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Estimation of deformation capacity of reinforced concrete columns failing in flexure

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ABSTRACT: This study aims to derive the formulas for estimating the deformation capacity of reinforced concrete columns subjected to flexure, shear and axial force with consideration of the confining effect of transverse reinforcements. Also, it is intended to clarify the required transverse reinforcements in plastic hinge zones for ensuring the adequate deformation capacity.

1 INTRODUCTION

In seismic design of reinforced concrete structures, it is important to ensure the adequate ductility of columns which are subjected to high axial force at lower stories in high-rise buildings. Recently, the ductility of columns has been frequently researched. The compressive ductility of columns has been able to be estimated through the formulas (see Sheikh et al. (1982), and Mander et al. (1988)). Although the code provisions have been proposed for ensuring the ductility of columns subjected to flexure, shear and axial force (see ACI code (1983) and NZ code (1982)), the deformation capacity of such columns can not be estimated by these code provisions.

This study aims to derive the formulas for estimating the deformation capacity of rectangular columns subjected to anti-symmetrical bending moment (flexure and shear) and axial force with consideration of the confining effect of transverse reinforcements. Also, it is intended to clarify the required transverse reinforcements in the plastic hinge zones for ensuring the adequate deformation capacity.

2 COMPRESSIVE STRAIN CAPACITY OF CONFINED CONCRETE

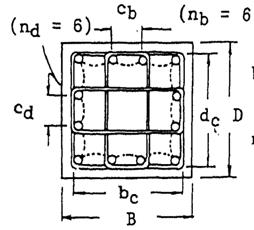
2.1 Definition of confining coefficient

Sheikh and Uzumeri (1982) proposed the formula to estimate the compressive strength of confined concrete. In Sheikh's formula, the confining effect affected by many variables, such as the configuration of transverse reinforcements, is taken into consideration. Although the compressive strength of the confined concrete can be accurately estimated, Sheikh's formula is complicated with many variables to apply to practical problems.

In this study, that formula is modified, as shown in equation (1). This equation is derived with the assumptions that longitudinal reinforcements are neglected and the treatment of the configuration of transverse reinforcements is simplified, i.e., (1) $P_{occ} = 0.85\ f_{co}\ b_c\ d_c$, (2) $\sigma_w = \sigma_{wy}$, (3) $c_b = b_c/n_b$, $c_d = d_c/n_d$: the assumption of the uniformed confinement (see Figure 1).

$$S_c = f_{cc}/f_{co} = 1 + K_c\ (1+s/2b_c)\ (1+s/2d_c) \times \sqrt{(p_{wc}\ \sigma_{wy})}/f_{co} \qquad (1)$$

where, Pocc : Axial strength of columns for the section of core concrete
$S_c=f_{cc}/f_{co}$: Confining coefficient
f_{cc}, f_{co}: Compressive strength of confined and plain concrete
$p_{wc} = a_w/(s\ b_c)$
a_w, s : A set of sectional area and spacing of transverse reinforcements
σ_w, σ_{wy} : Stress at ultimate capacity of columns, and yield strength of transverse reinforcements



b_c, d_c : Center-to-center distance of perimeter hoops
n_c, n_d : The number of confined arcs formed by ties at two sides facing each other

Figure 1 Definition of sectional dimensions

The coefficient K_c in the equation (1) is obtained through the above assumptions, as shown in equation (2). Then, the values of the coefficient K_c are calculated for various types of transverse reinforcements which are frequently used for the practical design, *as shown in Table 1. Also*, these values of K_c express the confining effect by the configuration of transverse reinforcements.

$$K_c = \alpha \{1 - (n_b c_b^2 + n_d c_d^2)/(5.5 b_c d_c)\} \quad (2)$$

where, $\alpha = 8.59 \times \sqrt{\beta}$
(unit of σ_{wy} and f_{co} : MPa)
$= 26.9 \times \sqrt{\beta}$
(unit of σ_{wy} and f_{co} : kgf/cm^2)
β : Coefficients for replacing volumetric ratio by transverse reinforcement ratio (pwc). For square columns with the configuration types 1 - 4, β = 2 (see Table 1).

2.2 Formulation of compressive strain capacity of confined concrete

Relations between measured values of ϵ_{cu} and the confining coefficient are shown in Figure 2. These ϵ_{cu} were measured at the position of the compressive reinforcements in plastic hinge zones of columns subjected to anti-symmetric bending moment and axial force at the upper limit of the deformation capacity, as shown in Figure 3. Also, this deformation capacity is defined as the point that shear force falls to 80% of the maximum shear force. Therefore, the ϵ_{cu} is defined as the compressive strain capacity of the confined concrete in plastic hinge zones.

Richart (1928) proposed the formula to estimate the compressive strain ϵ_{cc} at fcc of concrete, as shown in equation (3). Mander et al. (1988) used this formula to estimate the compressive strain ϵ_{cc} of reinforced concrete columns confined by transverse reinforcements.

$$\epsilon_{cc} = \epsilon_{co} [1 + 5 (S_c - 1)] \quad (3)$$

where, ϵ_{co}: Compressive strain at f_{cc} of plain concrete

In this paper, the compressive strain capacity of confined concrete (ϵ_{cu}) is formulated as equation (4) similar to the equation (3).

$$\epsilon_{cu} = \epsilon_{pu} [1 + 5 (S_c - 1)] \quad (4)$$

where, ϵ_{pu}: Compressive strain capacity of plain concrete

ϵ_{cu} *is calculated by the equation* (4) through the assumption ϵ_{pu}= 3, 3.5, 4 x10^{-3}, as shown in Figure 2. Then, by the comparison between experimental and calculated results, it is assumed that ϵ_{pu} = 4 x10^{-3}. Furthermore, these calculated results are scarcely affected by the axial force level and the amount of transverse reinforcements, as shown in Figure 4.

Table 1 The values of coefficient (K_c)

	Configuration Type 1	2	3	4	5
n_b (n_d)	2 (2)	4 (4)	6 (6)	10 (10)	6 (2)
Kc (:MPa)	3.32	7.76	9.23	10.4	5.43
Kc (:kgf/cm^2)	10.4	24.3	28.9	32.6	17.0

$3.71 \le h/D \le 4.52$ $0.35 \le p_w \le 1.07$ (%)
$0.22 \le N/(f_{co}BD) \le 0.60$ $408 \le \sigma_{wy} \le 1400$ (MPa)
$17.8 \le f_{co} \le 52.4$ (MPa) $3.0 \le p_w\sigma_{wy} \le 11.2$ (MPa)
The Number of Specimens : 15

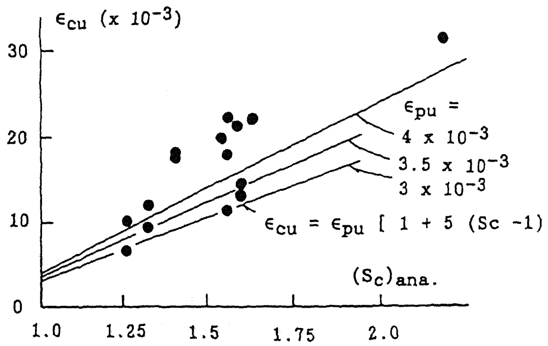
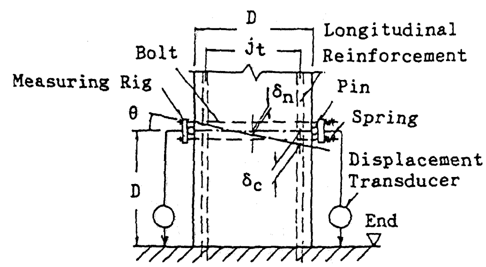
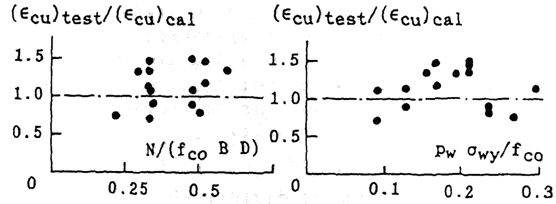


Figure 2 Compressive strain capacity of confined concrete



$\delta_c = \delta_n + \theta j_t/2$, $\epsilon_c = \delta_c / D$

Figure 3 Definition of deformation at column ends



(a)Effect of $N/(f_{co}BD)$ (b)Effect of $p_w\sigma_{wy}/f_{co}$
$p_w = a_w/(s B)$

Figure 4 (ϵ_{cu})test / (ϵ_{cu})cal

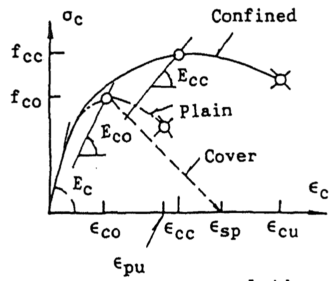


Figure 5 σ_c - ϵ_c relations of confined concrete

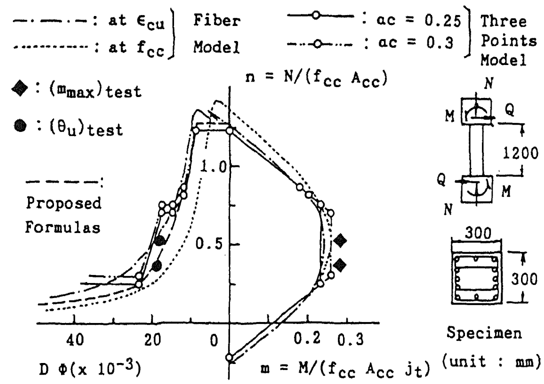


Figure 6 Analytical results of n - m and n - DΦ interactions

3 CURVATURE CAPACITY IN PLASTIC HINGE ZONES

3.1 Analysis by the fiber model

In this analysis, stress-strain relations of core concrete are calculated by Popovics's formula (1973) similar to Mander et al.'s analysis (1988). Although stress-strain relations of cover concrete are also calculated by Popovics's formula until its maximum strength, it is assumed that its stress linearly falls to the spalling strain (ϵ_{sp} = 3 ϵ_{co}) after its maximum strength (see Figure 5). Furthermore, it is assumed that stress-strain relations of reinforcements are elastic-plastic type.

Also, it is defined that the fiber strain of core concrete attains to ϵ_{cc} at the ultimate strength of columns and attains to ϵ_{cu} at the upper limit of the curvature capacity in plastic hinge zones.

Karatu et al.'s test specimens (1989) are analyzed through these assumptions. These specimens are reinforced concrete columns using high strength steel for transverse reinforcements, and subjected to anti-symmetric bending moment and constant axial force. One example of analytical results is shown in Figure 6. In this figure, bending moment, axial force and curvature are normalized, as follows.

$n = N/(f_{cc}\ A_{cc})$, $m = M/(f_{cc}\ A_{cc}\ j_t)$
M = Q h/2, N: Axial force, Q: Shear force
h : Clear height of columns
A_{cc}=b_c d_c: Sectional area of core concrete
j_t : Distance between longitudinal reinforcements
DΦ : Average curvature, Φ : Curvature
D : Depth of columns(= Measuring distance)

The measured values of the rotational angle (θ_u) and the ultimate strength are shown in Figure 6. These rotational angles were measured by the measuring device attached at hinge zones of the tested columns, as shown in Figure 3. The analytical results of n - DΦ relations at the upper limit of the curvature capacity and n - m relations at the ultimate strength adequately agree with the measured values.

3.2 Analysis by the three points model

This analysis aims to derive the formula for estimating the curvature capacity in plastic hinge zones. The section of core concrete is idealized into the three points model (see Yamada and Kawamura (1974)), as shown in Figure 7.1. Also, the followings are assumed for the simplicity of the analysis. (1) $\epsilon_{cu} \geq 3\ \epsilon_{co}$, (2) Stress-strain relations of core concrete and reinforcements are elastic-plastic type (see Figure 7.2), (3) $\epsilon_{sy} = \epsilon_{co}$ (ϵ_{sy} : Yield strength of longitudinal reinforcements), (4) An intermediate portion of longitudinal reinforcements in the section of columns is neglected.

n - m - $j_t\Phi$ relations in which each characteristic point corresponds to the strain state at the upper limit of the curvature capacity are analytically obtained, as shown in Figure 8.

n - m - DΦ relations at the above mentioned limit are calculated through this analytical method for the same tested columns in Figure 6. The calculated results in which the ratio of sectional area (α_c) equals 0.25 or 0.3 are shown. Both analytical results of n - DΦ relations adequately coincide with that by the fiber model, respectively. Also, the analytical results of n - m relations in the case of α_c = 0.25 comparatively agree with that by the fiber model. Therefore, it is assumed that α_c equals 0.25 in this study.

3.3 Formulation of curvature capacity

It is considered that the approximate curve of n -$j_t\Phi$ relations is obtained by connecting the characteristic points No.1 and No.4 (see Figures 6 and 8) analyzed by the three points model, on the basis of the observation of the analytical results by the three points model and the fiber model. Then, the

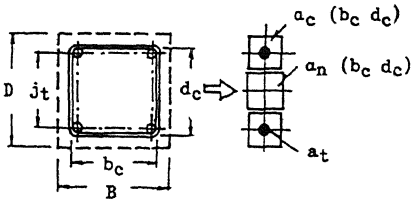
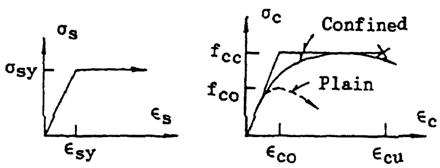
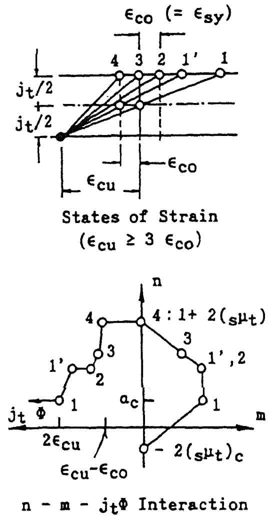


Figure 7.1 The three points model of the section of columns



(a) Steel (b) Concrete

Figure 7.2 Stress - Strain relations of steel and concrete



n, m and $j_t\Phi$ at compressive strain capacity in characteristic strain states

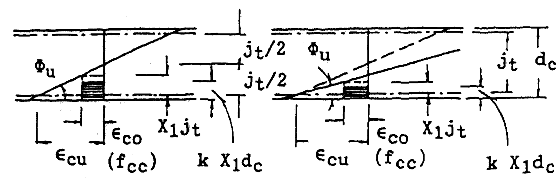
1	n	a
	m	$a/2 + ({}_s\mu_t)_c$
	$j_t\Phi$	$2\ \epsilon_{cu}$
1'	n	$a_c + a_n$
	m	$a_c/2 + ({}_s\mu_t)_c$
	$j_t\Phi$	$2\ (\epsilon_{cu} + \epsilon_{co})$
2	n	$a_c + a_n$
	m	$a_c/2 + ({}_s\mu_t)_c$
	$j_t\Phi$	$\epsilon_{cu} + \epsilon_{co}$
3	n	$a_c + a_n + ({}_s\mu_t)_c$
	m	$[a_c + ({}_s\mu_t)_c]/2$
	$j_t\Phi$	ϵ_{cu}
4	n	$1 + 2\ ({}_s\mu_t)_c$
	m	0
	$j_t\Phi$	$\epsilon_{cu} - \epsilon_{co}$

$({}_s\mu_t)_c = (a_t/A_{cc})(\sigma_{sy}/f_{cc}) = {}_s\mu_t/(S_c\ k_c)$

${}_s\mu_t = a_t\ \sigma_{sy}/(A_c\ f_{co})$

$k_c = A_{cc}/A_c$, $A_{cc} = b_c\ d_c$, $A_c = B\ D$

Figure 8 n - m - $j_t\Phi$ relations by the three points model at compressive strain capacity



(a) n = a_c (b) n < a_c

Figure 9 Concrete stress block

approximate curve is formulated as equation (5) shown in Figure 6. Furthermore, all longitudinal reinforcements are taken into account in equation (5).

$$j_t\ \Phi_u = \{2(1+{}_s\mu_{gc} - a_c)\epsilon_{cu}(e_{cu}-1)\} / \{n(e_{cu}+1)+(1+{}_s\mu_{gc})(e_{cu}-1)-2\ a_c\ e_{cu}\} \quad (5)$$

$$e_{cu} = \epsilon_{cu}/\epsilon_{co}\ , \quad {}_s\mu_{gc} = (a_g/A_{cc})(\sigma_{sy}/f_{cc})$$

where, a_g, σ_{sy} : Total sectional area and yield strength of longitudinal reinforcements

In the analysis by the three points model, the curvature capacity infinitely increases when the axial force level is smaller than a_c, as shown in Figure 6. Therefore, the equilibrium for the compressive stress block is considered for $n \leq a_c$, as shown in Figure 9. Also, it is assumed that the neutral axis corresponding to the compressive stress block coincides with that calculated by the three points model at $n = a_c$. Then, the curvature capacity for $n \leq a_c$ is obtained by equation (6), as shown in Figure 6.

$$j_t\ \Phi_u = 2\ a_c\ \epsilon_{cu}\ /\ n \quad (6)$$

4 DEFORMATION CAPACITY OF COLUMNS

The yield condition on n - m interaction of columns subjected to anti-symmetric bending moment and axial force is obtained on the basis of the three points model, as shown in Figure 10. The failure modes of such columns are classified into the following three modes. (1) Flexural-tensile mode, (2) Flexural-tensile-compressive mode, (3) Flexural-compressive mode. Also, these columns are idealized by the hinged model which consists of the plastic hinge zones at both ends and the rigid portion at middle, as shown in Figure 10. In this model, the rotational capacity (θ_u) equals $\Phi_u\ l_p$, where l_p : length of plastic hinge zones. Then, the deformation mechanisms of this model are derived on the basis of the plastic flow rule, as shown in Figure 10.

On the other hand, it is clarified that the rotational capacities (θ_u) of the plastic hinge zones nearly equal the deformation capacities (R_u) of tested columns regardless of the axial force level, as shown in Figures 11 (a) and (b) ($R_u = \delta_u/h$, δ_u : lateral deformation capacity). These rotational capacities were simultaneously measured at the upper limit of the deformation capacity by the same measuring devise which was used for measuring the compressive strain capacities (see Figure 3).

Therefore, the deformation mechanisms of tested columns are considered as the flexural-tensile mode below the balancing axial force and the flexural-compressive mode above that. Consequently, it is able to be assumed the following equation.

$$R_u = \theta_u = \Phi_u\ l_p \quad (7)$$

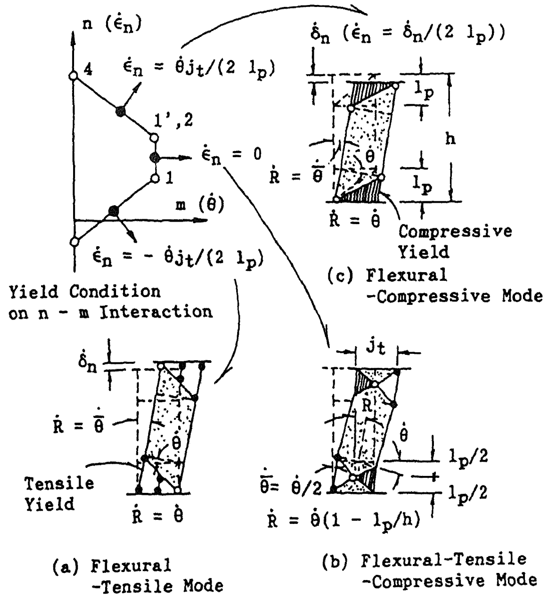


Figure 10 Deformation mechanisms of columns failed in flexture

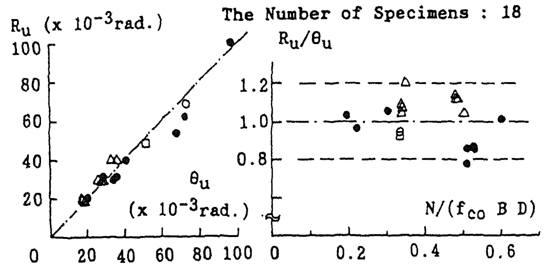


Figure 11 Relations between rotational angle and deflection angle

$3.0 \le h/D \le 6.0$ $\quad$ $0.27 \le p_w \le 1.75$ (%)
$0.08 \le N/(f_{co}BD) \le 0.60$ $\quad$ $278 \le \sigma_{wy} \le 1410$ (MPa)
$17.8 \le f_{co} \le 75.9$ (MPa) $\quad$ $1.0 \le p_w\sigma_{wy} \le 12.9$ (MPa)
The Number of Specimens : 64

	Normal Strength	High Strength
□	△	▲
⊞ ⊞ ▦ ◫	○	●

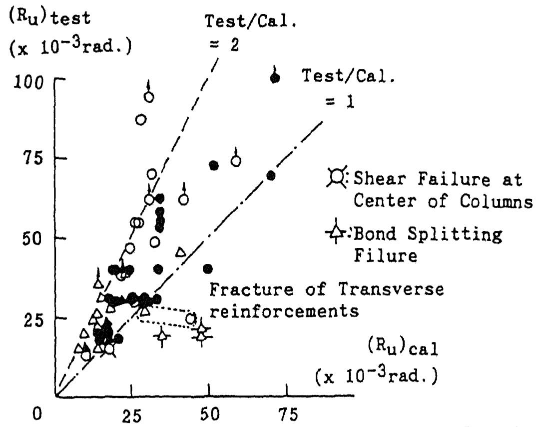


Figure 12 Comparison between calculated and experimental results

Furthermore, the length of plastic hinge zones is estimated by Yoshioka et al.'s formula (1979). That is

$$l_p = (h/D)\ (d/4) \tag{8}$$

where, $3 < h/D < 6$
d : effective depth of columns

Then, the proposed formulas for estimating the deformation capacity of columns failing in flexure are verified through the comparison between calculated and experimental results, as shown in Figure 12.

These experiments were carried out in Japan for 64 column specimens subjected to anti-symmetric bending moment and axial force. Also, transverse reinforcements of these specimens were made of normal strength steel(σ_{wy} = 278 - 521 MPa) and high strength steel(σ_{wy} = 787 - 1410 MPa) with the various types of the configuration. Furthermore, the values of main parameters are shown in Figure 12.

5 REQUIRED TRANSVERSE REINFORCEMENTS

Required transverse reinforcements in plastic hinge zones for ensuring the adequate deformation capacity can be obtained by the proposed formulas with a series of repeatedly calculating procedure.

Then, the following four parameters are investigated with regard to the required transverse reinforcements of square columns. The parameters are (1) the deformation capacity, (2) the configuration of transverse reinforcements, (3) the ratio of the spacing of transverse reinforcements to the depth of columns, (4) the compressive strength of concrete.

Relations between the required transverse reinforcements and these parameters are shown in Figures 13 (a)-(d). From these figures, it is able to obtain the required amount of transverse reinforcements.

Also, these calculated results are compared with the required transverse reinforcements by ACI and NZ code provisions in the above mentioned figures. It is shown that the required amount by NZ code corresponds to that obtained by the proposed formulas when the deformation capacity equals 20 x 10^{-3} rad. through 30 x 10^{-3} rad. under the conditions shown in Figure 13(a). However, it is considered that the required amount by NZ code is not conservative for high axial force level and poor confinement.

6 CONCLUSIONS

In this paper, the formulas for estimating the deformation capacity of reinforced concrete columns failing in flexure subjected to anti-symmetric bending moment and axial force are proposed. In these proposed formulas, the confining effect by the various types of transverse reinforcements can be taken into consideration. These formulas are based on the flexural theories and the plastic deformation mechanisms.

The deformation capacity estimated by these proposed formulas is conservative though good coincidence with experiments is observed. Also, the required transverse reinforcements in the plastic hinge zones for ensuring the adequate deformation capacity are clarified on the basis of the proposed formulas.

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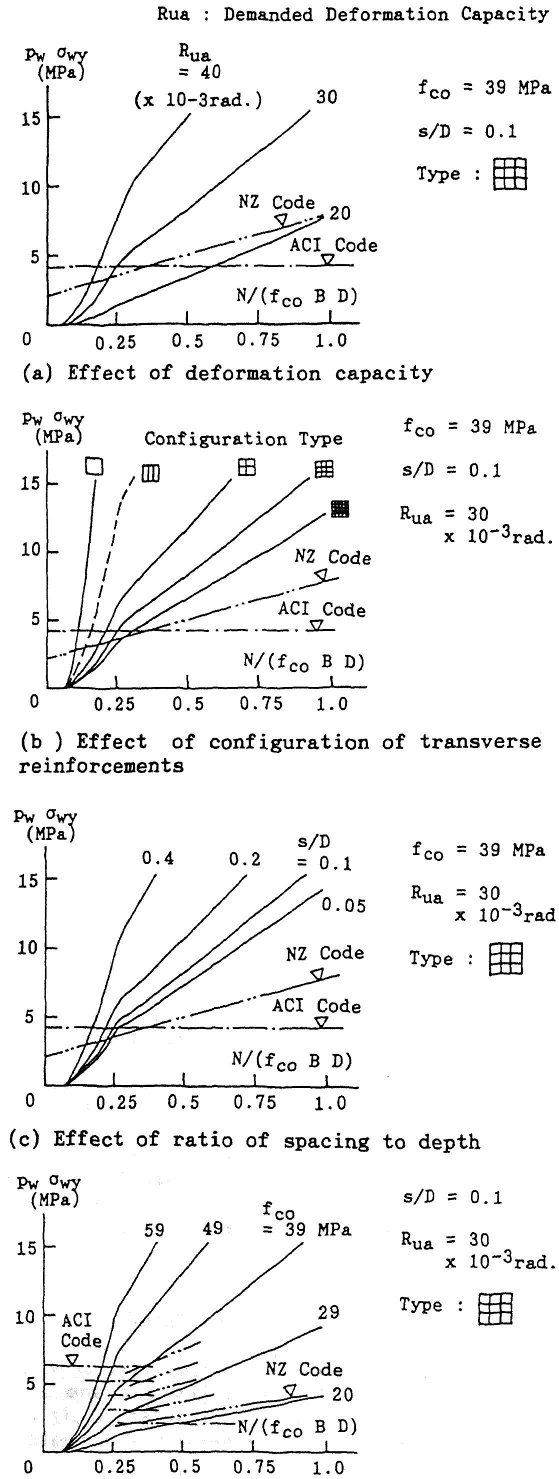

(a) Effect of deformation capacity

(b) Effect of configuration of transverse reinforcements

(c) Effect of ratio of spacing to depth

(d)Effect of compressive strength of concrete

Figure 13 Required transverse reinforcements