Simplified formula for estimating maximum strength of multistory framed shear walls and discriminant of their failure modes

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ABSTRACT: The simplified model and formula for estimating the maximum strength of the multistory framed shear walls are proposed, and the discriminant on their failure modes is also proposed. The validity of the simplified formula and the discriminant are ascertained by a supplementary experiment.

1. OBJECT

In the previous paper (Mochizuki and Onozato 1990) the authors proposed the macro model for estimating the maximum strength of the multistory framed shear walls (hereafter, referred to the shear walls). The analytical results using this model agree well with the observed results in experiment. However the analytical procedure needs many iterative calculations, and is not adequate as the practical design formula of the shear walls.

The objectives of this paper are firstly to propose the simplified formula without iterative calculation for the maximum strength of the shear walls and the discriminant of their failure modes, and secondly to ascertain the validity of the simplified formula and the discriminant by a supplementary experiment.

2. SIMPLIFIED MODEL

The simplified formula is derived from the simplified model of the shear walls. Fig.1 shows the simplified model for exhibiting the resisting mechanism at the maximum strength. This model consists of upper and lower beams having large sectional area to simulate the multistory shear wall, two columns with shear resisting capacity, compressive struts @ and © with same inclination angle of 45 deg., and vertical and horizontal reinforcing bars.

These members in the model are assumed to have the following properties.
1) Both beams are rigid, and do not fail.
2) Column under compression is under flexural yielding at the bottom end. Column under tension is under tensile yielding at the bottom end, and the shearing force at the bottom end is ignored.
3) Struts @ are under yielding, and their yield strength is taken as 0.63Gs. Struts © are ignored because the restraint force to the struts © disappears due to the extension of horizontal crack along the lower beam.
4) All vertical and horizontal reinforcing bars are under yielding.

In 3), the value of 0.63Gs is the effective compressive strength of concrete which was proposed by the authors (Mochizuki et al. 1990)

3. SIMPLIFIED FORMULA FOR MAXIMUM STRENGTH

The following equations hold between the external forces and assumed stresses acting on the simplified model shown in Fig.1.

\[ Q_{el} = Q_v + Q_c, \quad Q_w = Sc \cdot \xi \cdot L \]  \hspace{1cm} (1), (2)

The equilibrium of moment at the bottom end of the column under compression is expressed including \( \xi \) as follows:

4321
From consideration on the stress distribution in the neighborhood of the bottom end of the column under compression, in which the column is assumed as a cantilever column subjected to uniform load, the shearing force $Q_c$ is expressed as follows:

$$ Q_c = 2M_c - St $$

(4)

$M_c$ and $Q_c$ are approximated by the following Eqs. (5) and (6), respectively.

$$ M_c = N_y \cdot D / 2, \quad Q_c = \sqrt{2M_c - St} = \sqrt{N_y \cdot D - St} $$

(5), (6)

Substituting Eqs. (1), (2), (5) and (6) to Eq. (3), and then the following equation on $\xi$ is obtained.

$$ \frac{\xi^2}{2} \cdot \gamma \cdot \xi - \left( \frac{S_v}{2\gamma} - \frac{\eta + N_y \cdot D \cdot St - N / 2 \cdot N_y (D / 2 + 1)}{2\xi} \right) = 0 $$

(7)

where $\eta = h' \cdot r / L$. Solving the above equation, $\xi$ is expressed as follows;

$$ \xi = -\eta \pm \sqrt{\eta^2 + \frac{S_v}{2\gamma} - \frac{\eta + N_y \cdot D \cdot St - N / 2 \cdot N_y (D / 2 + 1)}{2\xi}} $$

(8)

Where, in the case of $\xi > 1.0$ the value of $\xi$ is regarded as 1.0. This is based on the fact that for putting $N_t = N_y$ the stress of the struts must be distributed on the large width corresponding to $\xi > 1.0$ to satisfy the equilibrium of moment, but if $N_t < N_y$ the value of $\xi$ does not exceed one.

The equilibrium of moment at the bottom end of the column under tension is expressed including $N_c$ as follows;

$$ M_c = N \cdot L / 2 - Q_{col} \cdot h' + N_c \cdot L $$

$$ + \gamma \cdot \xi \cdot \xi (1 - \xi / 2) \cdot L - S_v \cdot L^2 / 2 = 0 $$

(9)

From consideration on Eqs. (1), (2), (5) and (6) $N_c$ is expressed as follows:

$$ N_c = \eta + N_y \cdot D \cdot St + (\eta + \xi / 2 - 1) Q_w + S_v \cdot L / 2 + N / 2 - N_y \cdot D / 2 $$

(10)

Using $N_c$ in Eq. (10), $M_c$ is calculated from the yield strength formula of column, that is,

$$ M_c = M \cdot N_c $$

(11)

Then, from Eq. (4) using Eq. (11) the second approximate value of $Q_c$ is obtained as follows;

$$ Q_c = M \cdot N_c \cdot St $$

(12)

Finally, the maximum shear strength $Q_{col}$ of Eq. (1) is calculated as the summation of Eq. (2) and Eq. (12) without iterative calculation.

4. ANALYTICAL RESULTS

The analyses using the simplified formula were executed for the authors’ one hundred and fourteen specimens, the JCI’s thirty specimens and the other researchers’ sixty specimens. The JCI’s specimens are selected by the JCI’s committee for verification of the macro model of the shear walls.

Fig. 2 shows the relationship between $Q_{col}$ and $Q_{col}$ for all the specimens. Figs. 3 and 4 show the relationships between $Q_{col} / Q_{col}$ and the parameters $\sigma_s$ and $\rho_g$, respectively. Figs. 2-4 show that the simplified model and formula are valid.
5. DISCRIMINANT OF FAILURE MODES

The authors define the failure modes of the shear walls as follows:

1) Shear failure mode
   \[ \frac{N_t}{N_y} = 1.0, \quad R_b = 5.0 \times 10^{-9} \text{ rad.} \]

2) Flexural-shear failure mode
   \[ \frac{N_t}{N_y} = 1.0, \quad R_b \geq 25.0 \times 10^{-9} \text{ rad.} \] \hspace{1cm} (13)

3) Flexural failure mode
   \[ \frac{N_t}{N_y} = 1.0, \quad R_b \geq 10.0 \times 10^{-9} \text{ rad.} \]

Fig. 5 shows the typical patterns of the failure modes. The above definitions are based on the following facts: the shear failure mode occurs when the column under tension is not under tensile yielding and its behavior is brittle, and the flexural-shear or the flexural failure mode occurs when the column under tension is under tensile yielding and their behaviors are both ductile. In the simplified model of Fig. 1 the column under tension is under tensile yielding when the value of \( \xi \) in Eq. (8) is smaller than 1.0. But the tensile yielding of the column does not necessarily occur in the case that the value of \( \xi \) is smaller than 1.0 when the flexural yield strength of the column is small.

\[ R_b (\times 10^{-9} \text{ rad.}) \]

\[ \begin{array}{cccccccccc}
0 & 0.2 & 0.4 & 0.6 & 0.8 & 1.0 & (1.2) & (1.4) & (1.6) & (1.8) \ \\
0 & 5 & 10 & 15 & 20 & 25 & 30 & & & \\
\end{array} \]

Fig. 6 Relationship between \( R_b \) and \( \xi \)

Fig. 6 shows the relationship between the values of \( R_b \) and \( \xi \). The number of specimens in the figure are fifty six, of which the Q-R relationships are mentioned in detail, among two hundred and four specimens used in the analysis.

From consideration of the definition on the failure modes and Fig. 6, the discriminant of the failure modes is proposed as follows:

1) Shear failure mode
   \[ 0.8 < \xi \leq 1.0 \]

2) Flexural-shear failure mode
   \[ 0.4 < \xi \leq 0.8 \] \hspace{1cm} (14)

3) Flexural failure mode
   \[ \xi \leq 0.4 \]

6. VERIFICATION BY EXPERIMENT

Here, a supplementary experiment is shown for verification of the simplified formula and the discriminant. Fig. 7 shows the dimension, bar reinforcement, and loading method of the specimens. The specimens have upper and lower beams, which are sufficiently reinforced and stiffened to simulate the multistory shear wall, and two columns reinforced not to fall in shear. The specimens are subjected to alternately reversible horizontal forces acting on the upper beam by an actuator.

Fig. 7 Specimen
Table 1 Properties and experimental results of specimens

<table>
<thead>
<tr>
<th>Specimen</th>
<th>t (cm)</th>
<th>P_k (kgf/cm²)</th>
<th>sOy (kgf/cm²)</th>
<th>sOy (kgf/cm²)</th>
<th>Q_ω (tf)</th>
<th>Q_c (tf)</th>
<th>R_b (x 10^-3 rad.)</th>
<th>F</th>
<th>Failure mode</th>
</tr>
</thead>
<tbody>
<tr>
<td>90SW-01</td>
<td>3.1</td>
<td>4.62</td>
<td>3614</td>
<td>1.00</td>
<td>3050</td>
<td>320.9</td>
<td>36.2</td>
<td>41.4</td>
<td>0.88</td>
</tr>
<tr>
<td>90SW-05</td>
<td>3.1</td>
<td>2.31</td>
<td>3666</td>
<td>0.99</td>
<td>3050</td>
<td>293.2</td>
<td>20.3</td>
<td>19.1</td>
<td>1.07</td>
</tr>
<tr>
<td>90SW-04</td>
<td>3.3</td>
<td>2.31</td>
<td>3606</td>
<td>0.95</td>
<td>3050</td>
<td>293.2</td>
<td>19.5</td>
<td>19.2</td>
<td>0.99</td>
</tr>
<tr>
<td>90SW-05</td>
<td>3.4</td>
<td>1.47</td>
<td>3614</td>
<td>0.92</td>
<td>3050</td>
<td>335.9</td>
<td>26.9</td>
<td>27.7</td>
<td>0.97</td>
</tr>
<tr>
<td>90SW-06</td>
<td>3.4</td>
<td>1.47</td>
<td>3614</td>
<td>0.93</td>
<td>3050</td>
<td>335.9</td>
<td>26.2</td>
<td>26.7</td>
<td>0.95</td>
</tr>
</tbody>
</table>

t: Thickness of wall (measured)  
P_k: Gross longitudinal reinforcement ratio of column  
P_s: Shear reinforcement ratio of wall  
sOy: Yield strength of longitudinal bar of column  
sOy: Yield strength of reinforcing bar of wall  

Table 1 shows the properties of the specimens, the observed maximum strength, and the value of ξ. Figures 8 show the Q-R relationships for specimens 90SW-01, 90SW-06, and 90SW-04. These observed results in the experiment agree well with the calculated results by the simplified formula and the failure modes predicted by the discriminant.

7. CONCLUSIONS

The conclusions of this paper are summarized as follows:

1) The simplified formula for estimating the maximum strength of the shear walls is adequate.
2) The discriminant of the failure modes of the shear walls is also adequate.

8. REFERENCES


9. NOTATIONS

M: Moment acting on shear wall  
Mc: Flexural yielding strength of column under compression  
Mu: Flexural yield strength of column  
N: Vertical force acting on shear wall  
Nc: Axial force at bottom end of column under compression  
Nt: Axial force at bottom end of column under tension  
Ny: Tensile yield strength of column  
Q_w: Calculated maximum shear strength of shear wall  
Q_ω: Shearing force of struts @  
Q_c: Shearing force of column under compression  
Q_ω: Observed maximum shear strength of shear wall  
γ: Inflection point height ratio of bending moment  

R: Story angle of shear wall  
R_b: Maximum angle of shear wall defined in Fig.5  
S = 0.63 - O_y/2  
S = P_s.sO_y.t  
S = min(S_c, S_v)  
S = (S_c - S_h)  
O_y: Compressive strength of concrete  
ξ: Effective horizontal width of struts @

4324