Displacement based design of RC structures

J.P. Moehle

*University of California at Berkeley, Calif., USA*

ABSTRACT: The importance of structural displacement as a main determinant of structural and nonstructural damage during an earthquake is identified. Methods for estimating displacement amplitude and distribution in simple and complex structures are reviewed. Applications of displacement information to design of nonstructural elements, structural elements, and complete structures is reviewed. It is concluded that a design approach based on structural displacement is viable and effective.

1 INTRODUCTION

The tradition of designing structures to resist externally applied loads has led to earthquake resistant design approaches in which ductility demands are derived based on calculated force demand-capacity ratios. These approaches have focussed design attention away from the importance of structural deformation as a main determinant of damage in structures subjected to earthquakes. This paper develops some fundamental ideas of earthquake resistant design and evaluation using structural displacements directly. It is assumed throughout that design will be based on relatively simple analysis concepts and results. The discussion focuses on applications to reinforced concrete systems, but it is more generally applicable.

2 DISPLACEMENT AS A DESIGN PARAMETER

The idea of using structural displacement information directly in earthquake resistant design and evaluation is not new. Based on a limited analog study, Muto, et al. [1960] wrote that maximum inelastic displacements of SDOF (single-degree-of-freedom) systems are not significantly different from those of elastic systems having the same initial period and damping, and that this tendency "...may suggest employment of a structural design method based upon the maximum displacement." More recent numerical studies indicate that the equal displacement rule is valid only for specific period ranges and demand-capacity ratios [Shimazaki and Sozen, 1985; Qi and Moehle, 1991]. Techniques are proposed in these references for relating inelastic and elastic displacement demands as a function of the strength, period, and ground motion character.

A fundamentally important aspect of these studies is that displacement amplitude of SDOF systems can be estimated using relatively simple elastic models, thereby providing an avenue for design based directly on displacement information. Although the methods presented are generally applicable, in the present paper it will be assumed for clarity of presentation that the equal displacement rule provides an adequate measure of inelastic displacement amplitude. When using the rule for concrete structures, it is suggested that the displacement be read from a displacement spectrum using an initial period based on half the gross-section stiffness (to account for expected stiffness near yield) and damping in the range between two and five percent of the critical value.

3 DISPLACEMENT CONCEPTS FOR MULTI-STORY FRAMES

Numerical and experimental studies of planar frames of moderate height have indicated that displacement response is dominated by response in an apparent first mode [Sozen, 1981; Moehle, 1984]. Saiidi and Sozen [1979] demonstrated that this predominant component of the displacement response could be modeled using a SDOF oscillator having hysteretic properties similar to those of the constituent elements of the frame. This finding suggests that the global displacement maxima of multistory systems may be estimated using simplified response spectrum methods as discussed previously for equivalent SDOF systems. Numerous case studies confirm this view [Moehle, 1984; Qi and Moehle, 1991; Shimazaki and Sozen, 1985].

In a laterally-loaded multistory framed structure having column strengths exceeding the beam strength, the lateral distortion will vary over the height approximately as illustrated in Fig. 1b. For this shape, roof displacement may be estimated as being equal to approximately 1.25 times the equivalent SDOF displacement.

Design requirements will depend not only on total drift but on the drift profile. Elastic analyses may provide insight into drift distribution, but may result in non-conservative estimates due to concentrations of
inelastic action. Based on numerical studies, Qi and Moehle [1991] reported that for five-story frames the maximum interstory drift was approximately 1.5 times the roof value for frames designed to resist lateral load; in contrast, frames having uniform strength over height had maximum drift ratios as high as 1.8. For ten-story frames, the respective ratios were 2.1 and 2.8. Sozen [1981] reported measured drift ratios for earthquake simulation tests of small-scale nine- and ten-story frames ranging from 1.3 (for regular structures) to 2.0 (for irregular frames) times the roof drift. In the analyses and tests reported above, the columns were designed so the sum of column strengths at a joint exceeded the sum of beam strengths at all but the roof level. The composite results suggest that a reasonably conservative approach is to assume interstory drifts in regular structures may be as large as twice the roof drift.

Quantitative information on drift distribution in structures having columns relatively weaker than beams is limited [Schulz, 1990; Kelly, 1977]. Drift in such structures tends to concentrate in a single "weak" story (Fig. 1c). A conservative approach is to assume the maximum interstory displacement is equal to the maximum roof displacement, resulting in a relatively high interstory drift estimate for multistory structures. This approach provides a suitable deterrent for new designs having soft stories, but is unsatisfactory for existing construction where realistic solutions are needed for whatever conditions are encountered. Nonlinear static or dynamic analyses may be required to provide insight into drift profile in such structures, though the results should not be interpreted as being absolute.

4 DISPLACEMENT AND DESIGN

The relation between deformation and damage is a well-established concept in many fields of engineering. Some possible applications of displacement information in earthquake resistant design are discussed below. It is assumed that to some acceptable degree (a) the design earthquake ground motions are known, (b) the resulting global displacement to each motion can be calculated, and (c) the profile of displacement is estimable.

4.1 Nonstructural Elements and Non-Lateral Load Resisting Elements

Nonstructural damage, disrupted operations, and life-safety considerations make protection of nonstructural elements a high priority. Damage to these elements will depend on both drift and nonstructural details [Sakamoto, et al., 1984; Wang, 1987]. Attention need also be paid in design to structural elements that are not considered in design as part of the lateral load resisting system. Examples in the latter category include some floor systems and complete existing framing systems in seismically rehabilitated structures. These "vertical load carrying elements" are susceptible to damage due to earthquake induced lateral deformations, and must be checked to ensure they can continue to support vertical loads under the imposed lateral deformations.

When demand-capacity ratios are used to estimate ductility demands in a structure, the demand and the capacity refer to the lateral load resisting elements. The ratio provides no direct information regarding performance of the non-lateral load resisting elements. Analyses of expected behavior of nonstructural elements and structural elements not considered part of the lateral load resisting system should be carried out by directly considering the expected displacements.

4.2 Lateral Load Resisting Elements

The displacement based approach to design of reinforced concrete structural elements is based on the idealized deformation components of Fig. 2. Flexural deformation is modeled by elastic curvature over height and inelastic curvature concentrated in a plastic hinge of equivalent length \( l_p \) at the base. The ultimate displacement capacity with this model is

\[
\delta_u = \frac{\phi_u l_p^2 \phi_u}{3} + \left( \frac{\phi_u - \phi_u}{2} \right) \frac{l_p (l - l_p)}{2}
\]

In Eq. 1, \( \phi_u \) = yield curvature, \( l \) = pier height, and \( \phi_u \) = ultimate curvature. Satisfactory correlation with laboratory test results is obtained using \( l_p = 0.5h \) where \( h \) = the section depth.

The flexural curvature distribution of Fig. 2 can be simplified without significant loss of accuracy as illustrated in Fig. 3. Elastic curvatures outside the plastic hinge zone are ignored, and the curvature near the base is centered at the base (Fig. 3c). The ultimate displacement capacity with this model is

\[
\delta_u = \phi_u l_p l
\]

The approximation produces a satisfactory estimate of the ultimate displacement capacity for common ranges of section curvature ductility and member aspect ratio (Fig. 4).

Assuming plane sections remain plane, curvature in Eq. 2 can be expressed as

\[
\phi_u = \frac{e_u}{\beta h}
\]

in which \( e_u \) is limiting strain of a point on the cross section located a perpendicular distance \( \beta h \) from the neutral axis. Substituting Eq. 3 in Eq. 2, substituting 0.5h for \( l_p \), and dividing by member length \( l \) results in Eq. 4.

\[
\frac{\delta_u}{l} = \frac{1}{2\beta} e_u
\]

Equation 4 directly relates strain in a member to the effective drift ratio for the member.

As an application of Eq. 4, consider an unconfined reinforced concrete beam. Flexural response is
calculated according to ACI 318-89 [1989] except maximum usable concrete compression strain is taken equal to $0.004$. Concrete strength is $4000$ psi, and yield strength of elasto-plastic Grade 60 reinforcement is assumed equal to $75$ ksi. Figure 5 plots results in which drift ratio is defined by Eq. 4. (In a building, the beam drift ratio conservatively may be assumed to be equal to the interstory drift ratio.) The results indicate a maximum drift ratio capacity equal to approximately 0.01 for the permitted range of longitudinal reinforcement ratios. A design that controls interstory drift to this value is likely to result in satisfactory flexural response of the beam.

4.3 Global Response

Pounding between adjacent structures has been identified as a contributor to damage and collapse during earthquakes. In order to determine the likelihood of pounding, it is important that a direct analysis of expected displacements be made. It is not clear that widely used code equivalent lateral force procedures and minimum building separations provide adequate assurance against pounding damage.

Second order (P-delta) effects associated with large displacement response may result in displacement amplitudes exceeding those estimated by conventional analyses, and can therefore increase the potential for damage and collapse [Carr and Moss, 1980]. Considering a SDOF structure consisting of a mass atop a cantilever, it may be shown that the effective lateral load strength is

$$V = \frac{M_p}{l} - p \frac{\delta}{l}$$

in which $M_p$ = plastic moment strength and $P$ = the vertical load. The effective resistance is most significantly affected when the base shear strength $M_p/l$ is low or the lateral drift ratio $\delta/l$ is high. Some studies of inelastic seismic response of frames satisfying current code strength requirements indicate that P-delta effects will not significantly impact response if drift levels (ignoring P-delta effects) are below 0.01 [Carr and Moss, 1980].

In order to effective control pounding and P-delta effects, a design approach should focus on those aspects of design that directly control displacement response. Conventional force-based approaches to design may be unsuitable for this purpose.

5 DRIFT AND DUCTILITY APPROACHES TO DESIGN

The displacement approach to design as described previously utilizes expected structural displacements directly for evaluation of behavior of structural and nonstructural elements. A commonly used alternate and equally simple approach, the ductility based approach, uses displacement information indirectly with information on system strength to derive strength and ductility requirements. The fundamental principal of the approach is illustrated in Figure 6. If in a particular period range it is assumed that the elastic and inelastic displacements are equal, then ductility demand is equal to the ratio of the computed elastic force demands to the force capacity, that is, the demand capacity ratio. Where ductility demands are deemed to be excessive, it is common practice using this method to increase the strength of the structure so as to reduce the demand-capacity ratio, and hence, the ductility demand.

The displacement and ductility based approaches are effectively the same. However, the nature of the information used in the ductility based and displacement based approaches can influence decisions made in the design process. As an example, consider the existing structure illustrated in Fig. 7a. The columns possess a load-deformation behavior represented by curve "a" in Fig. 7b, with yield occurring at a unit displacement and failure occurring at displacement $\delta_0$ equal to 3 units, such that the available displacement ductility is $\mu_0 = 3$. The structure possesses an initial period $T_0$, with resulting displacement demand $\delta_0$ and acceleration demand $A_0$ read from the linear elastic response spectra of Fig. 7c. For this example, because the fundamental period exceeds the predominant ground motion period it can be expected that the elastic displacement spectrum adequately represents the inelastic displacement maxima. As indicated in Fig. 7b, the displacement demand $\delta_0$ exceeds the deformation capacity of the columns, so that redesign (retrofit) of the structure is in order. It will be assumed in the following discussion that the structure will be retrofit by addition of external structural elements rather than by direct modification of the columns.

In a ductility based approach the apparent goal in redesign of the structure (Fig 7) is to reduce the demand-capacity ratio (because doing so will reduce the nominal ductility demand). This goal may suggest that the successful redesign is one that (a) adds strength and (b) avoids significant stiffness increase because an increase in stiffness results in an increase in "demand." A plausible solution based on this specious goal is illustrated in Fig. 7b and 7c, where the structure strength is increased to $V_s$ and the stiffness is increased only slightly resulting in an initial period $T_s$. As illustrated in the figure, this solution may not adequately change the deformation demand on the columns and therefore might not provide the desired degree of safety. Specifically, for the period range of the subject structure, modifications that influence strength and not stiffness cannot be effective for protecting the critical columns.

A displacement based approach indicates directly that if the columns are to be protected the structure must be stiffened so as to reduce the displacement demand. A solution satisfying this more realistic goal is illustrated in Fig. 7, where the redesign results in a significant increase in stiffness (load-displacement relation "c"), with corresponding reductions in initial period $T_s$ and displacement demand $\delta_0$. 4299
The example illustrated in the preceding paragraphs is obviously simplified. Seldom can the structure strength and stiffness be modified independently as has been assumed. Furthermore, there are cases (e.g., short period structures situated on soft sites [Qi and Moehle, 1991]) where strength has a significant influence on deformation demand. Nonetheless, the example illustrates the advantage of operating directly with deformation quantities as opposed to demand-capacity ratios. The particular case considered here illustrates further that seismic strengthening frequently is more a matter of stiffening than it is of strengthening.

The ductility based approach is well suited to cases where inelastic response is distributed uniformly over height; in such cases the local demand-capacity ratios provide a reasonably accurate picture of ductility distribution and magnitude. Where inelastic response is not uniformly distributed the local demand-capacity ratios do not provide information on local ductility demands, and the displacement based approach may be preferred. As an example, consider a bridge pier founded on a flexible foundation (Fig. 8). The elastic rotational stiffnesses of the foundation and of the pier are assumed to be equal, and the pier strength is assumed to be one fourth of the elastic force demand. During an earthquake, the maximum displacement of the superstructure is determined by the combined flexibility of the pier and foundation. If the ductility based approach is used, the pier displacement ductility demand may erroneously be estimated to be equal to the demand-capacity ratio (that is, $\mu_s = 4$). If displacements are viewed directly, as in Fig. 8, the correct displacement ductility for the pier ($\mu_s = 7$) is obtained.

A comprehensive approach to analysis of seismic response requires simultaneous consideration of displacements, forces, and ductilities, regardless of the basic approach used. Lateral displacements result in P- delta effects that may be important. Forces associated with inelastic flexural response determine actions in shear and other less-ductile response modes, and should be used in a capacity design approach to establish required strengths [Park, 1986]. Flexural ductility plays an important role in capacity design where behavior in nonductile modes (e.g., shear in reinforced concrete columns) depends on the ductility level. For these reasons, displacements and forces should be considered when using the ductility based approach, and forces and ductilities should be considered when using the displacement based approach.

6 CONCLUSIONS

Studies of the inelastic response of simple and complex structures have resulted in the development of uncomplicated tools for estimating maximum lateral drift during a strong earthquake. Two approaches to design and evaluation using drift information are available. A ductility based approach uses displacement information indirectly, establishing ductility requirements as a function of the provided strength and the strength required for elastic response. A displacement based approach uses displacement information directly. The latter approach has been the main subject of the present paper.

The displacement based approach can be used to establish proportions and layout that will control drift demand, and to determine structural and nonstructural details that will ensure adequate performance. Several examples illustrating its application have been presented. The examples demonstrate that the displacement based approach is a simple and effective tool for design.

REFERENCES


Figure 4 - Comparison of Ultimate Displacement Capacities Calculated Using Idealized (Eq. 1) and Simplified (Eq. 4) Curvature Distributions

Figure 5 - Rotational Capacity of Reinforced Concrete Beam Sections

Figure 6 - Displacement Ductility Based on Equal Displacement Rule
Figure 7 - Example Redesign of Existing Frame

Figure 8 - Bridge Pier with Flexible Foundation