Earthquake Engineering, Tenth World Conference © 1992 Balkema, Rotterdam. ISBN 90 5410 060 5

On the lateral strength of multistory masonry walls with openings and horizontal reinforcing connections

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ABSTRACT: Aim of the paper is to perform a complete analysis of the lateral strenght of the multistory masonry walls with various kinds of horizontal reinforcing connections. This walls are the essential resistant elements of the masonry buildings under the seismic action. This analysis, in line with previous studies, will be performed in the framework of the limit analysis theory by using the unilateral no-tension model for the masonry material. A suitable stress analysis, taking in account the loading history up to the collapse, will be developed and some simplifying assumptions will be analyzed in order to determine the axial load in the horizontal connections at the ultimate state of the wall. The paper presents, in conclusions, a detailed example showing the calculation method.

1 INTRODUCTION

A double order of multistory walls with openings, weakly connected to wooden or iron beam floors, is the main resistant structure of a typical hystorical plain masonry building. Different systems of reinforcement can be arranged to improve its seismic strenght and, first of all, to prevent the out of plane collapse of the exterior walls.

The most common system of reinforcement consists in the insertion into the original building structure of suitable horizontal devices as:

- steel ties passing trough the piers and running inside the floors with anchor plates at the heads;
- steel ties in perforation cemented to the masonry;
- reinforced concrete tie beams at story level with new floors in reinforced concrete;
- steel *H* beams above the openings embedded inside the masonry walls.

The paper, that moves in the framework of the limit analysis applied to the masonry structure [1],[2],[3], will examine the effect, on the lateral strenght of the building, of the insertion of the steel ties passing trough the masonry piers and running inside the floors. The effect of the reinforcing system above the openings will be also analyzed.

The rigid in compression no-tension model for the masonry material is the main assumption used in this paper to establish a simplified model of the structure collapse. In the evaluation of the limit deformation of the masonry walls, usually with piers of large width, elastic strains under horizontal imposed forces can be considered negligible with respect to fracture ones. Thus, under the action of the seismic horizontal forces, the piers of the walls will remain practically vertical as long as their local turnover failure does not occur. The horizontal displacement of the failed piers will be therefore due only to their rotations around their toes.

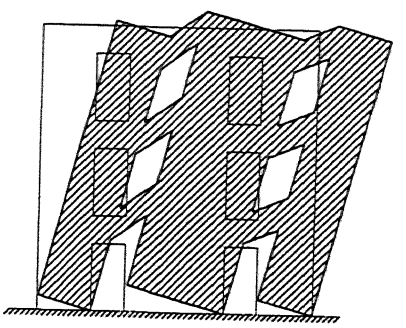

fig.1

The presence of the horizontal connections as architraves and tie rods, will produce then an interaction among the piers as long as the beam sidesway mechanism of the whole wall (fig.1) is not attained. In the analysis of this interaction, it is assumed that the platbands, for lack of fixing, and the architraves in plain masonry, are able to sustain only compressive forces. On the contrary the steel ties can sustain only tractions. Both architraves and ties will develop elastic strains respectively in compression or extension.

2 THE SEISMIC LOADS TRANSFER FROM THE NEUTRAL TO THE ACTIVE MASONRY WALLS

Let us consider the simple floor plan of fig.2. In the picture the reinforcing tie rods placed at each floor level and the direction of the floor beams are drawn.

The openings in the walls have a regular pattern, both in vertical and horizontal direction: therefore within the walls it is possible to distinguish between vertical

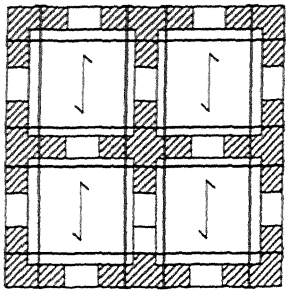

fig.2

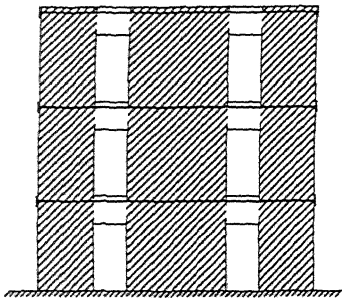

fig.3

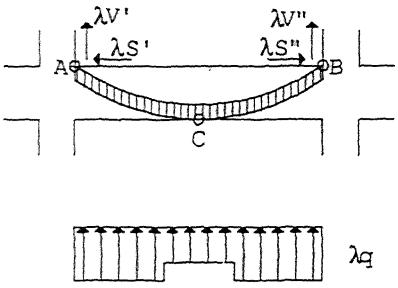

fig.4

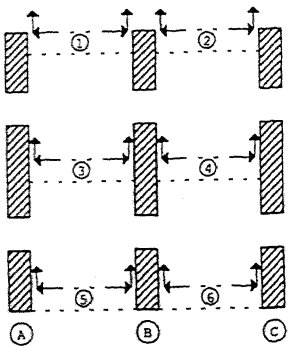

fig.5

elements called "piers" and horizontal ones called "architraves" (fig.3).

The piers will be connected by means of masonry architraves, eventually reinforced by *H* steel platbands, and by steel ties passing trough the masonry at the floor levels.

The building is subjected to vertical fixed dead loads and to horizontal forces gradually increasing with the load factor λ: these forces are representative of the seismic action and are applied to the various stories.

The steel ties passing trough the walls avoid the out of plane collapse of the walls and connect the various piers together, increasing the inplane strenght of the walls. According to the different role explicated by the walls, we can distinguish:

- neutral walls, i.e. walls subjected to out of plane horizontal forces;
- active walls, i.e. walls subjected to inplane horizontal forces.

In the old masonry buildings the horizontal loads move from the neutral to the acting walls by the setting up of a suitable horizontal arch effect acting at the floor level in the neutral walls (fig.4):

The horizontal forces acting on these horizontal arches consist of three different rates:

- the load due to the masses of the same horizontal arches;
- the load due to the masses of the vertical bands contiguous to the openings in the neutral walls and transferred by bending and compression;
- the load due to the masses of the floors that are supported by the same neutral walls.

Let us evaluate the shear forces λV' and λV" and the thrusts λS' and λS" transmitted by each horizontal arch to the contiguous active walls(fig.4). Because of the small displacements at the springings, the thrusts in the horizontal arch will be minimum and the arch can be approximately considered as a three pin structure with the two hinges A and B at the abutements and the internal hinge C at the midspan of the arch.

In the fig.5 a simple sketch of the load transfer from the neutral to the active walls is given. In this figure the horizontal arches are numbered and the active walls are labelled with a letter.

The loads transferred from the neutral walls to the active ones can be subdivided into three distinct groups:

1 - in plane actions directly absorbed by the piers of the active walls as $\lambda V'_3$ and $\lambda V'_5$ on the wall A, $\lambda V''_3$, $\lambda V''_5$, $\lambda V'_4$ and $\lambda V''_6$ on the wall B, etc.

2 - actions out of plane of the active walls, directly absorbed by the steel ties ortogonal to the seismic forces (as $\lambda S'_1$, $\lambda S''_2$, $\lambda S'_3$, etc.) or by the piers of the neutral walls (as $\lambda S''_1$- $\lambda S'_2$, etc.).

3 - inplane actions absorbed by the steel ties running parallel to the active walls as $\lambda V'_1$,$\lambda V''_1$, $\lambda V'_2$, $\lambda V''_2$.

The above stated considerations enable to compute immediately a first rate of the total traction in the ties. A second rate is typical of the external ties. With reference for instance to the wall A (fig.6), the thrust $\lambda S'_1$ tries to produce the local failure of the corner with a hinging around the vertical axis O.

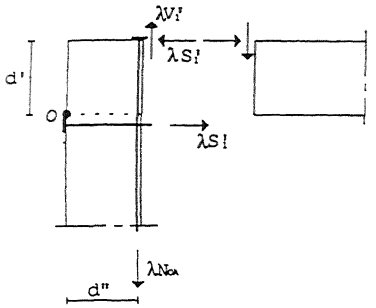

fig.6

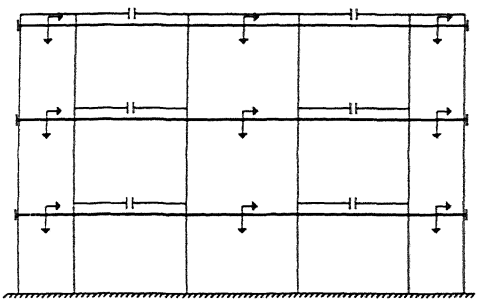

fig.7

The first two rates of the total traction in the tie are therefore:

$$\lambda N_{0A} = \lambda V'_1 + d'/d'' \ \lambda S'_1$$

The steel tie, not cemented to the masonry, transfers its traction to the pier at the opposite side od the wall by means of the anchor plates. Of course the total net horizontal force transmitted to the whole wall A by the external neutral wall is equal to $\lambda V'_1$ because of the counter thrust acting at the hinge O.

3 THE MECHANICAL MODEL OF THE ACTIVE WALL AT THE COLLAPSE

The lateral strength of the masonry walls depends on three different main effects. The first more relevant is due to the lifting of the vertical dead loads that occurs during the development of the beam sidesway mechanisme (fig.1); the piers, in fact, behave as simple vertical cantilevers and at the collapse rotate at their toes [4],[5],[6]. The second effect is due to the energy dissipation in shear that, during the failure of the wall, occurs in the masonry panels placed above the openings and sustained by the architraves or the platbands. The third effect is due to the plastic dissipation of energy at the plastic hinges that develop in the steel platbands.

The second and the third are negligible if the panels connecting the piers have small height and the steel *H* platbands are absent or not sufficiently fixed inside the walls.

Let us consider a plane multistory wall with a regular array of openings, with N_p stories and N_m piers (fig.7).

The wall is subjected to the action of fixed dead loads and imposed horizontal loads, gradually increasing with the load factor λ. The piers are connected by means of masonry architraves, eventually reinforced by steel beams, and also by steel ties passing through the masonry at the floor levels

The architraves will be able to support only compressive forces. On the contrary, the steel ties can sustain only tractions. Both the architraves and ties can develop elastic strains. The rigid-in compression no-tension model for the masonry material is the main assumption used in this paper to obtain a simplified model of the masonry building. In the masonry walls, usually characterized by piers of large width, and subjected to fixed dead loads and horizontal imposed forces, the elastic strains are negligible. We can therefore take in the account only the deformations due to the masonry fracture. Consequently, under the action of seismic horizontal forces, the piers will remain rigid as long as the local turnover fail does not occur. The horizontal displacements of the failed piers will be therefore due to their rotations around their toes.

At a generic stage of loading, on a pier act:

- vertical loads G_{ij}, applied to the pier i at the story j, that represent the weigth of the masonry and of the floor sustained by the pier. The position of the loads G_{ij}, with respect to the bottom right toe, are defined by the arms b_{ij};
- horizontal imposed seismic loads $\lambda G'_{ij}$ including the inertial forces due to the masses G_{ij}, and the horizontal forces transmitted by the neutral walls to the piers by means of the above pointed out transfer system. The elevations of the various stories, where the forces are applied, are indicated by z_j;
- actions trasmitted by the horizontal connections.

The fig.8 sketches all the forces acting on the various piers at ultimate state of the wall. With the reference, for instance, to the pier i, the forces $N_{i-1,j}$ and $N_{i,j}$ represent respectively the thrusts transmitted by the precedent or subsequent unilateral horizontal connection.

Analogously $M_{0\,i-1,j}$ and $M_{0\,i,j}$ are the ultimate moments and $T_{0\,i-1,j}$ and $T_{0\,i,j}$ are the ultimate shear due to the steel beam eventually reinforcing the precedent or the subsequent masonry architrave.

According to the positions

$$M_i^S = \sum_{j=1}^{N_p} G_{ij}\, b_{ij} + \sum_{j=1}^{N_p} (M_{0\,i-1,j} + M_{0\,i,j}) + \sum_{j=1}^{N_p} T_{0\,i-1,j}\, B_i$$

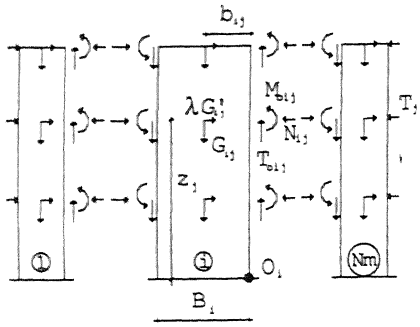

fig.8

$$M_i^R = \sum_{j=1}^{N_p} G'_{ij} z_j \; ; \quad M_i^N = \sum_{j=1}^{N_p} N_{ij} z_j \; ; \quad M^T = \sum_{j=1}^{N_p} T_j z_j \tag{2}$$

the quantities M_i^S, M_i^R, M_i^N, M^T respectively represent:

- the stabilizing moment of all the forces acting on the pier i;
- the turnover moment of the imposed horizontal loads;
- the moment of the axial loads in the architrave with respect to the pier base and relative to the span i;
- the moment of the tractions in the steel ties with respect to the pier base.

Each masonry pier i is characterized by a proper lateral strength λ_{0i} defined by the following limit value of the multiplier λ:

$$\lambda_{0i} = M_i^S / M_i^R \tag{3}$$

Under the gradually increasing forces the local failure in the weakest pier is attained when the load factor λ reaches the value $\lambda^*_{0i} = \text{Min}\,(\lambda_{0i})$. The full lateral strength of the wall, on the contrary, will be attained when all the single piers have reached their proper failure condition under the limit value λ_0 of the load factor λ.

At the collapse of the whole wall the equilibrium equations around the toes of the piers are:

first pier

$$- M_1^S + \lambda_0 M_1^R - M_1^N + M^T = 0$$

- pier i

$$- M_i^S + \lambda_0 M_i^R + M_{i-1}^N - M_i^N = 0 \tag{4}$$

- last pier

$$- M_{Nm}^S + \lambda_0 M_{Nm}^R + M_{Nm-1}^N - M^T = 0$$

The (4) represent the set of the solution equations in the unknowns λ_0, $M_i^N (i=1,N_m)$ and M^T.

Summing up this equations we obtain the collapse multiplier λ_0:

$$\lambda_0 = \sum_{i=1}^{N_m} M_i^S / \sum_{i=1}^{N_m} M_i^R \tag{5}$$

It is worth to point out that the value λ_0 is included between the minimum and the maximum value of the local collapse multiplier λ_{0i} defined in the (3).

4 STRESSES IN THE STEEL TIES AT THE COLLAPSE OF THE WALL

Once the collapse multiplier λ_0 has been previously evaluated, the system of the equation (4), has a simple infinity of solutions because of the redundance of the internal horizontal costraints connecting the piers.

Because of the unilateral character of the horizontal connections,we can associate to the (4) the other conditions

$$M_i^N \geq 0 \; (i=1,N_m), \qquad M^T \geq 0 \tag{6}$$

The above condition allows us to prove that at least one of the unknowns M_i^N, M^T must be equal to zero. In fact let us suppose that at the collapse the traction in the ties is not equal to zero: it means that the horizontal fiber of the wall along any tie is stretched out. If in any span the compressions in the architraves where different from zero, the same horizontal line should become shorter; hence to accept the stretching of the fiber,yelds that at least in one span the compression must be zero. Likewise if all the compressions are different from zero, the traction in the steel ties must be necessary equal to zero. Since at least one of the unknowns must be zero, the system (4) has a univocal determined solution.

A preventive evaluation of the unknowns equal to zero is necessary to get the actual solution, as firstly proposed in [7].

Once the collapse factor λ_0 has evaluated, the equations (4) can be written in a more suitable form:

$$\begin{aligned} M_1^N - M^T &= M_1^R (\lambda_0 - \lambda_{01}) \\ M_i^N - M_{i-1}^N &= M_i^R (\lambda_0 - \lambda_{0i}) \\ M^T - M_{Nm-1}^N &= M_{Nm}^R (\lambda_0 - \lambda_{0Nm}) \end{aligned} \tag{7}$$

It is possible to find the zero unknowns of the system (7) directly examining the sign of the known terms. In fact if in the equation i this sign is positive, the unknown i can not be zero, otherwise the unknown i-1 should be negative against the conditions (6). So the examination of the sign of the unknown terms in every equation permits to exclude at least one unknown in this sequential search.

It is worth to point out that if the sign of the all the known terms where the same, all the unknowns should be excluded in the search. But at least one sign must be different from the other ones because the global multiplier λ_0 is included in the range of the local multipliers λ_{0i}. Then at the end of the search at least one possible unknown equal to zero must remain.

If after the initial search only one unknown can be shown to be zero, the system (7) has a univocal solution immediately available by the elimination of the null unknown. Let us suppose, on the contrary, that the possible unknowns are two, for instance the kth

and l^{th} with $l > k$. Summing up the equations from k+1 to l, it is possible to obtain one equation in wich only the unknowns k and l appear having different sign. Then examining the sign of this last known term we can establish which of the two unknowns can be zero; so also in this case at least one possible unknown equal to zero remain at the end of the search.[1]

If after the initial search the possible zero unknowns are three or more, it is possible to exclude them one by one adopting the above stated technique, as long as at least one remains.

The solution of the system of equations (7) and the searching technique for the null unknowns have required the development of a specific calculation code implemented on a computer.

The solution of the equilibrium equations (7) allows the evaluation only of the global moment M^T of all the tractions acting in the steel ties at the various levels. For the evaluation of the traction in the single steel ties, the above equilibrium equations are insufficient because of the redundance of the internal horizontal constraints along the height. Nevertherless a simplified method based on the analysis of the behaviour of the walls before the collapse can now be proposed.

We have to point out firstly that the efforts in the horizontal constraints involve elastic strains and therefore can not be put in the relation with the collapse mechanism.The unknowns efforts in the connections are therefore associated to the deformations of the wall that occur during the loading before the collapse.

After this remark let us analyze the evolution of the wall deformations during the progress of the loading. At the beginning the masonry walls will exhibit only small elastic strains and the interaction among the piers, transmitted by the connections, will be negligible because the horizontal displacements of the piers will be nearly equal each others.

On the other hand during the loading in some piers the local overturning condition will be attained and passed.

The horizontal displacements in this last piers will thus become much larger than the displacements of the other piers not yet failed. The magnitude order of the displacements of the piers depend on the rigid rotation at their toes and on the elastic deformations of the horizontal connections. The stretching of the long steel ties passing through the masonry will produce, by means of the rigid rotation at the toes, horizontal displacements of the piers.

This last displacements will increase gradually and will become, approaching the collapse of the wall, much larger than the displacements due only to the elastic deformations of the piers themselves. Thus by assuming this simplified model for the masonry behaviour, we can neglect the displacements of the piers due to their elastic deformation and take in account only the displacements of the piers depending on their rigid rotation. The distribution along the height of the traction in the steel ties will be linearly varying. Particularly, if the ties are all equal, as usual, we have:

$$T_j = T_1 z_j / z_1 \qquad (8)$$

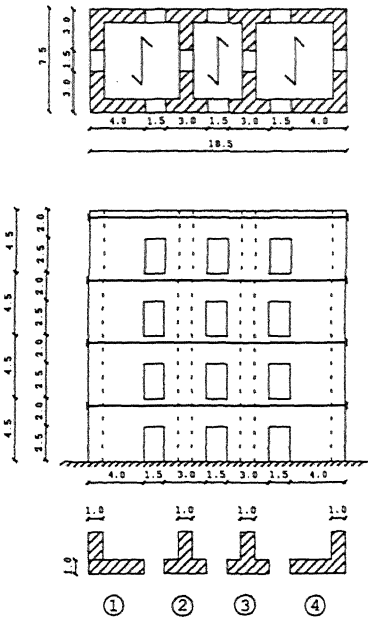

fig. 9

With reference for instance to the wall A of the fig.3 with the position (2) we get:

$$T_{Aj} = M^T{}_A \; z_j \Big/ \sum_{i=1}^{N_p} z_i^2 \qquad (9)$$

The traction (9) represents a third rate to the global traction N_{Aj} acting in the steel ties of the wall A. Summing the rates (1) to (9) we obtain at the collapse of the wall:

$$N_{Aj} = \lambda_0 V'_{1j} + d'/d'' \; \lambda_0 \; S'_{1j} + T_{Aj} \qquad (10)$$

5 A NUMERICAL EXAMPLE

The fig. 9 shows the floor plan of a simple masonry building together with the direction of the floor beams. We suppose a distributed floor load equal to 0.8 t/m^2 and a masonry specific weight equal to 1.6 t/m^3.

Under a seismic load acting from the left to the right, the active walls of the building are the two equal walls sketched in the fig.9 together with an horizontal cross section of the four piers at the first level. At each floor level the thickness of all the masonry panels is reduced by 0.10 m.

The vertical load G_{ij} (see fig.8) acting on the pier i at the level j is evaluated adding the following rates:

pier weigth equal to 1.6 t/m^3 x 4.5 m x A_{ij} where A_{ij} is the area of the pier cross section

- floor loading equal to 5.50 m x 1/2 x 0.8 t/m^2 x l_i where l_i is the width of the floor strip sustained by the pier

tab. A

Pier	1		2		3		4	
Floor	G	G'	G	G'	G	G'	G	G'
1	60	79	51	51	51	51	60	41
2	56	73	48	48	48	48	56	39
3	52	67	45	45	45	45	52	37
4	48	61	42	42	42	42	48	35

- masonry panel loading equal to 2.00 m x 1.6 t/m^3 x s_j x d_i x 1/2 where s_j is the thickness of the masonry panels and d_i is the width of the contiguous openings.

The horizontal load $\lambda G'_{ij}$ is evaluated adding to λG_{ij} the actions transmitted by the neutral walls and not yet included in λG_{ij}. The floor beams are directly sustained by the active walls. Thus the only actions to consider are the ones due to the masses of the same neutral walls. With reference to fig.4, for each horizontal arch we obtain:

$$\lambda V'_j = \lambda V''_j = \lambda\ 19.2 s_j\ t.\ ;\ \lambda S' = \lambda S'' = \lambda\ 30.3 t.$$

Therefore for each pier we have

- piers 2 an 3

$$\lambda G'_{ij} = \lambda G_{ij}$$

because the masses of the contiguous neutral walls have been already included in the terms λG_{ij}.

- pier 4

$$\lambda G'_{ij} = \lambda G_{ij} - \lambda V'_j$$

- pier 1

$$\lambda G'_{ij} = \lambda G_{ij} + \lambda V'_j$$

because at each level the steel ties transfer to the pier 1 the action $\lambda V'_j$, transmitted to the pier 4 by the contiguous neutral wall.

The table A summarizes these preliminary results (the forces are in tons)

We suppose that no *H* steel beam reinforces the openings. Therefore the data of the table A allow to calculate immediately the local collapse multiplier λ_{0i} of the piers (see (2), (3)):

$$\lambda_{01} = 0.208\ ;\ \lambda_{02} = \lambda_{03} = 0.138\ ;\ \lambda_{04} = 0.142$$

The global collapse multiplier λ_0 of the wall is (see (5)):

$$\lambda_0 = 0.162$$

Thus the equilibrium equation system (7) yelds:

$$\begin{aligned} M^N_1 - M^T &= -M^R_1\ 0.046 \\ M^N_2 - M^N_1 &= M^R_2\ 0.024 \\ M^N_3 - M^N_2 &= M^R_3\ 0.024 \\ M^T_0 - M^N_3 &= M^R_4\ 0.020 \end{aligned}$$

Looking at the sign of each known term and, taking in account the conditions (6) give:

$$M^T \neq 0\ ;\ M^N_2 \neq 0\ ;\ M^N_3 \neq 0\ ;\ M^T \neq 0\ .$$

Thus we conclude:

$$M^N_1 = 0$$

The solution of the system yelds

$$M_T = 135.29\ \text{tm}$$

The tractions T_j in the steel ties due to the connecting the various piers together in the beam sidesway mechanism, are valuable by means (9):

$$T_1 = 1.00\,t\ ;\ T_2 = 2.00\,t\ ;\ T_3 = 3.00\,t\ ;\ T_4 = 4.00\,t\,.$$

In order to obtain the global tractions N_j we must add the rates (1) preventing the out of plane collapse of the neutral wall contiguous to the pier 4:

$$N_j = T_j + \lambda_0 V'_j + \lambda_0 S';$$

thus we finally obtain :

$$N_1 = 8.91t.\quad N_2 = 9.60t.\quad N_3 = 10.29t.\quad N_4 = 10.98t.$$

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