A study on the ultimate strength and ductility of circular reinforced concrete columns with multi-directional diagonal reinforcements

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Abstract: Since columns on lower stories of high-rise reinforced concrete buildings are subjected to heavy axial and lateral load, they will probably make brittle failure when a severe earthquake occurs. To provide such columns with superior ductility, a new circular column reinforcing method whereby diagonal reinforcements are arranged in multiple directions is proposed in the present paper. A column whose reinforcements are installed in accordance with this system is referred to as MXRC. The experimental and analytical studies on the strength and ductility of MXRC were conducted by comparing it with columns of a conventional structure; that is, reinforced concrete column (RC) and steel reinforced concrete column (SRC). The results indicate that MXRC not only has higher ultimate strength and ductility than RC but also is nearly equal in these two properties to SRC.

1. INTRODUCTION

Columns on lower stories of high-rise building must withstand heavy axial and lateral load when a severe earthquake takes place. Therefore, to impart high ductility to such columns, a method for reinforcing such columns needs to be developed.

A circular column reinforcing method whereby diagonal reinforcements are arranged in multiple directions is proposed in this paper. It consists of installing highly ductile spiral hoops bearing axial load and significantly strong diagonal reinforcements bearing lateral load. This method is rational and effective in shear design and can reduce the amount of shear reinforcements necessary for circular columns.

In accordance with this method proposed, diagonal reinforcements bent at a certain angle to the axial direction are installed on two circumferences in multiple directions, as shown in Photo 1 and Fig. 1. In
<table>
<thead>
<tr>
<th>Cross Section</th>
<th>RC</th>
<th>MXRC</th>
<th>SRC</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \eta = 1 )</td>
<td>No. 1</td>
<td>No. 2</td>
<td>No. 3</td>
</tr>
<tr>
<td>( \eta = 2 )</td>
<td>No. 4</td>
<td>No. 5</td>
<td>No. 6</td>
</tr>
</tbody>
</table>

\( \eta \) : Shear Span Ratio.  
RC : Reinforced Concrete.  
MXRC : RC with Multi-Directional Diagonal Reinforcements.  
STC : Steel-Reinforced Concrete.

Fig. 2. Test Program

Fig. 3. Test Specimens of MXRC

In addition, spiral hoops are arranged at certain spacings so that the hoops enclose the diagonal reinforcements already installed. The group of diagonal reinforcements and spiral hoops and reinforced concrete columns composed of the group are referred to as the MX reinforcement core and MXRC, respectively.

The strength and ductility of MXRC were quantitatively investigated by comparing it with columns of a conventional structure comprising RC and SRC and these two properties of the three types of columns were theoretically evaluated. Moreover, the experimental results obtained were compared with calculated values.

2. TEST PROGRAM AND SPECIMENS

To investigate the strength and ductility of MXRC, a test program was prepared by employing RC, MXRC, and SRC as specimens and the shear span ratio \( \eta \) as the parameter as shown in Fig. 2.

The detailed arrangement, dimensions, and middle part and bottom cross sections of MXRC are shown in Fig. 3. The diameter of the specimens tested was \( D = 300 \text{ mm} \) and their height was \( h = 600 \text{ mm} \) and \( h = 1200 \text{ mm} \). As a result, the shear span ratio \( \eta \) \((h/2D)\) was 1.0 and 2.0. The horizontal reinforcements (deformed bars) were placed at 30 mm spacings. Their yield strength was \( f_y = 5090 \text{ kgf/cm}^2 \). The deformed bars used as vertical reinforcements and MX reinforcements were eight and sixteen in number, respectively.

The cross section of RC and SRC is shown in Fig. 2 for the purpose of comparison with the MXRC cross section. The RC, MXRC, and SRC specimens were designed so that their compression strength was the same; that is, the MX reinforcements in the RC specimen were replaced with reinforcements parallel to the vertical axis and the MX reinforcements in the SRC specimen were replaced with steel frames having the same cross-sectional area as the MX reinforcement.

The failure mode test specimens whose shear span ratio was \( \eta = 1.0 \) were assumed to make shear failure and those whose shear span ratio was \( \eta = 2.0 \) were assumed to make flexural failure. The axial force applied to the specimens whose shear span ratio was \( \eta = 1.0 \) was \( N = 75.5 \text{ tf} \) and that applied to the specimens whose shear span ratio was \( \eta = 2.0 \) was \( N = 65.5 \text{ tf} \).

Six test specimens were tested in the present study. The specifications of the test specimens used and the mechanical properties of the reinforcements, steel, and concrete used are shown in Table 1.

3. EXPERIMENTAL RESULTS

3.1 Failure modes

The crack patterns and failure modes of all the test specimens are shown in Photo 2.

[Shear span ratio \( \eta = 1.0 \)]: Flexural cracks, diagonal cracks, and crush of covering concrete at the top and bottom of the column occurred in that order in all the specimens tested before the maximum strength was reached. In addition, the horizontal reinforcements (spiral hoops) yielded under tensile stress. After the maximum strength had been reached, covering concrete peeled off and the spiral hoop was exposed. Judging from the experimental behavior mentioned above, the failure mode was identified as shear failure. Since the spiral hoop did not break in the test, it is considered to have confined the core concrete portion of the column.

[Shear span ratio \( \eta = 2.0 \)]: Flexural cracks, crush of covering concrete at the top and bottom of the column occurred in that order in all the specimens tested before the maximum strength was reached. In addition, the horizontal reinforcements (spiral hoops) yielded under tensile stress. After the maximum strength had been reached, covering concrete peeled off and the spiral hoop was exposed. Judging from the experimental behavior mentioned above, the failure mode was identified as shear failure. Since the spiral hoop did not break in the test, it is considered to have confined the core concrete portion of the column.

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### Table 1  Mechanical Properties of Reinforcements, Steel, and Concrete.

<table>
<thead>
<tr>
<th>Specimens</th>
<th>Longitudinal Bars</th>
<th>Multi-Directional Diagonal Reinforcements</th>
<th>( A_t )</th>
<th>( \sigma_y )</th>
<th>( #A )</th>
<th>( #B )</th>
<th>( #C )</th>
<th>( f_c )</th>
<th>( D )</th>
<th>( h )</th>
<th>( \eta )</th>
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<tr>
<td>No. 1 R C</td>
<td>8-D13 4150 16-D13</td>
<td>-----</td>
<td>4150</td>
<td>329</td>
<td>300</td>
<td>600</td>
<td>1.0</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>No. 2 MXRC</td>
<td>8-D13 4150 16-D13</td>
<td>-----</td>
<td>4150</td>
<td>329</td>
<td>300</td>
<td>600</td>
<td>1.0</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>No. 3 SRC</td>
<td>8-D13 4150 16-D13</td>
<td>200x5</td>
<td>2220</td>
<td>329</td>
<td>300</td>
<td>600</td>
<td>1.0</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>No. 4 R C</td>
<td>8-D13 4223 16-D13</td>
<td>-----</td>
<td>4423</td>
<td>278</td>
<td>300</td>
<td>1200</td>
<td>2.0</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>No. 5 MXRC</td>
<td>8-D13 4223 16-D13</td>
<td>-----</td>
<td>4423</td>
<td>269</td>
<td>300</td>
<td>1200</td>
<td>2.0</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>No. 6 SRC</td>
<td>8-D13 4223 16-D13</td>
<td>200x5</td>
<td>3397</td>
<td>258</td>
<td>300</td>
<td>1200</td>
<td>2.0</td>
<td></td>
<td></td>
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<td></td>
</tr>
</tbody>
</table>

Note: \( \eta \) : Shear Span Ratio, \((=h/(2\cdot D))\)
- \( A_t \) : Number and Diameter of Longitudinal Bars.
- \( \sigma_y \) : Yield Strength of Longitudinal Bars.
- \#A : Paralel Reinforcements.
- \#B : Multi-Directional Diagonal Reinforcements.
- \#C : Section of Steel.

\( f_c \) : Compressive Stress of Concrete,
\( D \) : Diameter of Column Section,
\( h \) : Length of Column.

**R C** : Reinforced Concrete,
**MXRC** : R/C with Multi-Directional Diagonal Reinforcements,
**SRC** : Steel-Reinforced Concrete.

#### Photo 2. Crack Patterns and Failure Modes.

(a) \( \eta = 1 \) \( [h=600\text{mm}] \)
(b) \( \eta = 2 \) \( [h=1200\text{mm}] \)

and bottom of the column, and diagonal cracks occurred in that order in all the specimens tested before the maximum strength was reached. Moreover, the flanges of steel and vertical reinforcements yielded under compression and tensile stress.

After the maximum strength had been reached, the spiral hoop in the plastic hinge zone yielded under tensile stress. Judging from the experimental behavior mentioned above, the failure mode was identified as flexural failure.

Covering concrete of RC peeled off from its whole surface while it peeled off only in the plastic hinge zone of the top and bottom of MXRC and SRC. Shortly prior to completion of the test, the vertical reinforcement buckled and the spiral hoop broke in the plastic hinge zone. However, this behavior did not result in a rapid reduction in strength.
3.2 Load-displacement curve

The envelope curve, i.e., the relationship between the load $Q$ and the joint translation angle $R$ is shown in Fig. 4 and the experimentally obtained maximum strength $\exp Q_{\text{max}}$ of the specimens in Table 2. [Shear span ratio $\eta = 1.0$]

The maximum strength and joint translation angle were $\exp Q_{\text{max}} = 32.6$ tf and $R = 1.5 \times 10^{-2}$ rad., respectively, for RC; $47.5$ tf and $3.1 \times 10^{-2}$ rad. for MXRC; and $37.2$ tf and $7.5 \times 10^{-2}$ rad. for SRC. When the maximum strength was reached, the joint translation angle of MXRC was significantly larger than that of RC and smaller than that of SRC.

The strength of MXRC was observed being reduced, but the load-displacement curve indicated that this behavior of MXRC was stable and that it has excellent ductility. [Shear span ratio $\eta = 2.0$]

The maximum strength and joint translation angle were $\exp Q_{\text{max}} = 27.2$ tf and $R = 2.0 \times 10^{-2}$ rad., respectively, for RC; $27.1$ tf and $3.0 \times 10^{-2}$ rad. for MXRC; and $25.2$ tf and $0.9 \times 10^{-2}$ rad. for SRC. When the maximum strength was reached, the joint translation angle of MXRC was significantly larger than that of both SRC and RC. SRC exhibited the smallest joint translation angle of all three types of columns.

After the maximum strength had been reached, the strength of RC was sharply reduced because of bond-splitting failure along the vertical reinforcement. On the other hand, the strength of SRC was not reduced. The strength of MXRC was observed being reduced, but the load-displacement curve indicated that this behavior of MXRC was stable and that it has excellent ductility.

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**Table 2** Experimental Results and Calculated Theoretical Values.

<table>
<thead>
<tr>
<th></th>
<th>$\exp Q_{\text{max}}$</th>
<th>the $Q_{\text{max}}$</th>
<th>$R_{\text{U}}$</th>
<th>the $R_{\text{U}}$</th>
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<tr>
<td>No. 1</td>
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<td>0.93</td>
<td>30</td>
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<tr>
<td>No. 2</td>
<td>47.5</td>
<td>44.0</td>
<td>1.08</td>
<td>77</td>
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<tr>
<td>No. 3</td>
<td>37.2</td>
<td>39.5</td>
<td>0.94</td>
<td>100</td>
</tr>
<tr>
<td>No. 4</td>
<td>27.2</td>
<td>24.4</td>
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<td>50</td>
</tr>
<tr>
<td>No. 5</td>
<td>27.1</td>
<td>23.0</td>
<td>1.18</td>
<td>120</td>
</tr>
<tr>
<td>No. 6</td>
<td>25.2</td>
<td>22.3</td>
<td>1.13</td>
<td>120</td>
</tr>
</tbody>
</table>

Note: $\exp Q_{\text{max}}$: Maximum Shear Strength of Experimental Results.
the $Q_{\text{max}}$: Maximum Shear Strength of Calculated Theoretical Value.
$R_{\text{U}}$: Critical joint translation angle of Experimental Results.
the $R_{\text{U}}$: Critical joint translation angle of Calculated Theoretical Value.

$(1) = \exp Q_{\text{max}} / \text{the } Q_{\text{max}}$

$(2) = \exp R_{\text{U}} / \text{the } R_{\text{U}}$
MXRC and SRC indicated that this behavior of MXRC and SRC was stable and that they have excellent ductility.

4. ANALYTICAL RESULTS

4.1 Ultimate strength in shear and flexural modes

Fig. 5 shows a comparison of experimental and analytical results obtained for the ultimate shear strength of the three column types. The experimental results are indicated by a black circle and the analytical results by a solid line. The calculated ultimate strength of the MX reinforcement and steel frame is shown in the shaded block. The experimental and analytical data are shown in Table 2.

The ultimate shear strength of the reinforced concrete columns whose shear span ratio \( \eta = 1.0 \) was calculated by the cumulative strength theory based on plasticity and that of the reinforced concrete columns whose shear span ratio was \( \eta = 2.0 \) was calculated by moment-curvature analysis. In both cases, the ultimate shear strength of MXRC and SRC was calculated by adding the strength of the MX reinforcement and steel frame to that of the reinforced concrete section as shown by Eqs. (1) and (2) below.

\[
\text{MXRC}_{U} = \text{RC}_{U} + \text{MX}_{U} \quad \text{and} \quad \text{SRC}_{U} = \text{RC}_{U} + \text{S}_{U} \quad (1)
\]

In both cases of the shear span ratio \( \eta = 1.0 \) and \( \eta = 2.0 \), the theoretically calculated values for the three types of columns well agreed with experimentally obtained values. When shear failure occurred in the three types of columns (\( \eta = 1.0 \)), the ultimate shear strength of MXRC was larger than that of RC and SRC. On the other hand, when the failure mode was flexural failure (\( \eta = 2.0 \)), the ultimate shear strength of RC was slightly larger than that of MXRC and SRC. This is because differences in strength between the three types of columns depend on the arrangement of MX reinforcements and steel frames on the cross-section of the column. The ultimate shear strength of MXRC that made shear failure was nearly equal to the flexural strength calculated by moment-curvature analysis and that of MXRC that made flexural failure was equal to the flexural strength calculated by moment-curvature analysis.

4.2 Ductility in shear and flexural modes

To estimate the ductility of the three types of columns, the critical joint translation angle \( R_U \) was defined as the joint translation angle at which 80% of the maximum strength \( Q_{\text{max}} \) can be obtained, as shown in Fig. 6. The relationship between the critical joint translation angle experimentally obtained and that obtained analytically is shown for each column type in Fig. 5. A comparison of the experimentally obtained critical joint translation angle \( \exp R_U \) and theoretically obtained critical joint angle \( \text{the} R_U \) of each column is shown in Table 2. Eq. (3) can be obtained by statistically treating the \( \exp R_U \) value of the column that made shear failure.

\[
\text{the} R_U = v \cdot R_U \quad (3)
\]

\[
v = (0.87 + 0.57) \cdot (0.08p_w + 0.24) \cdot (-2.0 + 1.7) \cdot (0.19 + 0.16) \quad (4)
\]

\[
R_U = 7.16 \cdot 10 \text{ rad.} \quad (5)
\]

where \( \beta \) : Diagonal reinforcement ratio (= number of MX reinforcements / total number of longitudinal reinforcements)

\( p_w \) : Horizontal reinforcement ratio (= spiral hoop ratio)

\( \text{: Experimental Value,} \)

\( \cdots \text{: Calculated Theoretical Value,} \)

Fig. 5. Correlation between Ultimate Strength (Q) and Critical Joint Translation Angle (R_U), and RC, MXRC, SRC
\( n \) : Axial force ratio \\
\( P_0 \) : Steel ratio \\
The \( R_{UJ} \) value of each column which made flexural failure was calculated by the moment-curvature analysis. The critical curvature \( \phi_{UJ} \) was defined as the curvature at the value of which 80% of the maximum flexural strength \( M_{\text{max}} \) can be obtained. The relationship between the critical curvature \( \phi_{UJ} \) and the critical joint translation angle \( R_{UJ} \) is defined using the plasticity length \( L_p \) and is given by the following equation.

\[
\theta_{R_{UJ}} = L_p \phi_{UJ} \tag{6}
\]

The experimentally and theoretically obtained critical joint translation angles \( \exp R_{UJ} \) and the \( R_{UJ} \) of MXRC that made shear failure were larger than those of RC and smaller than those of SRC. On the other hand, the theoretically obtained \( \theta_{R_{UJ}} \) of MXRC was almost equal to that of SRC and larger than that of RC. The experimentally and theoretically obtained critical translation angles \( \exp R_{UJ} \) and the \( R_{UJ} \) of MXRC that made flexural failure were larger than those of RC and equal to those of SRC. The results mentioned above have indicated that the ductility of MXRC is nearly equal to that of SRC in shear and flexural failure.

5. CONCLUSIONS

(1) The experimentally and theoretically obtained ultimate shear strength values of MXRC were larger than those of conventional RC and SRC in shear failure and the ultimate strength values of the three types of columns that made flexural failure were nearly equal to each other.

(2) The experimentally obtained \( \exp R_{UJ} \) critical joint translation angle \( R_{UJ} \) of MXRC was larger than that of conventional RC and smaller than that of conventional SRC in shear and flexural.

(3) The theoretically calculated ultimate shear strength of RC, MXRC, and SRC that made shear and flexural failure fell within the range of 93% to 118% of the experimentally obtained values. Therefore, it may be concluded that the ultimate shear strength values theoretically obtained by the cumulative strength theory and moment-curvature analysis in shear and flexural failure well agree with the experimental results.

(4) The theoretically obtained critical joint translation angle \( \theta_{R_{UJ}} \) of RC was nearly equal to its experimentally obtained critical joint angle \( \exp R_{UJ} \). The \( \theta_{R_{UJ}} \) value of MXRC is almost equal to its \( \exp R_{UJ} \) value in shear failure while the former is smaller than the latter in flexural failure. Moreover, the \( \theta_{R_{UJ}} \) value of SRC was smaller than its \( \exp R_{UJ} \) value in shear and flexural failure.

Fig. 6. Definition of the Critical Joint Translation angle, Ru

The MXRC column proposed exhibits higher strength and ductility than conventional RC (reinforced concrete column) and is nearly equal in these two properties to conventional SRC (steel-reinforced concrete column).

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