Earthquake Engineering, Tenth World Conference © 1992 Balkema, Rotterdam. ISBN 90 5410 060 5

Shear failure of R/C members after flexural yielding

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ABSTRACT: Shear failure of R/C members after flexural yielding is attributable to the reducing effective strength of concrete and the increasing inclination of truss action inside hinge regions. Consideration of these effects leads to a design procedure, which is accepted in the 1990 Japanese Design Guidelines for R/C buildings.

1. INTRODUCTION

Reinforced concrete beams and columns often fail in shear after flexural yielding. We should have a rational and reliable design procedure to prevent such failure until each constituent member reaches prescribed deformation capacity. Most approaches to this end, however, have been empirical and still failed to have good agreement with experimental results. The object of this paper is to propose a shear design procedure based on the truss - strut model. This procedure is accepted in the 1990 Japanese Design Guidelines for R/C buildings.

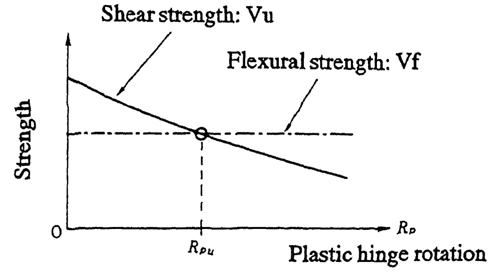


Fig. 1 Decrease of shear strength

2. ASSUMPTIONS

We may explain flexural shear failure using Fig. 1: inelastic rotation of the hinge region will decrease the shear strength, whereas it will not affect the flexural strength so much. At the crossing point R_{pu}, shear failure will occur. In this paper, we ascribe this reduction to:

(1) the reduction of aggregate interlocking in hinge regions due to widening of flexural shear cracks, and

(2) the reduction of the effective compressive strength of concrete in hinge regions due to densely intersecting large flexural shear cracks.

Let us consider a member with hinge regions at both ends. We assume that the length of hinge regions is 1.5 D (D: total depth of the member) as shown in Fig. 2 (a). We ignore any dead and live loads. Treatment of these loads is discussed by Ichinose (1992).

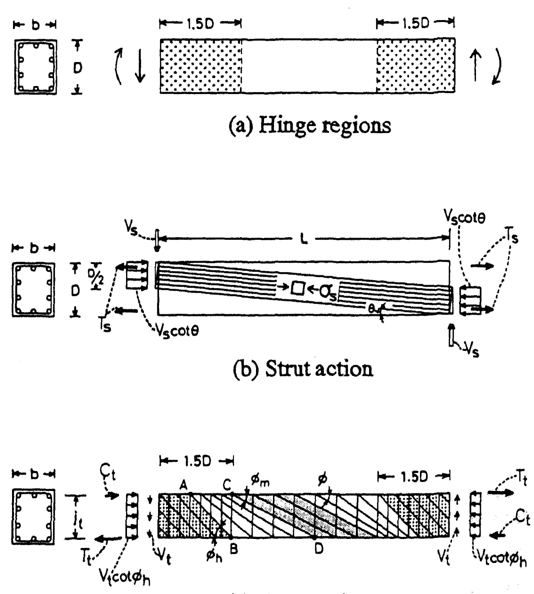


Fig. 2 Shear resisting actions of a member with hinge regions at both ends

2.1 *Inclination of truss and strut actions*

We assume that superimposing the truss and strut actions gives the shear strength of a member as was done by Shohara (1981) and Minami (1981). We call Fig. 2 (b) a "strut action," which does not require shear reinforcement. We call Fig. 2 (c) a "truss action," which requires shear reinforcement. The regions shaded by rough and fine dots in Fig. 2 (c) terminate inside and outside the hinge regions, respectively. We denote the angles of the truss actions in the roughly and finely dotted regions by ϕ_h and ϕ_m, respectively. Between these regions, we have the white region ABDC, where the inclination ϕ varies continuously between ϕ_h and ϕ_m. The compressive stress of concrete also varies in this region.

Note that the truss action does not correspond necessarily to crack patterns. Shear cracks in R/C members often occur very sparsely, but the stress in concrete may be continuous as in Figs. 2 (b) and (c).

Assumption 1: In the finely dotted region of Fig. 2 (c), we assume

$$\cot\phi_m \leq 2 \quad (\text{or } \phi_m \geq 26.5 \text{ deg}) \tag{1}$$

as was proposed by Thurlimann (1979). Truss action with $\cot\phi_m > 2$ would make shear cracks so wide as to prevent aggregate interlocking.

Assumption 2: In the roughly dotted regions of Fig. 2 (c), we assume that the upper limit of $\cot\phi_h$ may be between 2 and 1 depending on inelastic rotation of the hinge regions, Rp. We denote this limit by λ and assume that λ is given by Fig. 3. In short, we assume

$$\cot\phi_h \leq \lambda \tag{2}$$

In the 1990 Japanese Guidelines, ductile members are normally required to have the following deformation capacity including elastic deformation. These values are also plotted in Fig. 3 neglecting elastic deformation.

1. Beams connected to shear walls :	0.025 rad.
2. Other beams :	0.020 rad.
3. Columns :	0.015 rad.

As illustrated by Nielsen (1986), ϕ_h and ϕ_m cannot be larger than 45 deg. Then, ϕ_h of the ductile beam must always be 45 deg. (see Fig. 3 at Rp = 0.02), whereas ϕ_m can be between 26.5 and 45 deg.

Assumption 2 comes from the reduction of aggregate interlocking in the hinge region. Inelastic rotation widens flexural shear cracks in the hinge region. Consider the case of Rp = 0.02 rad. In many experiments, number of dominant flexural shear cracks is about 5 or less. Then, one of those cracks may widen up to $Rp/5$ = 0.004 rad, as shown in Fig. 4. If the distance from the tip of the crack to the centroid of the member is 500 mm, the crack width at the centroid will be about 0.004 rad x 500 mm = 2 mm. Aggregate interlocking will almost disappear when the crack is so wide (Walraven 1981). Then, the direction of the compressive stress in the concrete must coincide with that of the flexural shear crack, about 45 deg. This means that the inclination of truss action in the hinge region must be about 45 deg at Rp = 0.02 rad. Thus, we have $\lambda = 1$ for $Rp \geq 0.02$ rad as in Fig. 3. For $0 < Rp < 0.02$ rad, we interpolate between $\lambda = 2$ and 1.

We assume that the inclination of strut action θ may not be affected by inelastic deformation. This is because the strut action is not related to straining of shear

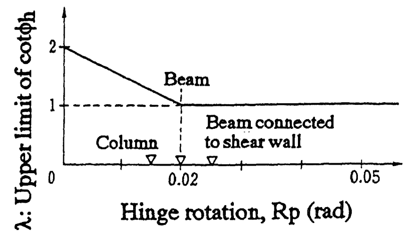


Fig. 3 Upper limit of $\cot\phi_m$ in hinge region.

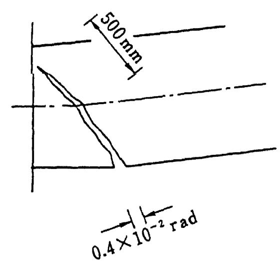


Fig. 4 Flexural shear crack in hinge region.

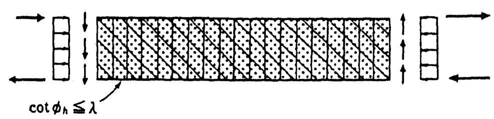


Fig. 5 Truss action (uniform shear reinforcement)

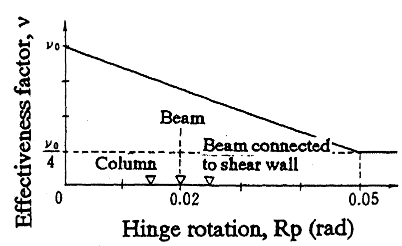


Fig. 6 Effectiveness factor of concrete in hinge region.

reinforcement.

In the case of a member with uniform shear reinforcement, equilibrium and the lower bound theorem requires $\cot\phi_m = \cot\phi_h$, which yields an uniform truss model shown in Fig. 5. This is nothing but a special case of Fig. 2 (c). Refer to Ichinose (1992) for detail.

2.2 *Effective strength of concrete*

We ignore tensile strength of concrete. We assume that the compressive stresses of concrete induced by the truss and strut actions must be less than the effective strength $\nu\sigma_B$, where σ_B is the uniaxial compressive strength of concrete and ν is the effectiveness factor assumed as follows:

Assumption 1: Outside hinge regions, we assume the following effectiveness factor proposed by Nielsen (1984).

$$\nu = \nu_O = 0.7 - \sigma_B/200 \quad (\sigma_B \text{ in MPa}) \qquad (3)$$

Assumption 2: Inside hinge regions, we assume Fig. 6 for the effectiveness factor. This allows for the decrease of effective strength due to densely intersecting large flexural shear cracks in hinge regions. A loading excursion (monotonic or cyclic) may also affect the effective strength, but we ignore this effect for simplicity.

2.3 *Strength of flexural reinforcement*

We assume that flexural reinforcement has an infinitely large yield strength. This assumption first means that the truss action in Fig. 2 (c) always fails owing to the compressive crushing of the web concrete and/or the yielding of the web reinforcement, not because of the yielding of the top and bottom flexural reinforcement.

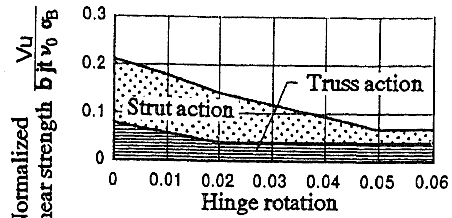
This assumption secondly means that the top and bottom flexural reinforcement can carry all the stresses of the top and bottom stringers of the truss action, which stresses in reality will be carried partly by concrete. Thus we may assume that the distance between the top and bottom stringers of the truss action, j_t, is equal to that between the top and bottom flexural reinforcement.

This assumption thirdly means that the tensile forces of the strut action in Fig. 2 (b) can be so large that the depth of the action will be $D/2$, making its shear force the largest (Nielsen 1984). This is a dangerous assumption because in reality the depth of strut action is affected by the amount of flexural reinforcement and axial force. However, the shear strengths observed in experiments agree with or exceed the calculated shear strengths with this assumption, as shown in Ichinose (1992), as long as we assume the effective strength of concrete as Eq. 3. We may thus justify this assumption.

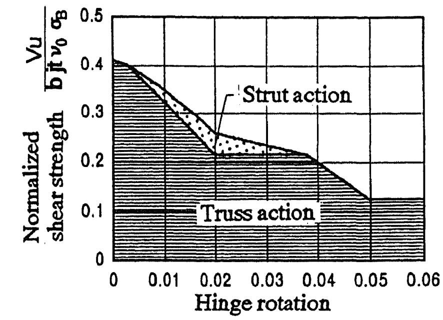
3. ANALYSIS, VERIFICATION & DESIGN CHART

The preceding assumptions lead us to the relationship between the inelastic rotation of the hinge regions, Rp, and the normalized shear strength as exemplified in Fig. 7. Strut action dominates in members with shorter length and light shear reinforcement as shown in Fig. 7 (a). Truss action dominates in members with longer length or heavy shear reinforcement as in Fig. 7 (b). Note that uniform shear reinforcement is assumed in Figs. 7 (a) and (b): if the shear reinforcement is heavier inside hinge regions than outside them, the contribution of the truss action decreases more smoothly and the contribution of the strut action becomes smaller.

An envelope curve of a observed shear force vs. deflection angle (V-R) relationship is shown in Fig. 16 with a broken line. We define the crossing point of the envelope curve of the observed V-R and 0.8 V_{exp} (V_{exp}: observed strength) the "observed deformation capacity", R_{exp}.



(a) Member with shorter length and light shear reinforcement



(b) Member with longer length or heavy shear reinforcement

Fig. 7 Examples of decreasing shear strength

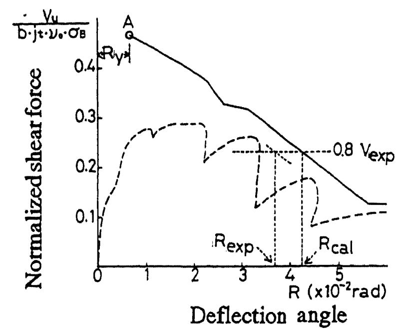


Fig. 8 Definition of deformation capacity

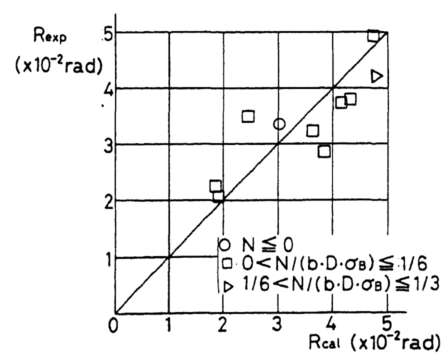


Fig. 9 Verification of deformation capacity

The calculated Vu-Rp relationship is shown in Fig. 8 with a solid line where the starting point A is moved rightward by the yielding deflection angle Ry, because R consists of elastic deformation Ry and inelastic hinge rotation Rp. We define the crossing point of the calculated V-R and 0.8 $Vexp$ the "calculated deformation capacity", $Rcal$.

Yoshioka (1983) collected experimental data of R/C members with a variety of parameters. Among these data, those failed in shear after flexural yielding are selected and compared with the calculation in Fig. 9. Three kinds of symbols indicate the levels of average axial stress. The calculated and the observed deformation capacity shows good agreement.

The preceding assumptions also lead us to the relationship between the normalized shear reinforcement and the normalized shear strength for given inelastic rotation of the hinge regions, as exemplified in Fig. 10. In this figure, n is defined as follows:

$$n = p_{wh}/p_{wm} \quad \text{when } 1 \leq p_{wh}/p_{wm} \leq 2/\lambda \qquad (4)$$

$$n = 2/\lambda \quad \text{when } p_{wh}/p_{wm} > 2/\lambda \qquad (5)$$

where p_{wh} and p_{wm} are the shear reinforcement ratios inside and outside the hinge regions. If n=1 (uniform shear reinforcement), the point D coincides with the point B. If Rp = 0 (non-ductile member), Fig. 10 reduces to Fig. 11.

Designers must calculate the necessary shear reinforcement for a given shear force and required deformation capacity. They can obtain it as follows:

a) Calculate ν and λ for prescribed Rp.

b) Assume the ratio $n = p_{wh}/p_{wm}$ within the range $1 \leq n \leq 2/\lambda$.

c) Calculate $V_u/(b.j_t.\nu\sigma_B)$, the vertical axis of Fig. 10.

d) Find $p_{wm}\sigma_{wy}$, the horizontal axis of Fig. 10.

Thus, Fig. 10 can be the design chart. The lines DB and BC are concave upwards and can be replaced conservatively by straight lines.

4. CONCLUSIONS

(1) We may attribute flexural shear failure to the reduction of aggregate interlocking and that of effective compressive strength of concrete in hinge regions.

(2) Based on the truss - strut model in Fig. 2, we may calculate imaginary shear strength of a ductile member as a function of plastic hinge rotation as shown in Fig. 7.

(3) We may calculate necessary shear reinforcement from Fig. 10 for a given normalized shear stress.

ACKNOWLEDGEMENTS

The original version of this work was submitted to a private group chaired by Prof. H. Aoyama of Tokyo Univ. (Aoyama 1986). Afterwards, this version was revised into the current form and submitted to the Working Group on Shear of Architectural Institute of Japan chaired by Prof. F. Watanabe of Kyoto Univ. The author thanks the discussion by both the groups.

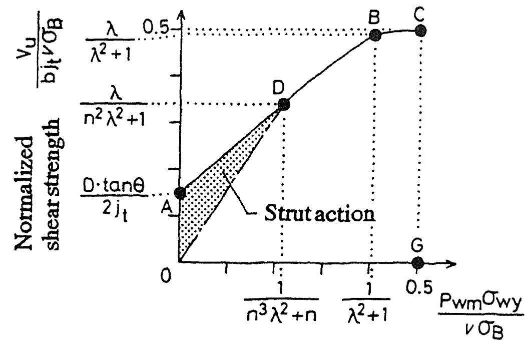


Fig. 10 Shear strength of member

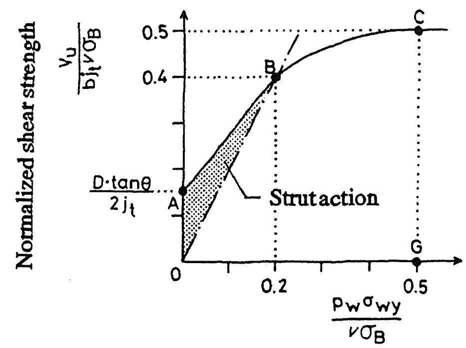


Fig. 11 Shear strength of non-ductile member

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