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Dam stability and time-dependent coefficient of friction

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ABSTRACT: The stability of the concrete gravity dams in the case of strong ground motions is an important problem with new designed dams and also in observing old dams which are in exploitation. This problem was studied by observing the contact between the dam body and the foundation floor. Based upon the theoretical and experimental research work, analytical methods were studied to calculate and to assess the non-linear behaviour of the foundation rock-dam contact. Coefficient of friction plays a very important role in the stability analysis. In the literature the static coefficient of friction $\mu(stat)$ and the dynamic coefficient of friction $\mu(dyn)$ are assumed to be constant and that $\mu(dyn) \approx 0.8\mu(stat)$. It has been astablished that the constant coefficient of friction in that range does not give the appropriate results. The method for the variation of the friction coefficient during the time is given.

1.0 INTRODUCTION

The body of a concrete gravity dam is considered as a body consisting of individual functional blocks divided by expansion joints which enable independent displacement of individual blocks during a strong earthquake. In the event of very strong motions, relative displacements of the dam structure may occur with respect to the ground, so that the structure loses its static stability and slides or even rotates. In the conventional stability analysis such cases are regarded as ultimate conditions of failure. If the structure is observed from the aspect of dynamic behaviour, it can be established that such displacements are not necessarily treated as failure and the structure can still perform its basic function for which it had been designed.

In our research work an analytical model was constructed, by means of which the behaviour of the structure over a period of time was observed. Through the study of the behaviour of simple models we tried to recognise the factors which are important for dam stability. We used the theory of rigid body on rigid floor. Floor excitation was given in horizontal and vertical direction. Special attention was paid to the coefficient of friction which has a very important effect on the behaviour of the body. Conventional supposition that the coefficient of friction should be constant, is suitable only for static analysis. In the dynamic stability analysis we use the variable coefficient of friction, which is, as we said, "time dependent". That means that we followed the variation of friction coefficient through the time period and we described it through the time variable influence of different contact conditions on the floor, direction and velocity of motion and magnitude of floor excitation.

2.0 RIGID BODY ON RIGID FLOOR

The basic mechanism of motion of rigid body on rigid floor at strong ground motion was studied more than a hundred years ago by Milne and Perry (Ref.1 and Ref.2). This phenomenon has been studied by a number of researchers since that time and was presented by Ishiyama 1983 (Ref.3).

In conventional observation of stability the problems of overturning or sliding would occur under the following conditions:

$$X'' \geq \frac{B}{H} g \quad .....(A)$$

$$\mu \leq \frac{B}{H} \quad ......(B)$$

where:
X'' = horizontal floor acceleration
B,H = width and height of the body
g = gravitational acceleration
μ = coefficient of friction

In fact the equation (A) describes only the beginning of rotation around one of the edges of the body. If we suppose that at this moment the edge of the body would also slide (B), the problem of motion becomes very difficult.

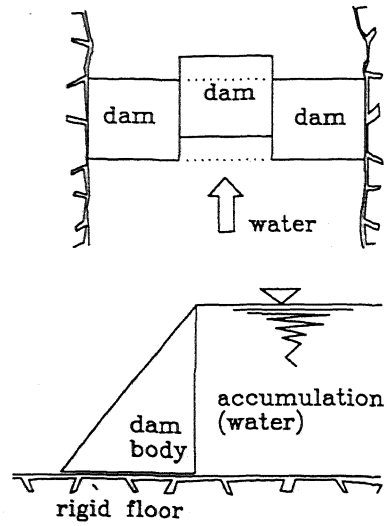

Fig.1 Definition of the problem

2.1. BASIC EQUATIONS OF MOTION

Each type of motion can be described with more equations. We give here only the basic equations for acceleration of an unsymmetric body in case of slide and slide-rotation. The position of the center of gravity of the body can be described by following variables :

x, x', x'' - displacement, velocity and acceleration in horizontal direction,
y, y', y'' - displacement, velocity and acceleration in vertical direction,
$\theta, \theta', \theta''$ - rotation angle, rotation velocity and rotation acceleration.

The basic assumptions in this theory are:
- the motion of the body is always in the x-y plane,
- the body and the floor are rigid,
- the body is unsymmetric,
- the floor is always horizontal,
- floor excitation is in horizontal (X'') and/or in vertical (Y'') direction,
- additional forces which describe hydrostatic and hydrodynamic forces are constant in directions and magnitudes,
- the time interval of the observation is very short. The body occupies the same position during that time interval.

Slide:

$$\boxed{x'' = -X'' - \mu\left(S(g+Y'') - \frac{V}{m}\right) + \frac{P}{m}}$$

$y'' = y' = y = 0$
$\theta'' = \theta' = \theta = 0$
where: $S = \mathrm{sgn}\,(-x^{\circ})$ for $x'^{\circ} \neq 0$
and $S = \mathrm{sgn}\,(-X'')$ for $x'^{\circ} = 0$. Index $^{\circ}$ describes the contact floor - body.

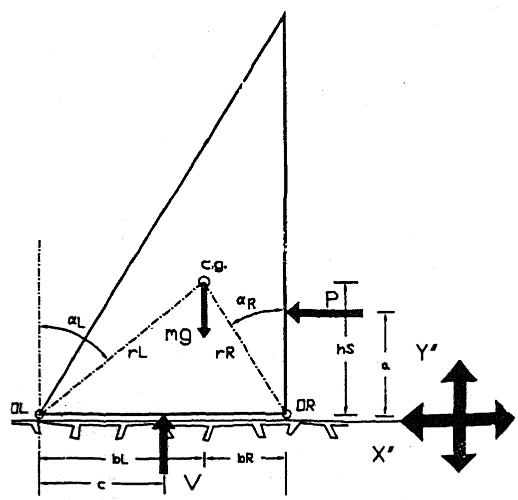

Fig. 2 Description of the body

Slide - rotation :

$$x'' = -X''-\mu\left(S(g+Y''-\theta_i'^2 r_i\cos(\alpha_i-|\theta_i|)+S'\theta_i'' r_i\sin(\alpha_i-|\theta_i|))-\frac{V}{m}\right)+\frac{P}{m}$$

Angular acceleration of the body is given for i = L,R (left and/or right edge) as:

$$A = r_i((g + Y'')-r_i\theta_i'^2\cos(\alpha_i-|\theta_i|))(\mu\cos(\alpha_i-|\theta_i|)-\sin(\alpha_i-|\theta_i|))$$

$$B = -\frac{P}{m}(r_i\cos(\alpha_i-|\theta_i|) - a) - \frac{V}{m}(r_i\sin(\alpha_i-|\theta_i|) - c)$$

$$C = (i^2 - S'r_i^2\sin(\alpha_i-|\theta_i|)(\mu\cos(\alpha_i-|\theta_i|)-\sin(\alpha_i-|\theta_i|))$$

$$\theta_i'' = \frac{A + B}{C}$$

where r_i is the distance between the pole of rotation and the center of gravity of the body, i is the radius of gyration of the body about the center of gravity, α_i is the angle between the vertical line and the line connecting the base edge and the center of gravity of the body at rest. $S=\mathrm{sgn}(x'^\circ)$ if $x'^\circ \neq 0$, $S=\mathrm{sgn}(-X'')$ if $x'^\circ=0$ and $S'=\mathrm{sgn}(\theta)$ if $\theta\neq 0$, $S'=\mathrm{sgn}(\theta')$ if $\theta=0$ and $\theta'\neq 0$ or $S'=\mathrm{sgn}(X'')$ if $\theta=\theta'=0$. Given equations describe some types of motion of unsymmetric body relative to the floor. Through numerical solution of these differential equations all parameters are obtained, i.e. displacements and velocities in horizontal and vertical directions and rotation angles.

3.0 COEFFICIENT OF FRICTION

Coefficient of friction μ is usually defined as the quotient of the friction force (Ft) and the normal gravitational force (Fg), i.e.:

$$\mu = \frac{Ft}{Fg}$$

where Fg and Ft are experimentally determined. For a usual stability analysis such definition of the coefficient of friction is sufficient and it is called the static coefficient of friction. In the case of sliding the coefficient of friction is introduced as a dynamic (kinetic) coefficient of friction. Its order of magnitude is normally taken as 75% to 80% of the static coefficient and remains constant for the period of observation.

With earthquake floor excitation such definition of coefficient of friction is unsuitable and does not give the appropriate analytical results which might be compared to the measured displacements. Therefore, another definition of the coefficient of friction was introduced. It is called the time-dependent coefficient of friction which actually means that it is dependent on the current location on the floor area (the micro relief of contact surfaces of the body and of the floor has a great effect on the behaviour of the body). In addition to that, also the magnitude, the direction of floor excitation and the velocity of the body motion are important. So the coefficient of friction can be expressed and observed at every moment t:

$$\mu(t) = \frac{-\,(x'' + X'')}{S'(g + Y'' + y'')}$$

The variables X", Y" (floor excitations) and gravitational acceleration g on the right side of the equation are the input data for the stability analysis. Accelerations of the center of gravity x" and y" are obtained experimentally. This equation gives positive and negative values for $\mu(t)$ which means that the *coefficient of friction can be obtained* in positive and negative direction of body motion. Since μ can physically only be greater than 0, despite of the direction into which the body is moving, in further analysis only the absolute values have been considered.

With respect to the great number of data $|\mu(t)|$, which can largely deviate from the mean value, we tried to express the data by a polynomial of n-th degree

$$\boxed{|\mu(t)| = a_0 + a_1 t + a_2 t^2 + \ldots + a_n t^n}$$

where $a_0, \ldots, a_n$ are coefficients of a polynomial obtained on the basis of a linear regression where t means time.

In our case the results obtained by a polynomial of 4th degree were suitable (Fig.3) what was confirmed also *through experimental work.*

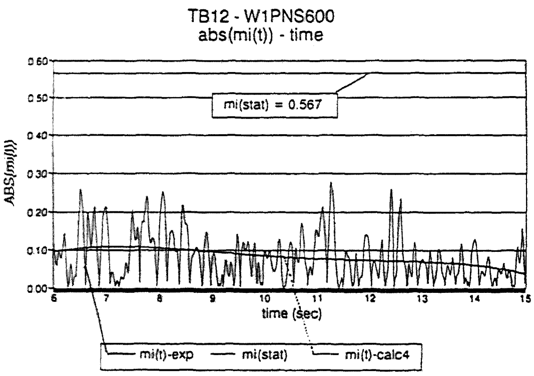

Fig.3 $|\mu(t)|$ and polyn. of 4th dgr

4.0 EXPERIMENTAL WORK

The basic goal of experimental work was to test and to confirm the suppositions and analytical approach when describing the motions of hypotheti*cal model of the dam* on *rigid floor.*

Our research work was divided into two different steps:

a) Testing under static loads where the behaviour of the contact between the body and the floor was observed by use of detachable mats.
b) Testing under dynamic loads where the horizontal component of harmonic and earthquake excitations was used. *The additional horizontal loads* as well as the contact parameters were varied.

The model, sized 72x50x50 cm, was made of concrete, grade 30 (Fig.4). The model was designed to enable an easy change of the contact conditions and additional loads. At the bottom it was possible to change the mat made of different materials, as well as the floor material. The concept of the model enabled combinations which could be marked as $M_{i,j,k}$, where:
M = model,
i = soil quality,
j = contact mat quality,
k = additional horizontal force.

The values of "i" and "j" could be 1(concrete), 2(steel) or 3(timber), respectively. The values of "k" could be 0(without) or 1(with additional horizontal force).

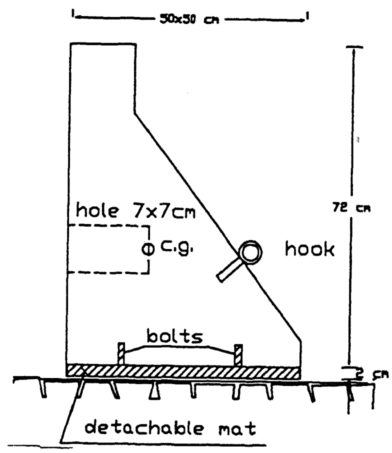

Fig.4 Model of the dam

Based upon the above combinations, it was possible to find the critical conditions for slide, rotations, slide-rotation or overturning of the specimens and to follow the behaviour of the specimens for a few seconds.

4.1 Testing on the vibroplatform

The model layout and the measuring equipment are shown in Fig.6. The motions of the model, relative to the vibroplatform, were observed. The effect of sliding and rotation was followed by measuring the horizontal and vertical displacements of the contact between the specimens and the floor. The acceleration of the center of gravity of the body in both directions was measured as the most important data for later input in the analytical procedure. Checking the input, the acceleration of the vibroplatform and the acceleration at the top of the model were used for control. As dynamic excitations were used:

- sinusoidal excitations of frequency 5.0 Hz and peak accelerations of 0.3g - 0.5g,
- earthquake excitations of original time history of the earthquakes in Monte Negro in 1979, Petrovac N-S, having a peak acceleration of 0.23g - 1.0g.

During the experiments a special problem was to keep the body moving only in one vertical plane. The tendency of torsional rotation round the vertical axis has been noticed for several times. The irregularity of such motion is probably caused by the micro relief of the floor and eccentricity of the center of gravity of the body against the vertical plane.

5. RESULTS

Over hundred experiments with different contact conditions and excitations were made. These experiments were followed by analytical work. Through numerical integration of the differential equation of the body motion we obtained the residual displacements of the dam body (which has to be theoretically treated as an unsymmetric rigid body) relative to the rigid floor. By introducing the new defined coefficient of friction into the numerical analysis of body motion, we obtained the results shown in Fig.5. The results show that the displacements of the body, obtained by analytical procedure, are similar to the experimental ones. This is a general remark. Sliding was the predominant type of motion in all cases. The analytically obtained displacements are in the range of the experimentally measured ones. We are satisfied with these results particulary due to the stohastic character of the input data (definition of the earthquake, probability of the arise, intensity, frequency, etc.). We think that such prognosis could be used in practise as a basis for structural solutions in the construction of dams.

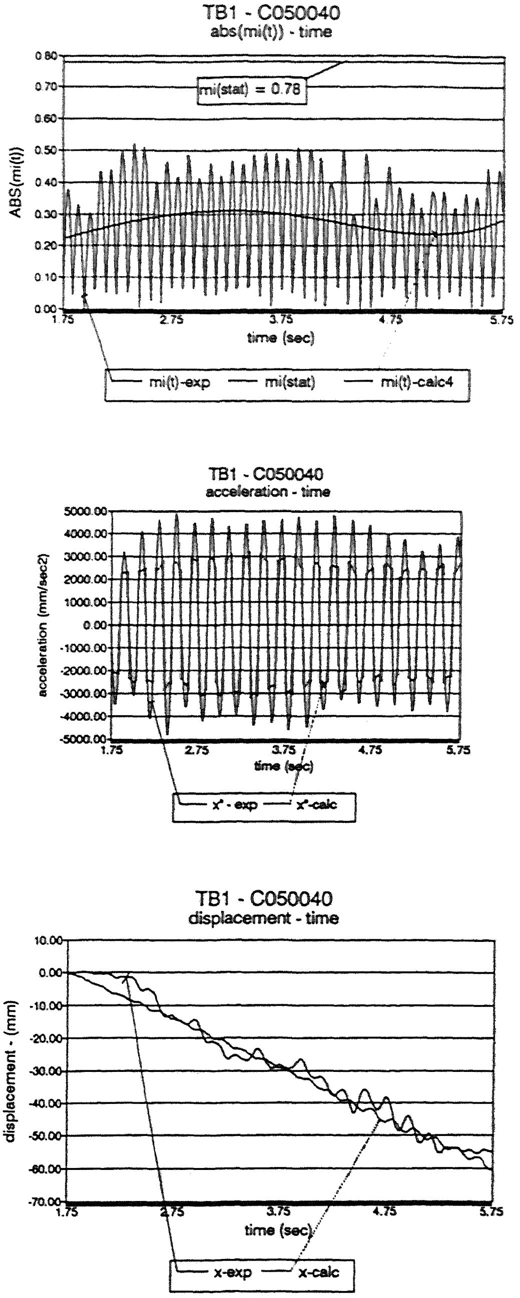

Fig.5 Comparison of the measured and analytically obtained results

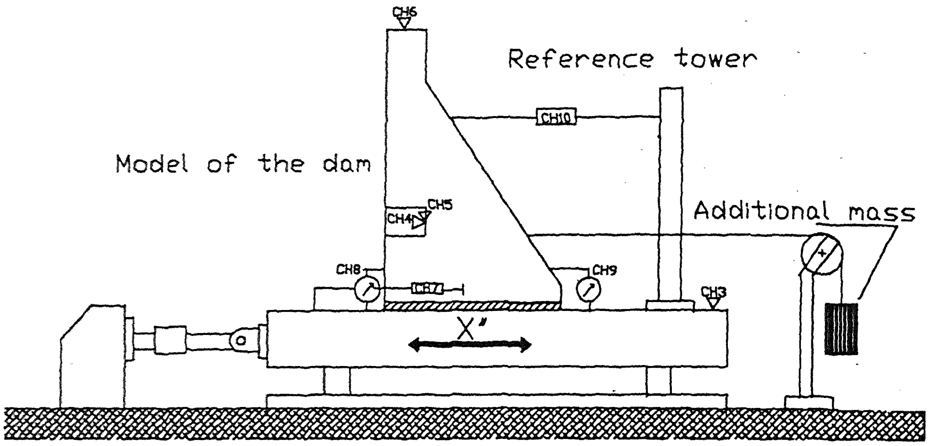

Fig.6 Model on the vibroplatform

6. CONCLUSIONS

We think that conventional measurement of μ(dyn) is not sufficient for the purposes of earthquake engineering. The velocity of the loading or acceleration of the mass of the body, which is in some kind of motion, is very important and has a great influence on the magnitude of the coefficient of friction. With the classical experiment for μ(dyn) the velocities of the body are not as high as in the case of the body which moves on the floor at an earthquake. On the other hand, the body moves on the same area more times in both directions (forwards and backwards). It comes to rubbing and the body moves on some kind of powder as a rest of the micro relief of the contact area which was produced through moving of the body on the floor. So the average value of μ(t) has to be lower than the value of μ(dyn) obtained in a conventional way. According to our experiments μ(t) should be in the range of 15% - 30% of the μ(stat), sometimes even lower.

The presented methodology for stability analysis could be used also for concrete gravity dams of lower and middle height ($h \le 80$ m) where the assumption of the rigid body of the dam on the rigid rock floor is still appropriate.

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