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Parametric optimal design of steel structures

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ABSTRACT: Structural optimization procedures are used to perform parametric studies of seismically loaded structures. The structures are analyzed and optimized in the time domain and are subjected to a variety of behavioral and design constraints. Optimization procedures provide a reliable means for comparing structural systems since these procedures ensure that the optimal structures are designed using the exact same criteria for story drifts, displacements, member stresses, natural frequencies, load cases, etc. The parameters to be varied and studied are: differing lateral bracing schemes, differing damping values, multiple versus single time history loadings, and different support conditions. The optimal designs which are subjected to the same behavioral design constraints but with variations of these parameters are compared and contrasted as to their performance, practicality, and cost or weight.

1 INTRODUCTION

The demand to produce reliable structural systems while also considering economics in trying to obtain a minimum cost design has led to a considerable amount of research in the field of optimum structural design as evident by Levy and Lev's compilation of an extensive list of published work (Levy and Lev 1987). As a result, numerous optimization algorithms of linear, nonlinear, and dynamic programming have been developed and used to solve structural problems with great success over the past 30 years. However, to date much consideration has only been given to static loading. Optimization for dynamic loads primarily covers two types of constraints; constraints on the natural frequencies and on quantities related directly to the dynamic response (i.e. displacement and stress). Prior to 1970, little consideration was given to dynamic structural response, and since then most of the work done has been to control natural frequencies since this problem does not involve the parameter of time. Optimization for dynamic displacement and stress constraints has been examined in a limited manner using approximations such as assuming pseudo earthquake loading or sinusoidal loads; thus, not requiring a time history analysis to be performed. Consequently, no method has been developed to handle any arbitrary loading using a time history analysis method.

The optimization of structural systems for sinusoidal loads has been examined by Icerman (1969), Fox and Kapoor (1970), and Mills-Curran and Schmit (1985) to name a few. Earthquake loading has been partially investigated by Venkayya and Khot (1975), Haug, Arora, and Feng (1977) and Cheng and Truman (1983).

An optimization algorithm has been developed that can find the optimum designs for plane frame steel structures subjected to a time history loading (Petruska 1991). An optimality criteria method is used with the objective function being the weight of the building. Possible constraints are displacement, story drift, stress, frequency, and side constraints. Examples will be presented to show the usefulness of this method to structural engineering applications.

2 STRUCTURAL MODEL

The building systems can include beam-columns (shear, bending, and axial deformed members) along with axial bracing. All members are assumed to be wide flange steel sections. A statistical, non-linear

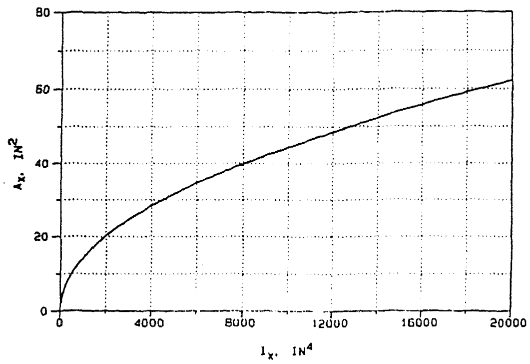

Figure 1. Cross-Sectional Area versus Moment of Inertia for AISC Wide-flange Sections (1 in. = 2.54 cm)

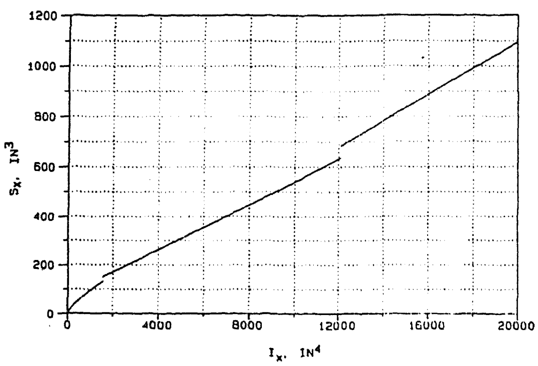

Figure 2. Section Modulis versus Moment of Inertia for AISC Wide-flange Sections (1 in. = 2.54 cm)

relationship is assumed to exist between the moment of inertia and the cross-sectional area and section modulus as shown in Figures 1 and 2.

The dynamic analysis is based upon an elastic stiffness and consistent mass system. The equilibrium equations of motion are

$$[M]\{\ddot{U}\} + [C]\{\dot{U}\} + [K]\{U\} = \{R\} \qquad (1)$$

where [M] is the mass matrix, [C] is the damping matrix, [K] is the stiffness matrix, {R} is the load vector, {U} is the displacement vector and each dot represents one differentiation with respect to time. Rayleigh damping is assumed.

The system of ordinary differential equations given in equation (1) can be approximately solved using any available numerical integration method. The optimization algorithm implements the central difference and Newmark's method. Newmark's method is preferred since it is unconditionally stable thus allowing larger time steps to be used, and thus greatly reducing computing time.

3 STRUCTURAL OPTIMIZATION

In structural optimization, generally the geometry of the structure and the loads are known. Structural optimization involves analyzing the structure to determine the response, followed by a resizing of the design variables so as to minimize the objective function while satisfying all constraints. Since the structures are usually indeterminate and the constraints nonlinear, the algorithms are iterative.

When the weight of the structure is the objective function to be minimized, the structural optimization problem can be stated as follows

$$\text{minimize } W_T = \sum_{i=1}^{m} \rho_i A_i l_i \qquad (2)$$

subject to

$$\underline{U}_j \leq U_j(\delta,t) \leq \overline{U}_j \quad j = 1,\dots,k \qquad (3)$$

here ρ is the density of the material, A is the cross-ectional area, l is the length of the member, U is the constraint which can be displacement, story drift, tress, or frequency, $\overline{U}$ is the upper bound limit on the constraint, $\underline{U}$ is the lower limit, k is the number of constraints, and m is the number of design variables.

An optimality criteria similar to the one used by Cheng and Truman (1983) was employed to solve the optimization problem. The gradients of the constraints with respect to the design variables for a specific time must be known, and are found by directly differentiating the equations for calculating the displacement response at each time step according to the numerical integration method employed. Once the gradients of the displacements are found, the gradients of the story drift and stress response can be determined. The gradients of the frequency constraints can also be found by directly differentiating the equation associated with the eigenvalue problem Petruska (1991).

4 COMPUTATION AND RESULTS

A computer program ODSBDYN-2D, (2 Dimensional Optimum Design of Steel Buildings under DYNamic loading) was developed to handle the optimization algorithm. Several examples will be presented to show how useful and effective the program can be. For all examples presented, E = 29000 ksi, ρ = 490 lb/ft^3, the lower limit on the moment of inertia is 290 in^4, the lower limit on the cross-sectional area is 2 in^2, and initially the structure is at rest and the base undergoes a transient acceleration. The time step used was 0.01 seconds which allows at least ten time steps per fundamental period of the structure, and Newmark's method was used to perform the numerical integration.

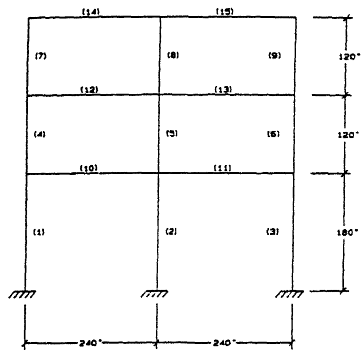

Figure 3. Three-story, Two-bay Frame (1 in. = 2.54 cm)

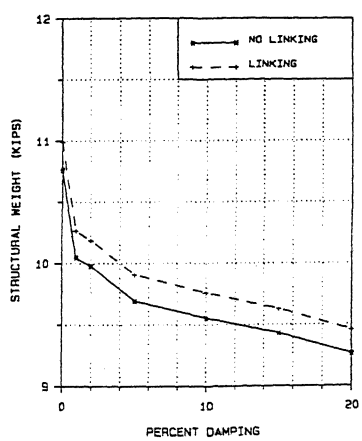

Figure 4. Weight versus Damping Ratio for Three-Story, Two-Bay Frame, Multi-Sine Loading (1 Kip = 4.45 kN)

The first example is the three-story, two bay structure shown in Figure 3. Displacement constraints are considered with the allowed absolute value for the horizontal displacement being 1.00, .65, and 2.35 inches for the first, second, and third floor respectively. A total weight of 33.33 kips was applied uniformly over each beam and was treated as non-structural weight. For this example, damping is neglected. Two base accelerations were considered: the N-S component of the El-Centro earthquake on May 18, 1940 and the N-S component of the San Fernando, California earthquake on February 9, 1971 taken at the Caltech Seismological Lab. The San Fernando earthquake was multiplied by a constant of 1.81 so that both earthquakes had the same peak accelerations. Three optimizations were performed; Case A is the El-Centro Earthquake, Case B is the San Fernando Earthquake, and Case C is both earthquakes considered as two separate load cases. The optimum designs are shown in Table 1.

TABLE 1. Optimum Design for Three-Story, Two-Bay Frame under Earthquake Loading. (1 in. = 2.54 cm, 1 kip = 4.45 kN)

Load	Case A	Case B	Case C
Element number	Moment of Inertia (in^4)		
1	1503.0	1390.7	1076.3
2	5694.3	4849.1	9706.3
3	1501.2	1441.8	1308.7
4	621.5	503.4	1004.8
5	1827.3	1526.8	1719.0
6	623.7	503.8	1007.9
7	355.0	399.1	397.1
8	1050.7	1055.9	1251.8
9	358.4	398.8	389.3
10	1887.0	1705.7	2384.9
11	1887.1	1742.4	2632.8
12	1035.0	1031.5	1769.0
13	1042.1	1028.3	1745.8
14	304.4	340.6	356.2
15	307.5	341.3	357.0
W^*_{Total} (kips)	11.867	11.525	13.516
Iteration	10	8	12
Active Displacement Constraints			
DOF Time (sec) Constraint Value (in.)	10,16 t=3.58 1.628 22 t=3.59 2.378 19 t=4.22 -2.328	10,16 t=7.23 1.652 19,25 t=7.43 2.355	25 E-C t=4.74 -2.360 19 S-F t=6.81 2.353

Case A El-Centro Earthquake
Case B San Fernando
Case C Load Case 1 El-Centro Earthquake
Load Case 2 San Fernando Earthquake

The results show that El Centro earthquake produces a slightly heavier structure than the San Fernando earthquake. However, when the structure is optimized using both earthquakes, an even heavier structure results since the building must satisfy the frequency characteristics of both earthquakes while still satisfying the displacement constraints. Note that the top floor constraint was active at different times for both earthquake load cases.

The next example considers the same frame as shown in Figure 1, but the effect of damping will be included. A multiple sinusoidal load defined as

$$a_g(t) = \begin{cases} -50\sin\pi t + 40\sin 2\pi t + \\ 30\sin 2.5\pi t + 45\sin 3\pi t, \\ 0 \le t \le 5\text{sec.} \\ 0, \quad t > 5\text{sec.} \end{cases} \tag{4}$$

was used for the base acceleration. Damping of the first two modes was considered. Table 2 shows the optimum design, along with Figure 4 showing the optimum weight versus percent damping of the first mode and Figure 5 showing the optimum center column stiffness as a function of floor level. Figure 4 also shows the optimum design for the linked structure. Linking forces all the members in a particular group to be the same size. For the linked case, all columns on a given floor are linked into one set of design variables. Figure 4 shows the largest decrease in weight occurring between 0 and 1% damping with the change being for the unlinked structure approximately 6.7%. The weight decreases nearly linearly between 1 and 5% and between 5 and 20% damping with the slope decreasing as damping increases. The figure also shows that the linked structure is heavier than the unlinked structure which is typical since each member is not free to take on its optimum value. Figure 5 shows that the largest decrease in stiffness occurs between the 0 and 5% damping range with only minor changes in stiffness occurring in the second and third floor for damping greater than 5%. Thus for this structure, it can be concluded that assuming 5 to 10% damping is enough damping to achieve moderate decreases in weight and column stiffness which is a realistic estimate of damping in such a structure under this loading.

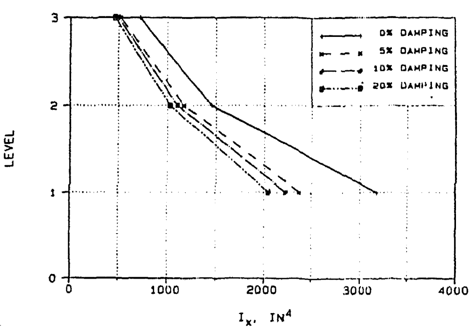

Figure 5. Center Column Stiffness for Three-Story, Two-Bay Frame for various Damping ratios Subject to Displacement Constraints, Multi-sine Loading. No Linking (1 in. = 2.54 cm)

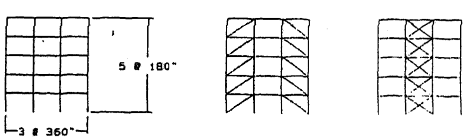

Figure 6. Five-Story, Three-Bay Unbraced and Braced Frames (1 in. = 2.54 cm)

The five-story, three bay frame shown in Figure 6 is considered next. The frame is examined for the two bracing schemes shown along with the moment frame and also for fixed and hinged support

TABLE 2. Optimum Design for Three-Story, Two-Bay Frame subject to Displacement Constraints for various Damping Ratios under Multiple Sine Loading.
(1 in. = 2.54 cm, 1 kip = 4.45 kN)

% Damping	0%	ξ_1=5% ξ_2=5%	ξ_1=10% ξ_2=10%	ξ_1=20% ξ_2=20%	ξ_1=10% ξ_2= 5%
Element number	Moment of Inertia (in^4)				
1,3	1409.1	1126.9	1113.9	1033.9	1114.3
2	3183.8	2370.0	2223.6	2047.2	2223.3
4,6	449.6	392.1	332.4	302.8	331.7
5	1455.1	1215.0	1104.9	1028.4	1106.0
7,9	305.8	290.0	290.0	290.0	290.0
8	716.9	524.5	498.8	466.9	499.0
10,11	1610.6	1256.9	1227.3	1136.5	1226.9
12,13	952.3	757.4	710.4	660.1	710.8
14,15	290.0	290.0	290.0	290.0	290.0
W^*_{Total} (kips)	10.764	9.694	9.555	9.263	9.555
Iteration	6	6	5	5	5
Active Displacement Constraints					
DOF Time (sec) Value (in.)	22 t=4.16 -2.349	19,25 t=3.83 2.351	22 t=4.23 -2.352	10 t=4.27 -1.652 22 t=4.27 -2.352	22 t=4.23 -2.352

conditions. The ground acceleration was defined as

$$a_g(t) = \begin{Bmatrix} 135\sin(2\pi t), & 0 \le t \le 1 \text{ sec} \\ 0, & t > 1 \text{ sec} \end{Bmatrix} \qquad (5)$$

The allowable horizontal displacement was 0.5 inches multiplied by the story number. A uniform weight of 100 lb/in was treated as non-structural weight on the floors. Damping was neglected and will be in the remainder of the examples presented. The results are shown in Figures 7, 8, and 9.

Figure 7 shows the column stiffness versus level for the braced frame for the fixed and hinged support. It can be seen that the support condition really only effects the first floor column stiffness by decreasing the stiffness by 10 to 20% when going from a hinged to a fixed support. The support condition is even less pronounced on the optimum weight. The optimum weight for the K braced frame with hinge supports was 34.1 kips while for the fixed support it was 33.9 kips or a decrease of 0.6%. Figure 8 shows the column stiffness for the moment frame. This example also considers the case where the upper limit on the moment of inertia was set at 28000 in^4. The effect of the support condition is much more apparent on this structure. Using a fixed support greatly decreases the first floor column size while decreasing the weight from 110.8 kips for the hinge supported structure to 85.1 kips for the fixed support or a savings of 30.2%.

Figure 9 shows the optimum weight of the structures versus iteration for the fixed support structure for the three bracing scheme. This figure shows the fast convergence of the algorithm and

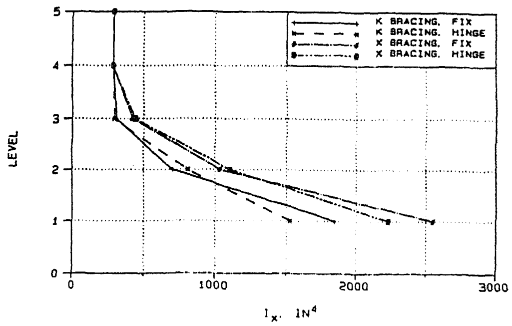

Figure 7. Column Stiffness for Five=Story, Three-Bay Braced Frame. Linking, Hinge and Fix Support. (1 in. = 2.54 cm)

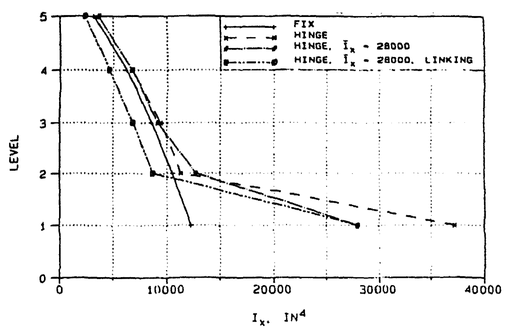

Figure 8. Column Stiffness for Five-Story, Three-Bay Frame, Hinge and Fix Support (1 in. = 2.54 cm)

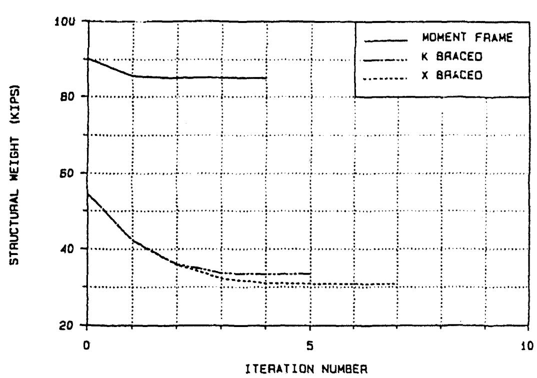

Figure 9. Weight Versus Iteration for Five-Story, Three-Bay Frame (1 kip = 4.45 kN)

compares the global optimum weight for the braced and unbraced structures. Adding the bracing results in approximately a 50% weight decrease. Thus it can be concluded that adding bracing significantly reduces the structural weight and that the support condition has very little effect on the braced frame since the axial bracing will efficiently transfer the loads but has a large effect on the moment frame.

TABLE 3. Optimum Design for Eight-Story Setback Frame (1 in. = 2.54 cm, 1 kip = 4.45 kN)

Constraints	Displ.	Displ. Story Drift	Displ. Stress
Level	Column Moment of Inertia (in^4)		
1	8886.4	9219.2	10244.5
2	6205.4	6006.3	6014.5
3	5018.2	4850.3	5685.8
4	3937.4	3826.0	4623.0
5	3038.5	3122.7	3188.6
6	3439.9	3720.4	3149.2
7	2244.4	2420.2	2015.0
8	1228.3	1431.4	1093.1
Level	Beam Moment of Inertia (in^4)		
1	6324.5	5475.8	5810.9
2	7505.6	7286.4	7162.5
3	6368.7	6007.6	6171.0
4	5051.3	4993.5	5140.9
5	3762.4	3892.3	3634.4
6	6566.8	7144.3	5982.3
7	4339.6	4815.8	3888.8
8	1032.2	1175.0	917.5
W^*_{Total} (kips)	58.85	59.88	59.98
Iterations	5	4	6
Active Displacement Constraints			
DOF Time (sec) Value (in)	61 t=0.78 -6.000	61 t=0.78 -6.011	61 t=0.78 -6.000
Active Story Drift Constraints			
Column # Time (sec) Value (in)		12,13,17, 18,20 t=0.78 -0.826	
Active Stress Constraints			
Member Time (sec Value (ksi)			1,6,8,11 14 t=0.77 30.0

The last example is the eight-story setback frame shown in Figure 10. Both the braced and unbraced frames are considered. This example is presented to show the algorithm's performance and the structures behavior when subject to several different types of constraints. For displacement constraints, the allowed lateral displacement is 0.75 inches multiplied by the story number. For story drift constraints, the allowed value is 0.825 inches. And for stress constraints, the absolute allowed stress is 30 ksi. The sinusoidal loading in equation 5 is used for the base acceleration. All the columns on a given floor are linked, all the beams on the same floor are linked and all the bracing on a given floor are linked. A uniform weight of 100 lb/in was treated as non-structural weight acting on the beams. The optimum design for the unbraced frame is given in Table 3 and for the braced frame in Table 4. The beam sizes for the braced frame are not shown in Table 4 because they have reached their lower bound value of 290 in^4. The results show an increase in column stiffness for the moment frame at the sixth level because the second bay stops at the fifth level, thus an increase in the member size is required because the additional support from the other bay no longer exists and also Also note as the behavior of the structure is restricted, the weight must increase in order to control all of the responses. For the moment frame,

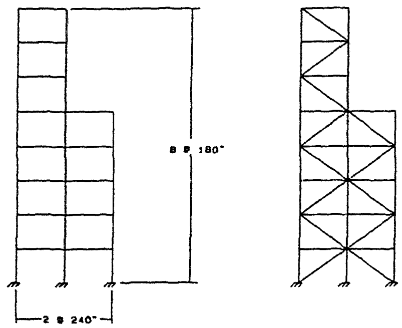

Figure 10. Eight-Story Setback Unbraced and Braced Frames (1 in. = 2.54 cm)

TABLE 4. Optimum Design for Eight-Story Setback Diagonally Braced Frame. (1 in. = 2.54 cm, 1 kip = 4.45kN)

Constraints	Displ.	Displ. Drift	Displ. Stress	Displ. Drift Stress
Level	Column Moment of Inertia (in^4)			
1	781.5	935.6	1156.2	1159.1
2	723.7	888.5	838.0	868.3
3	290.0	427.8	290.0	391.9
4	310.3	499.6	294.8	454.6
5	290.0	375.6	290.0	333.8
6	290.0	610.7	290.0	545.1
7	290.0	290.0	290.0	290.0
8	290.0	290.0	290.0	290.0
Level	Bracing Cross-Sectional Area (in^2)			
1	5.75	5.25	6.41	6.15
2	5.34	4.86	4.97	4.82
3	5.02	4.39	4.33	4.57
4	4.35	3.53	4.18	3.81
5	4.40	5.24	4.11	5.07
6	5.29	6.60	4.37	6.32
7	4.52	7.87	3.59	7.47
8	2.95	5.26	2.27	4.95
W^*_{Total} (kips)	24.30	24.09	22.45	24.14
Iterations	4	7	3	5
Active Displacement Constraints				
DOF Time (sec) Value (in)	61 t=0.77 -6.008		58 t=0.77 -6.001	
Active Story Drift Constraints				
Column # Time (sec) Value (in)		17,18,21 t=0.75 -0.825		17,18,21 t=0.74 -0.825
Active Stress Constraints				
Member Time (sec) Value (ksi)			1,4,10, 44 t=0.77 30.0	1,42 t=0.76 30.0

the column stiffness increases for the first five levels for the case with stress constraints in comparison to the displacement constraint problem while it decreases for the top three levels. However, for the drift constraint problem, the opposite occurs since greater column stiffness is required at the top to control the drift while greater column stiffness is required at the bottom to control the stresses. Finally, note that the addition of bracing to the frame resulted in nearly a 60% weight decrease for all constraint cases examined.

5 CONCLUSIONS

An optimality criteria method is shown to be feasible for optimizing steel frames subject to dynamic loads using a time history analysis method. Several parameteric studies were performed using the developed algorithm in order to study their effects on the optimal design for seismic events. The study showed that the use of bracing is very efficient in controlling the structural response producing significant weight decreases over the unbraced moment frame. The examples also showed that damping beyond five to ten percent of critical has only a little effect on the elastic optimal design which is significant since this is a typical amount of damping that a building can provide. Furthermore, multiple earthquake optimized structures generally require a heavier structural system than single earthquake optimized structures due to the need to avoid differing frequencies from the two seismic events. And finally, the support condition has only a minor effect on a braced frame while it significantly impacts the optimum design of a moment frame.

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