Earthquake Engineering, Tenth World Conference © 1992 Balkema, Rotterdam. ISBN 90 5410 060 5

A model of plasticity coupled to damage for RC frames

A.Cipollina, R.Febres-Cedillo & J.Flórez-López
Universidad de los Andes, Mérida, Venezuela

ABSTRACT: A model based upon the concepts of continuum damage mechanics and the theory of lumped-dissipation models for frames is proposed. The model takes into account plasticity as well as stiffness degradation effects. The model is identified for RC frames.

1 INTRODUCTION

Lumped-dissipation models (see for instance Maier et al. 1973, Cohn and Franchi 1979) can be described as the extension of standard matrix method for elastic frames to the modeling of material non-linear behavior. In this approach inelastic behavior is assumed to be lumped at some points of each member of the frame. These points are modeled as non-linear, energy -dissipation springs. Lumped -dissipation models have shown satisfactory accuracy and at the same time simplicity in the formulation. They are perhaps the most suitable for non -linear analysis of large structures.

On the other hand continuum damage mechanics (see for instance Lemaitre and Chaboche 1985) is the theoretical tool that allows the description of material properties degradation under thermo-mechanical loading. It is based upon the introduction of an internal variable that measures the effects of the density of microcraks or microvoids on the behavior of the material.

This paper intends to combine both theories and formulate a lumped-dissipation model that takes into account plasticity as well as damage effects in the behavior of frames. Although the model proposed in this paper is very general, it has been identified to describe the behavior of RC frames.

2 LUMPED-DISSIPATION MODEL OF A SINGLE BAR

For the sake of simplicity, let us consider a member of a planar frame in the small deformation case. Generalized stresses and deformations of the member in local coordinates are denoted respectively: $\langle M\rangle^T = (M_i, M_j, N)$ and $\langle \Phi\rangle^T = (\Phi_i, \Phi_j, \delta)$ (see figure 1).

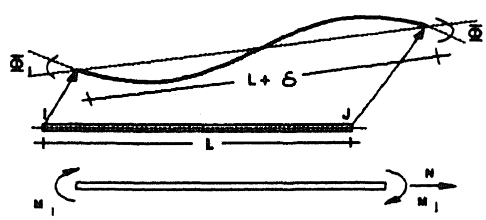

Figure 1: Generalized stresses and deformations of a frame's bar.

Generalized stress and deformation are related by expression (1) when the member has an elastic behavior.

$$\langle M\rangle = [S^\circ].\langle \Phi\rangle \qquad (1)$$

$$\text{where } [S^\circ] = \begin{bmatrix} S^\circ_{11} & S^\circ_{12} & 0 \\ S^\circ_{21} & S^\circ_{22} & 0 \\ 0 & 0 & S^\circ_{33} \end{bmatrix}$$

The matrix $[S^\circ]$ is the elastic stiffness matrix of the bar and its elements remain constant during the loading process.

The inverse of the stiffness matrix is the so called flexibility matrix:

$$\langle \Phi\rangle = [F^\circ].\langle M\rangle \qquad (2)$$

where $[F^\circ] = [S^\circ]^{-1}$

Under severe overloads the elastic model is obviously inadequate. In such a case the member undergoes damage, plasticity and other energy -dissipation phenomena.

In order to determine a more general stress-deformation relation that includes these effects, we consider the following "lumped dissipation" model of a member: "The member" is modeled as an assemblage of an elastic beam-column that we will call simply "the beam" and a set of zero length "inelastic hinges" that behave as non-linear springs (see figure 2). Energy dissipation is assumed to concentrate only on the hinges.

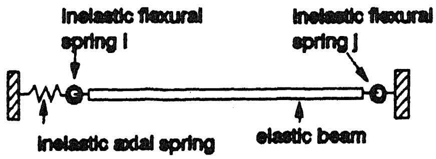

Figure 2: Lumped-dissipation model of a frame's bar.

Deformations of the bar can be split into beam's deformations $\langle\Phi^b\rangle^T =$ $(\Phi_i^b, \Phi_j^b, \delta^b)$ and springs's deformations $\langle\Phi^s\rangle^T = (\Phi_i^s, \Phi_j^s, \delta^s)$:

$$\langle\Phi\rangle = \langle\Phi^b\rangle + \langle\Phi^s\rangle \qquad (3)$$

where Φ_i^s and Φ_j^s denote the rotation of the flexural springs, and δ^s the deformation of the axial spring. Equilibrium between the elements of the member implies:

$$\langle M\rangle = \langle M^b\rangle = \langle M^s\rangle \qquad (4)$$

Where $\langle M^b\rangle^T = (M_i^b, M_j^b, N^b)$ and $\langle M^s\rangle^T =$ (M_i^s, M_j^s, N^s) denote respectively the generalized stress of the beam and springs.

3 STRESS-DEFORMATION RELATION OF A DAMAGED MEMBER.

3.1 Beam's stress-deformation relation.

This relation is (since the beam is assumed to remain elastic):

$$\langle M^b\rangle = [S^o]\langle\Phi^b\rangle \qquad (5)$$

Where $[S^o]$ is the elastic stiffness matrix defined in (1).

3.2 Springs's stress-deformation relation.

In order to characterize the spring's behavior we introduce three sets of "internal variables", two of them characterize the state of damage of the member. These are denoted by: $\langle D^+\rangle^T = \langle d_i^+, d_j^+, d_a^+\rangle$ and $\langle D^-\rangle^T =$ $\langle d_i^-, d_j^-, d_a^-\rangle$. The third set has the plastic deformations of the member: $\langle\Phi_p\rangle^T = \langle\Phi_{pi}, \Phi_{pj}, \delta_p\rangle$. Although these variables represent inelastic effects of the whole member, they affect only the behavior of the springs because of the lumped dissipation model that is being used.

Scalar parameters d_i^+ (d_j^+, d_a^+) and d_i^- (d_j^-, d_a^-) represent respectively the damage due to positive and negative stress of flexural spring i (flexural spring j and the axial spring). The use of two different damage parameters for each spring will allow us to represent unilateral effects due to load reversals (i.e. with change of sign), such as in some models of standard damage mechanics (see for instance Mazars 1986). Φ_{pi} and Φ_{pj} are respectively the plastic rotation of flexural springs i and j; and δ_p the permanent deformation of the axial spring.

Plastic deformations can be associated with the yield of the reinforcement in the case of RC beams. The damage variable d_i^+ characterizes the effect of the cracking in the lower part of the section on the moment-rotation behavior of the member and, d_i^- the effect of the cracking in the upper part. d_a^+ represents the damage due to tension axial loads and d_a^- the damage due to compressive axial forces.

The stress-deformation relation for inelastic hinges is obtained from standard continuum damage mechanics by using the lumping procedure described in Flórez-López 1992a and 1992b. We obtain then the following relation:

$$\langle\Phi^s - \Phi_p\rangle = [C(D^+)]\langle\langle M^s\rangle\rangle_+ + [C(D^-)]\langle\langle M^s\rangle\rangle_- \qquad (6)$$

where $[C(D)]$ for $\langle D\rangle = \langle D^+\rangle$ or $\langle D\rangle =$ $\langle D^-\rangle$ is a diagonal matrix whose non zero terms are:

$$C_{11} = \frac{d_i}{(1-d_i)S_{11}^o}\ ; \qquad C_{22} = \frac{d_j}{(1-d_j)S_{22}^o}\ ;$$

$$C_{33} = \frac{d_\alpha}{(1-d_\alpha) S^o_{33}} \tag{7}$$

and symbols $\langle x\rangle_+$ $\langle x\rangle_-$ denote *respectively the positive and negative* part of x, i.e.

$$\langle x\rangle_+ = \begin{cases} x & \text{if } x > 0 \\ 0 & \text{otherwise} \end{cases}$$
$$\tag{8}$$
$$\langle x\rangle_- = \begin{cases} x & \text{if } x < 0 \\ 0 & \text{otherwise} \end{cases}$$

It can be notice from (7) that a damage parameter d_i equal to zero represents a rigid spring without effect on the behavior of the member and, if it is equal to one a totally damaged spring. The latter case allows us to introduce the notion of "damaged hinge" in a frame (by analogy with "internal hinges" and "plastic hinges" in standard frame theories).

3.3 Flexibility matrix of a damaged member.

The stress-deformation relation of a damaged member can be determined by solving the system of equations defined by expressions (2),(3),(4),(5) and (6). We obtain then:

$$\langle \Phi - \Phi_p \rangle = [F(D^+)]\langle\langle M\rangle\rangle_+ + [F(D^-)]\langle\langle M\rangle\rangle_- \tag{9}$$

where flexibility matrices [F(D)] are given by:

$$[F(D)] = [C(D)] + [F^o] \tag{10}$$

It can be notice that deformations $\langle \Phi^a\rangle$ and $\langle \Phi^r\rangle$ where condensed and do not appear in the final stress-deformation relation (9).

3.4 Flexibility and stiffness matrix for non-reversal loading.

Only one set of damage parameters is necessary in the particular case of non-reversal loading. In such a case expression (9) becomes:

$$\langle \Phi - \Phi_p \rangle = [F(D)]\langle M\rangle \quad \text{or}$$
$$\langle M\rangle = [S(D)]\langle \Phi - \Phi_p \rangle \tag{11}$$

where $[S(D)] = ([C(D)] + [F^o])^{-1}$ is the stiffness matrix of a damaged member.

4 THERMODYNAMIC POTENTIAL AND STATE LAWS.

We postulate the existence of a thermodynamic potential which is a function of the state variables defined at the member level. State laws as well as the associated thermodynamic forces can be derived from this potential.

We chose the complementary strain energy U^* given by:

$$U^* = \frac{1}{2}\langle\langle M\rangle_+\rangle^T [F(D^+)]\langle\langle M\rangle_+\rangle + \frac{1}{2}\langle\langle M\rangle_-\rangle^T [F(D^-)]\langle\langle M\rangle_-\rangle \tag{13}$$

So that the state law (9) can be expressed as:

$$\langle \Phi_e \rangle = \langle \Phi - \Phi_p \rangle = \left(\frac{\partial U^*}{\partial M}\right) \tag{14}$$

Thermodynamic forces associated to damage parameters can now be defined:

$$\langle G^+\rangle = -\left(\frac{\partial U^*}{\partial D^+}\right); \quad \langle G^-\rangle = -\left(\frac{\partial U^*}{\partial D^-}\right) \tag{15}$$

and energy dissipation due to inelastic effects is given by:

$$\mathcal{D} = \langle G^+\rangle^T \langle \dot{D}^+\rangle + \langle G^-\rangle^T \langle \dot{D}^-\rangle + \langle M\rangle^T \langle \dot{\Phi}_p\rangle \geq 0 \tag{16}$$

The latter relation indicates that damage evolution in frames should not be expressed as a function of the generalized stress $\langle M\rangle$ but as a function of the new thermodynamic forces $\langle G^+\rangle$ and $\langle G^+\rangle$.

5 INTERNAL VARIABLE EVOLUTION LAWS.

5.1

The relation between deformations and the history of stress has not been completely defined since equation (9) depends on some additional internal variables. Therefore three sets of new relations must be added. These are the "internal variable evolution laws".

5.2 *Time-independent plastic* deformation evolution law.

Time-independent laws can be obtained by using the same formalism of standard plastic models in continuum mechanics.

We introduce a set of "yield functions" which characterize the domain of "non-plasticity" of each spring:

$$f_i(M;\Phi_p,D^+,D^-) \le 0 \tag{17}$$

The non-plastic domain is defined as the set of stresses {M} such that the yield function f_i is negative.

Therefore, yield functions depend on the stress vector. Plastic deformations as well as damage variables can appear as parameters in these functions in order to describe hardening or softening of the non-plastic domain.

Plastic deformation evolution laws are then:

$$\dot{\Phi}_{pi} = \begin{cases} 0 & \text{if } f_i < 0 \text{ or } \dot{f}_i < 0 \\ \lambda_i \dfrac{\partial f_i}{\partial M_i} & \text{otherwise} \end{cases} \tag{18}$$

where plastic multiplicator λ_i is calculated by the following "consistence condition": $\dot{f}_i = 0$

5.3 Damage evolution law

The same formalism can be used to obtain time-independent evolution laws for the damage variables.

We introduce then the following "damage functions":

$$g_i^+(G^+;\Phi_p,D^+,D^-) \le 0$$

$$g_i^-(G^-;\Phi_p,D^+,D^-) \le 0 \tag{19}$$

which are functions of the thermodynamic forces associated to damage variables. Internal variables can appear as parameters in the damage functions as in the precedent case.

"Non-damage" domains for respectively positive and negative stress can now be introduced. These are defined as the sets of thermodynamic forces such that functions g_i are negative.

Damage evolution laws are then given by:

$$\dot{d}_i^+ = \begin{cases} 0 & \text{if } g_i^+ < 0 \text{ or } \dot{g}_i^+ < 0 \\ \rho_i^+ \dfrac{\partial g_i^+}{\partial G_i} & \text{otherwise} \end{cases} \tag{20}$$

"damage multiplicator" ρ_i^+ is calculated thanks to the "consistence condition": $\dot{g}_i^+ = 0$.

Evolution laws for damage variables $\{D^-\}$ are determined in a similar way.

5.4 Evolution laws identification for RC frames.

Evolution laws can be identified from numerical simulations with standard damage mechanics models or directly (as in this paper) from experimental results. These test should be performed, preferably, on beam-column joints if damage is the result of horizontal loading (for instance due to seismic effects).

However, as a first approximation we will identify functions f_i and g_i from test performed on simply supported beams of constant area A, inertia I, Young's modulus E and length 2l as shown in figure 3.

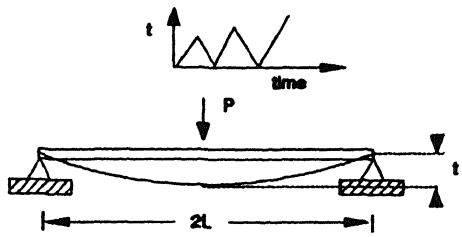

Figure 3 Evolution laws identification l=75 cm, A= 20x20 cm^2 reinforcement= 5b3/8', fy ≅ 4200Kg/cm^2, fc ≅ 250Kg/cm^2.

We will identify the evolution laws for the particular case of loading that do not change of sign. This case corresponds to the stress-deformation relationship (11).

We will neglect axial effects, so that only evolution laws for the flexural springs will be identified.

We assume $\Phi_{pi} = d_i = 0$, therefore the lumped dissipation model of the test has only one inelastic spring as shown in figure 4.

The analytical force-deflection relationship can be obtained from equation (11) taking into account that:

$$M_i = 0; \quad M_j = \frac{P\,l}{2}; \quad \Phi_j = \frac{t}{l}$$

$$N = 0; \quad d_a = 0; \quad \delta = \delta_p = 0$$

We obtain then:

$$P = \frac{4 - 4d}{4 - d}\left(\frac{6EI}{l^3}\right)(t - t_p) \qquad (21)$$

where $d = d_j$ and $t_p = l\ \Phi_{pj}$.

Elastic unloading allow the determination of the experimental values of the elastic slope Z, where

$$Z(d) = \frac{4 - 4d}{4 - d} Z_o; \qquad Z_o = \frac{6EI}{l^3} \qquad (22)$$

and the permanent deflection t_p for a given value of the total deflection as shown in figure 4.

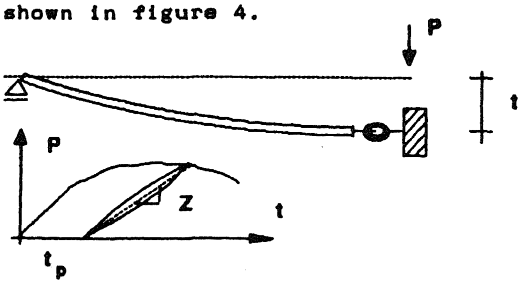

Figure 4 Lumped-dissipation model of the simply supported beam.

M_j vs Φ_{pj} and d_j vs G_j curves can be obtained from these data since:

$$d = 4(1 - \frac{Z(d)}{Z_o}) \ / \ (4 - \frac{Z(d)}{Z_o})$$

$$G = \frac{1}{2S_o}\left(\frac{M}{(1-d)}\right)^2 \qquad S_o = \frac{4EI}{l} \qquad (23)$$

Relations (23) follow from (22) and (15).

These curves are shown in figures 5 and 6.

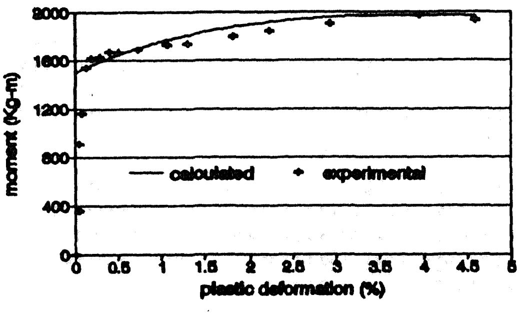

Figure 5 Stress vs plastic deformation in a simply supported beam.

We propose the following phenomenological expressions to describe the behavior of the beam:

$$f_j = |M_j| - \left[\frac{1 - d_j}{4 - d_j}\right](c\ \Phi_{pj} + 4M_y) \qquad (24)$$

$$g_j = G_j - \left(G_{cr} - q\ \frac{\ln(1-d)}{(1-d)} \right)$$

where c, M_y G_{cr} and q are member and material dependent constants.

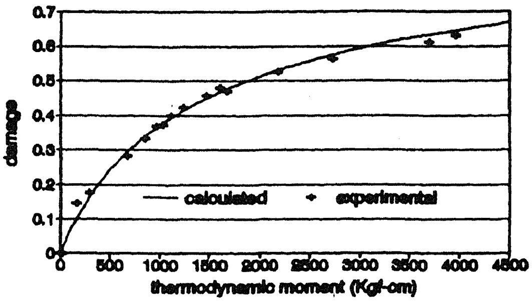

Figure 6. Thermodynamic moment associated to damage vs damage in a simply supported beam.

The second term of f_j in (24) gives the "size" of the "non-plastic" domain. The size is then the product of two functions, one of them depends on the damage the second on the plastic deformation.

It can be noticed that this model becomes the standard elasto- (perfect) plastic model when $\dot{d}_j = 0$ and $c = 0$.

If $\dot{d}_j = 0$ and $c > 0$ these equations describe the also usual bilineal plastic model with isotropic hardening.

In the general case the size of the "non-plastic domain" is the result of the "competition" between the hardening effect introduced by the function that depends on the plastic deformation and the softening effect induced by the term that depends on the damage.

Constants c, M_y, G_{cr} and q can be obtained from experimental results. Empirical relations between these constants and the characteristics of the member could be obtained from a large enough set of experimental data.

However a simpler way of calculate the constants consists in solving the following non-linear system of equations:

$$\begin{aligned} M &= M_{cr} && \Rightarrow d = 0 \\ M &= M_{p} && \Rightarrow \Phi_{p} = 0 \\ M &= M_{u} && \Rightarrow \frac{dM}{d\Phi^{r}} = 0 \qquad (25) \\ M &= M_{u} && \Rightarrow \Phi_{p} = \Phi_{pu} \end{aligned}$$

where M_{cr} is the cracking moment, M_p the plastic moment, M_u the ultimate moment and Φ_{pu} the corresponding plastic deformation. The latter can be obtained from the ductility factor of the cross section of the member. All of these constants can be calculated from standard reinforced concrete theory.

Comparison between experimental results and the model is shown in figures 5,6 and 7. The values of M_{cr}, M_p, M_u, Φ_{pu} and S_o were determined from experimental results. These values are:

$M_{cr} = 0$; $M_p = 1505$ Kg-m; $M_u = 1972$ Kg-m
$\Phi_{pu} = 3.954 \times 10^{-2}$; $S_o = 390806$ Kg-m

Constants q, G_{cr}, My and C were obtained by numerical resolution of (25). These constants are:

q = -1353 kg-cm; c = 267336.4kg-m
$M_y = 1866.61$ Kg-m; $G_{cr} = 0$

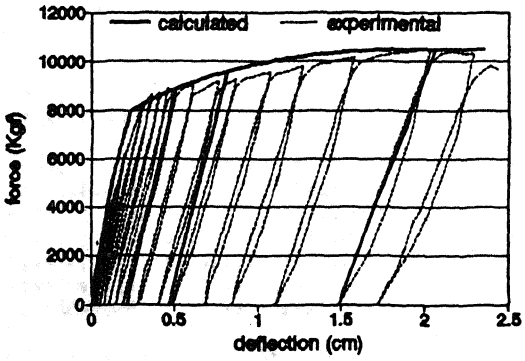

Figure 7 Force vs deflection curve in a simply supported beam.

6 CONCLUDING REMARKS

The model that was proposed (relations (9), (15), (18) and (20)) can be used for no-linear analysis of any kind of frame although in this paper the model was identified with very simple tests on RC members (relations (24)).

Only some (two in the simplest case, eight in the more general case) functions must be identified for each kind of frame.

Numerical analysis and implementation of the model in standard finite element programs is described in (Flórez et al. to appear).

Large deformations effects can be taken into account by using the adequate stiffness matrix which may depend on beam's deformations in equation (1).

REFERENCES

G. Maier, O. De Donato, L. Corradi "Inelastic analysis of reinforced concrete frames by quadratic programming" Simp. on Inelasticity and Nonlinearities in Struc. Concr., Univ. of Waterloo, Canada (1973)

Z.M. Cohn, A. Franchi. "STRUPL: A computer system for structural plasticity." J. Struct. Div. ASCE. 105(4). 789-804 (1979).

J. Lemaitre, J.L. Chaboche. "Mécanique des matériaux solides." Dunod Paris (1985).

J.Mazars "A model of unilateral elastic damageable material and its application to concrete", Proceedings, Fracture Toughness and Fracture Energy of Concrete ed. by Witmann F.H. Elsevier Pubs, 61-71 (1986).

J. Flórez-López "Modelos de daño concentrado para la simulación numérica del colapso de pórticos planos" Cuadernos del CeSiMo IT-0492 Universidad de Los Andes 1992.

J. Flórez-López "A simplified damage mechanics approach to non-linear analysis of frames" Cuadernos del CeSiMo IT-0392. Universidad de Los Andes 1992.

J. Flórez-López, A. López-Hinojosa, I. Terán-Perez "Some numerical aspects of a simplified damage mechanics problem" Cuadernos del CeSiMo. Universidad de Los Andes. To appear.