Finite element seismic analysis of dams including interaction effects
Computational model and validation

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ABSTRACT: In this paper a numerical-experimental study of the seismic analysis of double curvature arch dams under linear elastic conditions is presented. Both empty reservoir and full reservoir conditions taking into account fluid-structure interaction effects are considered. The numerical computations obtained via finite element methods are compared with experimental "in situ" tests on an existing dam, and also with results obtained in the experimental analysis of a 1:100 scale model.

1 NUMERICAL MODEL
1.1 Dam analysis

The dynamic analysis of double curvature arch dams is performed using the finite element method. After adequate discretisation using 3D solid elements the dynamic equilibrium equations can be written in the standard matrix form (Zienkiewicz 1970)-(Bathe 1982):

\[ \mathbf{M} \ddot{\mathbf{u}} + \mathbf{C} \dot{\mathbf{u}} + \mathbf{K} \mathbf{u} = -\mathbf{M} \dddot{\mathbf{u}}_g \]  

(1)

where \( \mathbf{M} \), \( \mathbf{C} \), and \( \mathbf{K} \) are the well known mass, damping and stiffness matrices, respectively, and \( \dot{\mathbf{u}}, \ddot{\mathbf{u}}, \dddot{\mathbf{u}} \) and \( \dddot{\mathbf{u}}_g \) are the ground acceleration at the dam base (Barbat and Miquel 1989).

The dam body is discretized using 20 node isoparametric hexaedral elements. Also 13 node triangular prisms have been used to model more accurately the interaction zones with the foundation.

The inclusion of the foundation in the discretisation is not simple due to well known wave radiation effects at the mesh boundaries in the soil zone and special techniques are needed to avoid artificial reflection of waves inside the mesh at these boundaries (Fox and Chopra 1985)-(Miquel et al. 1990). In this work a soil region of a depth equal to the dam height has been modelled with massless 3D solid elements. This has led to good numerical results as it will be shown in a later section.

Eq (1) has been solved using a modal decomposition technique based on the determinat search method (Bathe 1982). The maximum response in the dam has been obtained using a response spectrum.

1.2 Analysis of the fluid and coupling effects

The effect of the reservoir water has been taken into account by solving the wave equation

\[ \nabla^2 p = \frac{1}{c^2} \frac{\partial^2 p}{\partial t^2} \]  

(2)

where \( p \) is the water pressure and \( c \) is the wave speed. All the analysis has been carried out in the frequency domain.

The finite element discretization of the fluid equation leads to (Figure 2) (Fox and Chopra 1985), (Hall and Chopra 1980), (Miquel et al. 1990)

\[
\begin{bmatrix}
\mathbf{P}_1 \\
\mathbf{P}_2 \\
\mathbf{P}_3
\end{bmatrix} = \begin{bmatrix}
-\omega^2 \mathbf{G}_{11} + \mathbf{H}_{11} & -\omega^2 \mathbf{G}_{12} + \mathbf{H}_{12} \\
-\omega^2 \mathbf{G}_{21} + \mathbf{H}_{21} & -\omega^2 \mathbf{G}_{22} + \mathbf{H}_{22} \\
\int_{S} N \mathbf{g} \, dS
\end{bmatrix} \begin{bmatrix}
\mathbf{P}_1 \\
\mathbf{P}_2 \\
\mathbf{P}_3
\end{bmatrix} + \begin{bmatrix}
0 \\
0 \\
-5 \mathbf{a}_f
\end{bmatrix}
\]

(3)
where $\omega$ is the considered frequency, $G$ and $H$ the mass and stiffness matrices of the fluid respectively, and $S$ the rest fluid structure coupling matrix. The variables $P_1$ denote the pressures of the nodes in the contact fluid structure region and $P_2$ the rest of the fluid nodal pressures.

$$\omega^2 G_{ij} + H_{ij} \frac{\partial^2}{\partial x^2} + \omega^2_0 \alpha_0 S \frac{\partial}{\partial x} \right]$$

and coupling this equation with the discretized decoupled dam equation for each vibration mode, the following equation system is obtained (Fox and Chopra 1985), (Hall and Chopra 1980), (Miquel et al. 1990)

$$[\begin{array}{c} -\omega^2 G_{11} + H_{11} \\
\alpha^2(-\omega^2 G_{12} + H_{12})x + H_{11} \\
\frac{\partial}{\partial x} \left( \frac{\alpha^2(-\omega^2 G_{22} + H_{22})}{\alpha^2} \right) + \Phi' S \\
\frac{\partial}{\partial x} \frac{\alpha^2}{\alpha} \\
\frac{\partial}{\partial x} x \\
0 \\
\end{array} ] \begin{array}{c} \frac{\alpha^2}{\alpha} \\
\frac{\partial}{\partial x} \\
0 \\
\end{array}$$

and

$$\begin{array}{c} \begin{array}{c} P_1 \\
\gamma \\
y \\
\end{array} = \begin{array}{c} -S J_0 S \\
0 \\
-\Phi M J_0 S \\
\end{array} \end{array}$$

where the elements of $T$ matrix are

$$T_{11} = -\omega^2 + i \omega C_i + \omega^2 i$$
$$T_{1j} = 0 \text{ if } i \neq j$$

In above $\alpha$ are the eigenvalues of $S_e$ (Figure 2), $\Phi$ the eigenvectors of the dam and $\gamma$ the generalized coordinates of the dam, $\gamma$ the generalized coordinates of the cross section of the fluid and $\alpha^2 \Phi$ is a term taking into account the effect of infinite fluid boundaries (Miquel et al. 1990).

By solving the above matrix equation for different values of $\omega$, the peaks in the solution for $\gamma$ will indicate the resonant frequencies of the dam.

The maximum values of the coupled system using the response spectrum have been obtained with the assumption that the excess pressure measured on the upstream wall is proportional to the radial component of the acceleration at each point in the form

$$\mathbf{F}(\omega) = \mathbf{M}_e q \mathbf{A}_r(\omega)$$

Eq.(5) provides the values of the frequency dependent added mass $\mathbf{M}_e$, where and $\mathbf{A}_r(\omega)$ contain the amplitudes of the radial acceleration. This hypothesis has led to very good numerical results (Miquel et al. 1990) (Prove dinamiche, etc. 1980).

2 NUMERICAL-EXPERIMENTAL ANALYSIS

The previous formulation was used to study Lluset dam located in the noroeste of Spain over the Moares hydraulic power plant. This is a 84 height symmetric double curvature arch dam with three centers. The dam is owned by the Spanish hydroelectric company ENHER. Further information on the geometrical and material properties of Lluset dam can be found in (Fernández-Bollo 1989). The seismic loading considered is that defined by the Spanish Seismic Standard (see Figure 3) with $A_0 = 0.075$ and $T_0 = 0.5$ (Norma Simorresistente Española).

Experimental results obtained in the analysis of a 1:100 scale laboratory model of the same dam carried out at ISMES (Bergamo, Italy) were available for the maximum displacements and stresses and the first natural frequencies for both full and empty reservoir conditions. Also the first natural frequencies for the full reservoir case were experimentally measured "in situ" by using dynamite explosion tests in the vicinity of the actual dam and by using eccentric mass techniques in the dam body (Fernández-Bollo 1989).

2.1 Results for the empty reservoir case

Figures 4 and 5 show the two first vibration modes of the dam together with the discretization used. Table 1 shows the comparison of the first four natural frequencies values computed with values obtained experimentaly.
Figure 4 Llauget dam. First modal shape.

Figure 5 Llauget dam. Second modal shape.

Table I Comparison of experimental and computed first four frequencies for Llauget dam. Empty reservoir case.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Experimental values (rad/s$^2$)</th>
<th>Computed values (rad/s$^2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>20.357</td>
<td>23.091</td>
</tr>
<tr>
<td>2</td>
<td>29.217</td>
<td>28.003</td>
</tr>
<tr>
<td>3</td>
<td>31.793</td>
<td>33.724</td>
</tr>
<tr>
<td>4</td>
<td>42.097</td>
<td>45.011</td>
</tr>
</tbody>
</table>

Figure 6 shows the deformed shape of the dam under the response spectrum of Figure 3 applied in the direction of the valley.

Table II Comparison of experimental and computed displacements. Response spectrum acting in the direction of the valley.

<table>
<thead>
<tr>
<th></th>
<th>Experimental results</th>
<th>Computed results</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Rigid Foundation</td>
<td>Discretised foundation equal to dam height</td>
</tr>
<tr>
<td>Displacement along the valley</td>
<td>0.126 cm</td>
<td>0.175 cm</td>
</tr>
<tr>
<td>Displacement perpendicular to the valley</td>
<td>0.080 cm</td>
<td>0.035 cm</td>
</tr>
</tbody>
</table>

Table III shows the same comparisons when the response spectrum acts in the direction perpendicular to the valley.

Table III Empty reservoir case. Comparison of experimental and numerical displacements. Response spectrum acting perpendicular to the valley.

<table>
<thead>
<tr>
<th></th>
<th>Experimental results</th>
<th>Computed results</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Rigid Foundation</td>
<td>Discretised foundation equal to dam height</td>
</tr>
<tr>
<td>Displacement along the valley</td>
<td>0.216 cm</td>
<td>0.119 cm</td>
</tr>
<tr>
<td>Displacement perpendicular to the valley</td>
<td>0.132 cm</td>
<td>0.073 cm</td>
</tr>
</tbody>
</table>

Tables IV and V show the same type of comparison for the maximum horizontal and vertical stresses when the response spectrum acts along and perpendicular to the valley, respectively.
Table IV Empty reservoir case. Comparison of experimental and computed values of maximum vertical and horizontal stresses. Spectrum acting along the valley direction.

<table>
<thead>
<tr>
<th>Experimental results</th>
<th>Computed results</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Rigid Foundation</td>
</tr>
<tr>
<td>Maximum vertical stress</td>
<td>1.51 Kg/cm²</td>
</tr>
<tr>
<td>Maximum horizontal stress</td>
<td>1.70 Kg/cm²</td>
</tr>
</tbody>
</table>

Table V Empty reservoir case. Comparison of experimental and computed values of maximum vertical and horizontal stresses. Spectrum acting perpendicular to the valley direction.

<table>
<thead>
<tr>
<th>Experimental results</th>
<th>Computed results</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Rigid Foundation</td>
</tr>
<tr>
<td>Maximum vertical stress</td>
<td>1.53 Kg/cm²</td>
</tr>
<tr>
<td>Maximum horizontal stress</td>
<td>2.36 Kg/cm²</td>
</tr>
</tbody>
</table>

Table VI Full reservoir case. Natural frequencies for the first four modes. Comparison of computed and experimental values.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Computed Values</th>
<th>Experimental tests</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>17.1 rdn/s</td>
<td>16.75 rdn/s</td>
</tr>
<tr>
<td>2</td>
<td>21.5 rdn/s</td>
<td>23.66 rdn/s</td>
</tr>
<tr>
<td>3</td>
<td>26.5 rdn/s</td>
<td>25.7 rdn/s</td>
</tr>
<tr>
<td>4</td>
<td>36.5 rdn/s</td>
<td>37.13 rdn/s</td>
</tr>
</tbody>
</table>

Comparison of numerical and experimental results show an excellent agreement with maximum differences in the vicinity of 10%. Also the need for including the effect of a discretized foundation zone equal to the dam height in the computation is demonstrated.

Finally, Table VI and VII show the same comparisons for the full reservoir case. Note that the "in situ" tests provided a value of the first natural frequency of 16.9 rad/s which again corresponds very well with the computed value.

3 CONCLUSIONS

In this work a finite element numerical model for the dynamic analysis of double curvature arch dams taking into account fluid interaction effects has been presented. The accuracy of the model has been tested on the analysis of a dam for which experimental results are available.

ACKNOWLEDGEMENTS

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