Γ_{ff}

$$P = 0 \tag{12}$$

(c) Radiating boundary: Γ_{fr}

$$\nabla P \cdot n = -\frac{1}{c}\dot{P} \tag{13}$$

(d) Radiating boundary with incoming wave: Γ_{fi}

$$\nabla P \cdot n = -\frac{1}{c}(\dot{P} + 2\dot{P}_i) \tag{14}$$

where P_i is the prescribed incoming pressure wave.

6 THE SEMI-DISCRETE SYSTEM

Equations (1) and (6) together with boundary conditions (8) to (14) result in the following system of equations after spatial discretization:

$$\begin{array}{l} M_s\ddot{a} + C_s\dot{a} + p_s(a) = f_s - Q^T p \\ M_f\ddot{p} + C_f\dot{p} + K_f a = f_f + \rho_f Q\ddot{a} \end{array} \tag{15}$$

where

$$M_s = \mathbf{A}_e \int_{\Omega_s} \rho_s N_s^T N_s \, d\Omega$$

$$K_s = \mathbf{A}_e \int_{\Omega_s} B^T D B \, d\Omega$$

$$C_s = \alpha M_s + \beta K_s + \mathbf{A}_e \int_{\Gamma_{sr}} D_{rs} \, d\Gamma$$

$$p_s = \mathbf{A}_e \int_{\Omega_s} B^T \sigma(a) \, d\Omega$$

$$f_s = \mathbf{A}_e \int_{\Omega_s} \rho_s N_s^T b_s \, d\Omega + \mathbf{A}_e \int_{\Gamma_t} N^T t \, d\Gamma + \left[\mathbf{A}_e \int_{\Gamma_{si}} n D_{rs} \, d\Gamma \right] 2\dot{U}_i$$

$$M_f = \frac{1}{c^2} \mathbf{A}_e \int_{\Omega_f} N_f^T N_f \, d\Omega$$

$$C_f = \frac{1}{c} \mathbf{A}_e \int_{\Gamma_{fr}} N_f^T N_f d \, D\Gamma$$

$$K_f = \mathbf{A}_e \int_{\Omega_f} \nabla N_f^T \nabla N_f \, d\Omega$$

$$f_f = \mathbf{A}_e \int_{\Omega_s} \rho_f \nabla N_f b_f \, d\Omega + \left[\frac{1}{c} \mathbf{A}_e \int_{\Gamma_{fi}} N_f^T N_f \right] 2\dot{P}_i \, d\Gamma$$

$$Q = \mathbf{A}_e \int_{\Gamma_{sf}} N_s^T n N_f \, d\Gamma$$

The interaction matrix Q is rectangular and it will only have non–zero entries for the interface nodes.

7 TIME DISCRETIZATION

Newmark's scheme is probably the most widely used time–stepping algorithm for second order equations. The scheme may be written in different formats, but for nonlinear problems it is advantageous to formulate it as a prediction–multicorrection algorithm in terms of the incremental of the primary unknown (Hughes and Liu (1978)). The procedure is listed below using the notation corresponding to the single field structural problem:

Equation : $M_s\ddot{a}^{t+\Delta t} + C_s\dot{a}^{t+\Delta t} + p_s(a^{t+\Delta t}) = f^{t+\Delta t}$
Data : $\ddot{a}^t, \dot{a}^t, a^t, f^t$ and $f^{t+\Delta t}$
Unknows : $\ddot{a}^{t+\Delta t}, \dot{a}^{t+\Delta t}$ and $a^{t+\Delta t}$
Scheme :

(1) Set iteration counter $i = 1$ and predict on Δa_i

(2) Correct displacements, velocities and accelerations

$$a_i^{t+\Delta t} = a^t + \Delta a_i$$
$$\dot{a}_i^{t+\Delta t} = \frac{1}{\beta\Delta t}\Delta a_i - \left(\frac{\gamma}{\beta} - 1\right)\dot{a}^t - \Delta t\left(\frac{\gamma}{2\beta} - 1\right)\ddot{a}^t$$
$$\ddot{a}_i^{t+\Delta t} = \frac{1}{\beta\Delta t^2}\Delta a_i - \frac{1}{\beta\Delta t}\dot{a}^t - \left(\frac{1}{2\beta} - 1\right)\ddot{a}^t$$

(3) Compute the residual forces

$$\psi_i = f^{t+\Delta t} - M_s\ddot{a}_i^{t+\Delta t} - C_s\dot{a}_i^{t+\Delta t} - p_s(a_i^{t+\Delta t})$$

(4) If $\|\psi\| \leq \epsilon$ then the solution has been found, set $t = t + \Delta t$ and continue with step (1)

(5) If is it necessary, compute the effective matrix

$$\bar{K}_i = \frac{1}{\beta\Delta t^2}M_s + \frac{\gamma}{\beta\Delta t}C_s + K_s$$

(6) Update the incremental displacement

$$\Delta a_{i+1} = \Delta a_i + \bar{K}_i^{-1}\psi_i$$

(7) Set $i = i + 1$ and go to step (2)

It is well known that this scheme is unconditionally stable for $\gamma \geq 0.5$ and $\beta \geq 0.25(\gamma + 0.5)^2$. For linear problems without interaction, the predictor used in step (1) is of no relevance on the scheme's performance. However, in non-linear or coupled problems solved in a staggered way, special attention must be given to the predictor formulae used. In single field non–linear problems the prediction may affect the rate of convergence of the algorithm, but it will not change the stability properties. In coupled problems, both convergence and stability may be affected by the predictor selected. In this paper four different predictors were considered:

(a) $\Delta a_1 = 0$

(b) $\Delta a_1 = \Delta a^t$

(c) $\Delta a_1 = \Delta t\dot{a}^t + \frac{1}{2}(1 - 2\beta)\Delta t^2\ddot{a}^t$, so that $\ddot{a}^{t+\Delta t} = 0$

(d) $\Delta a_1 = \Delta t\dot{a}^t + \frac{1}{2}\Delta t^2\ddot{a}^t$, so that $\ddot{a}^{t+\Delta t} = \ddot{a}^t$

8 INTERACTION MODEL

The system of equations (15) can be written as:

$$\begin{bmatrix} M_s & 0 \\ -\rho_f Q & M_f \end{bmatrix}\begin{Bmatrix} \ddot{a} \\ \ddot{p} \end{Bmatrix} + \begin{bmatrix} C_s & 0 \\ 0 & C_f \end{bmatrix}\begin{Bmatrix} \dot{a} \\ \dot{p} \end{Bmatrix} + + \begin{bmatrix} K_s & Q \\ 0 & K_f \end{bmatrix}\begin{Bmatrix} a \\ p \end{Bmatrix} = \begin{Bmatrix} f_s \\ f_f \end{Bmatrix} \tag{16}$$

This system of equations can be discretized in time with the scheme presented above. It is a common practice to multiply the second equation by the factor $-\beta\Delta t^2/\rho_f$ first. Then a symmetrix effective matrix is obtained and the direct solution of the full coupled problem may be attempted. However, most of the existing software consists of single field analysers (for structures, fluids, soils, etc.). It would be very convenient to tackle the coupled problem using two (or more) existing single field codes (one for the structure and another for the fluid). The two software packages should be connected through the coupling terms:

$$f_{cs} = -Q^T P \quad \text{and} \quad f_{cf} = \rho_f Q\ddot{a} \tag{17}$$

In this procedure, the first field (equation) would be solved with the predicted values for the variables the other. Once the solution for the first field (equation) is obtained, its values are substituted in the second. In this way, both are treated independently. Many possibilities are immediately open:

- Codes dealing efficiently with single systems could be used,
- Completely different methodologies could be used for each field,
- Parallel computation with its advantages could be used.

This approach is economical and efficient in problems where the response of one field (the fluid) is of secondary importance compared to that of the other (the structure). Here, it is tempting to examine the performance of such staggered procedures. This type of procedure has been previously discussed (Felippa and Park (1980)) and it is known that unconditional stability cannot be achieved without substantial modification. Various stabilization schemes can be used (Felippa and Park (1980), Zienkiewicz and Hughes (1991)), most of them consisting in the addition of damping terms to the governing equations. However, all of these damping terms require transmission of duty information between the two fields, thus destroying the modularity (independence) of the single field analysers used, the main reason to use a staggered procedure in the first place. An alternative to this consists on iterating on the partitioned solution of the coupled system until convergence is reached. This can be seen as a block–iterative solution of the fully discretized system resulting from (16). This means that it is necessary to find a cost–minimization compromise between a few passes solution with small time steps and a more iterated solution process with larger time steps. This compromise may depend, among other things, in the degree of nonlinearity of the structural problem, which may require equilibrium iterations independently of the interaction effects. From the computational point of view, a one–pass solution with no iteration would be optimal, but stability considerations may prove this impractical.

The stability conditions of single field analysers are well established. However, coupled problems with partitioning schemes using the predictor–multicorrector algorithm shown above are not easily amenable to stability analysis. Stability depends then on many factors such as: the predictor formulae, the time integration scheme, the computational path, and the mesh partition. In the following a test example is presented to illustrate some of these dependencies bis–a–bis the iterated solution.

9 TEST EXAMPLE

To study the effect of several predictors on the stability of the iterative procedure using the predictor–multicorrector Newmark scheme written in displacement form, the following unidimensional test example was analysed using codes OMEGA and SHUTTLE developed by the authors.

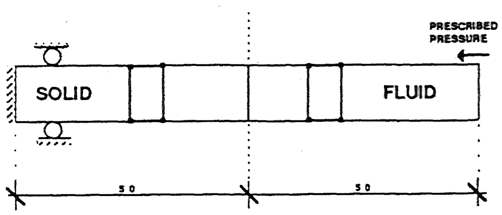

Figure 4. Unidimensional test example.

Figure 4 shows a sketch of the problem: it consists of a plane strain solid bar fixed at the left end, and coupled to a fluid domain. The solid material properties are: $E = 1$, $\nu = 0$ and $\rho_s = 1$. The speed of the compression waves is then $c_u = 1$. The fluid density is $\rho_f = 1$ and the speed of the pressure wave in the fluid is $c_f = 1$. The system is excited with a prescribed pressure at the right end expressed by the function: $P(t) = +100(1 + \sin(2\pi t/50 - \pi/2))$ for $t \leq 50$ and $P(t) = 0$ for $t > 50$. This generates a compressive wave travelling from right to left with speed $c = 1$. The wave travels along the fluid, enters the solid through the interface, travels along the solid, reflects at the left end, returns through both domains and reflects at the right end, starting the sequence again. The domain was discretized with 100 linear four–noded elements (50 solid elements and 50 fluid elements) of dimension 1x1x1.

Firstly, the problem was tackled with a one–pass procedure and in a "solid then fluid" sequence. This proved that predictors (a) and (b) are uncondicionally unstable with no iteration. Predictors (c) and (d) are conditionally stable with a critical time step size of $\Delta t \leq 0.5$. Secondly, the problem was solved using $\Delta t = 1$ and iterating with a tolerance of 10% in the iterative change of pressures. The number of iterations per step was 3.9, 4.4, 6.7 and 7.8 for predictors (a), (b), (c) and (d), respectively.

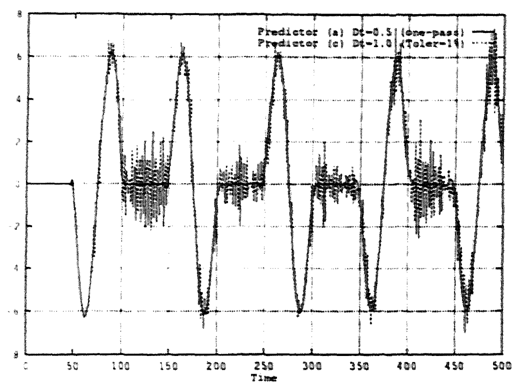

Figure 5. Acceleration at the interaction node.

Figure 5 shows the acceleration in the solid at the interaction node for predictor (a) with $\Delta t = 0.5$ (one–pass) and for predictor (c) with $\Delta t = 1.0$ iterating with a tolerance of 1% in the iterative change of pressures. The final time for both analyses was 500. The analysis using predictor (a) and no iterations obtained the best solution with 1000 resolutions while the other obtained a worse solution with 3973 resolutions !.

10 ANALYSIS OF A DAM

A dam–reservoir–ground system which includes all aspects of fluid–structure–soil interaction is solved as an example. The Koyna Dam (India), 107 meters high above fundation with a water level of 81.45 meters, is modelled. The material properties for the dam are: $E = 31.64$ [GPa], $\nu = 0.2$ and $\rho_s = 2690$ [Kg/m^3]. For the damage constitutive model a fracture energy G_f=100 [J/m^2] and a tensile strenght $f_t = 0.85$ [MPa] are used. The soil is considered elastic with $E = 18$ [GPa] and $\nu = 0.2$. The fluid properties are $\rho_f = 1019$ [Kg/m^3] and $c = 1439$ [m/s].

Figure 6 shows the finite element meshes used. The dam and soil are modelled as one domain discretized with 18 and 26 plane strain eight–noded elements, respectively. The fluid is discretized as a separate domain with 30 eight–noded elements. The left and right boundaries was modelled as radiating in both domains. The bottom boundary is modelled as radiating with an incoming seismic wave. No damping was considered apart from that provided by the radiating boundaries, and the nonlinear material behaviour of the dam.

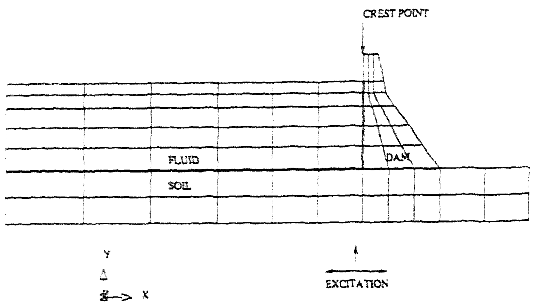

Figure 6. Koyna Dam–reservoir–ground system.

The gravity force is applied in a quasi-static fashion. The dynamic excitation consists of a horinzontal signal at the bottom boundary given by the function: $\dot{U}_i(x,t) = 0.035\sin(\pi t/0.192)$ for $t \leq 1.536$ and $\dot{U}_i = 0$ for $t > 1.536$. The iterative solution is obtained with a tolerance of 10% in the iterative change of pressures. The average number of iterations per time step is 2.3 with a maximum of 5.

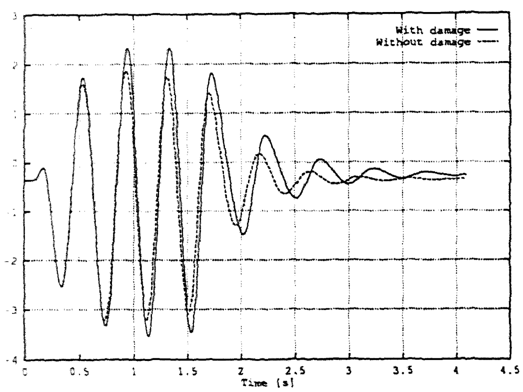

Figure 7. Horizontal displacement at the crest [cm].

Figure 7 compares the horizontal displacement computed at the crest of the dam with and without consideration of the nonlinear structural behaviour. In the case of nonlinear material the peak displacements and the final displacement after the oscillations die out are greater due to the occurrence of damage and the corresponding loss of structural stiffness. The damping of the oscillations after the excitation ends is due to the presence of the radiating boundaries, that preclude spurious reflections of the outgoing waves back into the domain. Note that the initial and final displacements are negative (towards the reservoir) because of the initial deformation of the soil; with a more rigid foundation, hydrostatic pressure would cause this displacement to be positive indeed.

11 CONCLUSIONS

The isotropic continuum damage model is shown to be particularly suitable for large scale transient problems due to its explicit format.

The seismic input via imposed velocities at "transparent" boundaries is found to be the most general, compared to the traditional imposed accelerations or imposed displacements.

Any partitioned solution attains unconditional stability if iterated to convergence, regardless the choice of predictors. In our experience predictor (a) requires less iterations, but unfortunately this predictor with a one–pass strategy is unconditionally unstable.

The accuracy of the conditionally stable one–pass procedures is very good. If time steps larger than the critical one are used, a large number of iterations is necessary to attain the same degree of accuracy.

The damping provided by the radiating boundaries reduces significantly the number of iterations needed to attain a given accuracy. This indicates that stability is improved by the inclusion of such boundaries.

With damping present in the system stability is improved, so larger time steps may be used. The time step size will probably be limited by accuracy considerations, specially if nonlinear behaviour is considered.

In partitioned analysis, the best computational path is the one followed by the excitation wave: soil–dam–fluid for seismic dam analysis.

It is found that the CPU time spent by the fluid solver is about one quarter of that spent by the structural solver. This ratio would indeed be lower for 3D problems.

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