Earthquake Engineering, Tenth World Conference © 1992 Balkema, Rotterdam. ISBN 90 5410 060 5

Finite element analysis of reinforced concrete structures under monotonic and cyclic loads

H.-G. Kwak
Korea Advanced Institute of Technology, Seoul, Korea

F.C. Filippou
University of California, Berkeley, Calif., USA

ABSTRACT: This paper deals with the finite element analysis of the monotonic and cyclic behavior of reinforced concrete beams and beam-column joint subassemblages. It is assumed that the behavior of these members can be described by a plane stress field. Concrete and reinforcing steel are represented by separate material models which are combined together with a model of the interaction between reinforcing steel and concrete through bond-slip to describe the behavior of the composite reinforced concrete material. A new smeared crack finite element model is proposed based on an improved cracking criterion derived from fracture mechanics principles. A new reinforcing steel model which is embedded inside a concrete element, but accounts for the effect of bond-slip is developed. Correlation studies between analytical and experimental results show the validity of the proposed models and identify the significance of various effects on the local and global response of reinforced concrete members.

1 INTRODUCTION

The assessment of the strength and stiffness of existing structures and newly designed critical structures such as offshore platforms, long-span bridges and nuclear power plants requires the development of advanced analytical methods capable of representing the behavior of the structure under all possible loading conditions, both, monotonic and cyclic, its time-dependent behavior, and, especially, its behavior under overloading.

The development of such models for reinforced and prestressed concrete structures is complicated by differences in short- and long-term behavior of the constituent materials, concrete and reinforcing steel. Moreover, reinforcing steel and concrete interact in a complex way through bond-slip and aggregate interlock. A general purpose model of the short- and long-term response of reinforced concrete members and structures should, therefore, be based on separate material models for reinforcing steel and concrete, which are then combined along with models of the interaction between the two constituents to describe the behavior of the composite reinforced concrete material. This is the approach adopted in this study.

2 MATERIAL MODELS

The following assumptions are made in the description of material behavior:

1. The stiffness of concrete and reinforcing steel is formulated separately. The results are then superimposed to obtain the element stiffness;
2. The smeared crack model is adopted in the description of the behavior of cracked concrete;
3. Cracking in more than one direction is represented by a system of orthogonal cracks;
4. The crack direction changes with load history (rotating crack model);
5. The reinforcing steel is assumed to carry stress along its axis only and the effect of dowel action of reinforcement is neglected;
6. The transfer of stresses between reinforcing steel and concrete and the resulting bond-slip is explicitly accounted for in a new discrete reinforcing steel model, which is embedded in the concrete element.

In the following, the material model of each constituent material and their interaction through bond-slip is briefly described. This is followed by examples of the monotonic and cyclic behavior of anchored reinforcing bars, beams and beam-column joints. Details of the models and their implementation can be found elsewhere (Kwak and Filippou 1990).

2.1 Concrete

The orthotropic model is adopted in this study for its simplicity and computational efficiency. The ultimate objective of this work is the response analysis of large structures under cyclic loads for which more rational and sophisticated models are prohibitively expensive.

The behavior of the model depends on the location of the present stress state in the principal stress space. In the biaxial compression region the model remains linear elastic for stress combinations inside the initial yield surface. Both the initial yield and the ultimate load surface are described by Kupfer's model (Figure 1).

For stress combinations outside the initial yield surface but inside the ultimate failure surface the behavior of concrete is described by a nonlinear orthotropic model. This model derives the biaxial stress-strain response from equivalent uniaxial stress-strain relations in the axes of

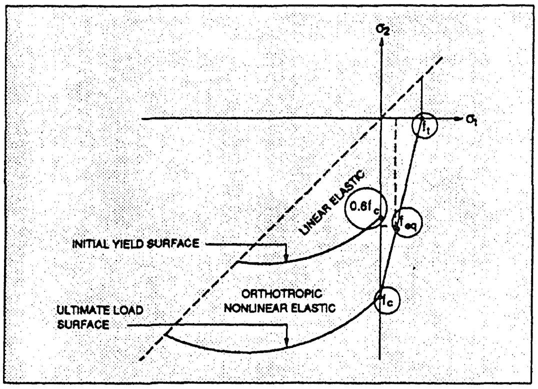

Figure 1. Strength Failure Envelope of Concrete

orthotropy (Darwin and Pecknold 1977). The uniaxial concrete stress-strain relation follows a piecewise linear modification of Hognestad's model.

When the biaxial stresses exceed Kupfer's failure envelope, concrete enters into the strain softening range of behavior where an orthotropic model describes the biaxial behavior (Darwin and Pecknold 1977). In this region failure occurs by crushing of concrete when the principal compressive strain exceeds a limit value. In defining the crushing of concrete under biaxial compressive strains a strain failure surface in complete analogy to Kupfer's stress failure envelope is used.

In the biaxial compression-tension and tension-tension region the following assumptions are adopted in this study: (1) failure takes place by cracking and, therefore, the tensile behavior of concrete dominates the response; (2) the uniaxial tensile strength of concrete is reduced to the value f_{eq}, to account for the effect of the compressive stress; in the tension-tension region the tensile strength remains equal to the uniaxial tensile strength f_t; (3) the concrete stress-strain relation in compression is the same as under uniaxial loading and does not change with increasing principal tensile stress. The last assumption holds true in the compressive stress range which is of practical interest in typically reinforced frame structures.

In the compression-tension and the biaxial tension region the proposed concrete model remains linear elastic for tensile stresses smaller than f_{eq}. Beyond the tensile strength the tensile stress decreases linearly with increasing principal tensile strain. Ultimate failure in the compression-tension and the tension-tension region takes place by cracking, when the principal tensile strain exceeds the value ε_o. The value of ε_o is derived from fracture mechanics concepts and depends on the finite element mesh size. In this way the analytical results retain objectivity for large finite element mesh size. At the same time this approach allows for the realistic representation of microcrack concentration near the tip of the crack without loss of accuracy for large finite elements. When the principal tensile strain exceeds ε_o, the material only loses its tensile strength normal to the crack, while it is assumed to retain its strength parallel to the crack direction.

2.2 Reinforcing steel with bond-slip

The simplest model of the bond-slip interaction between reinforcing steel and concrete is the bond-link element (Ngo and Scordelis 1967). This element connects one steel node with a corresponding concrete node which occupies the same physical location in the undeformed configuration of the structure. Consequently, the use of this element in the finite element analysis of RC structures imposes the following restrictions: (a) the finite element mesh must be arranged so, that a reinforcing bar is located along the edge of a concrete element and (b) a double node is required to represent the relative slip between reinforcing steel and concrete. These restrictions arise from the fact that the stiffness of the bond link element is associated with the relative displacement between steel and concrete. Consequently, the total displacement of, both, reinforcing steel and concrete is required at each node of the finite element mesh, so that the relative displacement and, consequently, the bond stress between steel and concrete can be determined.

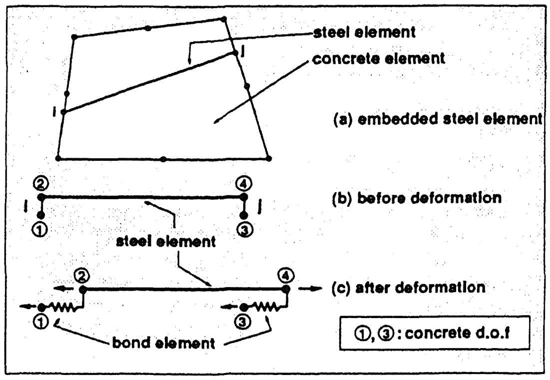

Figure 2. Reinforcing steel element with bond-slip.

In a complex structure, particularly in three-dimensional models, these requirements lead to a considerable increase in the number of degrees of freedom, not only because of the doubling of the number of nodes along the reinforcing steel bars, but also because the mesh has to be refined, so that the bars pass along the edges of concrete elements. The complexity of mesh definition and the large number of degrees of freedom has discouraged researchers from including the bond-slip effect in many studies to date.

To address these limitations of the bond link element a new discrete reinforcing steel model which includes the bond-slip deformation is proposed in this study. In this model the reinforcing bar is modeled by a truss element embedded inside the concrete element, as shown in Figure 2. In this case the finite element mesh configuration does not have to follow the arrangement of the reinforcing bars. At the same time the relative slip between reinforcing steel and concrete is explicitly taken into account in the model. The cyclic stress-slip relation of the bond links follows the modified model of Eligehausen in Figure 3 (Filippou et. al. 1983).

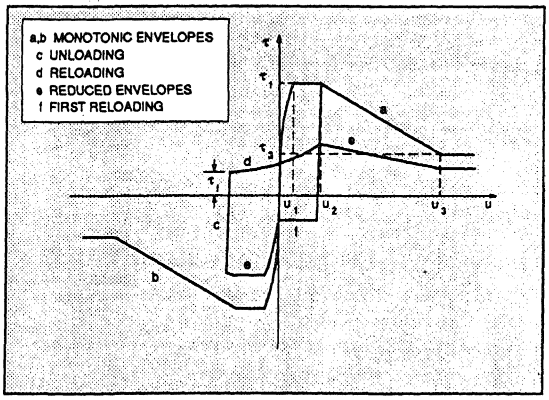

Figure 3. Cyclic stress-slip relation of bond links.

Since the finite element model only includes the concrete displacement degrees of freedom, the degrees of freedom which are associated with the reinforcing steel are condensed out from the element stiffness matrix, before it is assembled into the structure stiffness matrix. A second pass, however, is now required in order to satisfy the equilibrium of each reinforcing bar and determine the steel forces. In the second pass the current concrete displacement increments are imposed on the reinforcing steel-concrete interface. The resulting linearized system of equilibrium equations for the entire reinforcing bar is solved followed by the state determination and calculation of the resisting steel and bond forces. The updated steel and bond link stiffness along with the resisting forces are then transformed to the global concrete degrees of freedom to allow for the iterative solution of the nonlinear equilibrium equations of the entire structure.

3 APPLICATIONS

A number of correlation studies are conducted with the objective of establishing the ability of the proposed model to simulate the response of reinforced concrete beams and beam-column joint subassemblages. In order to independently test the reinforcing steel model with bond-slip, the response of anchored reinforcing bars under monotonic pull-out and cyclic push-pull loads is also studied. The latter study highlights the pronounced effect of cyclic bond-deterioration on the hysteretic response of reinforced concrete members.

3.1 Reinforcing bar under monotonic and cyclic loads

Several anchored reinforcing bars simulating anchorage and loading conditions in interior beam-column joints of moment resisting frames which are subjected to a combination of gravity and high lateral loads were tested (Viwathanatepa, Popov and Bertero 1979b). #6, #8 and #10 reinforcing bars were anchored in well confined concrete blocks and were subjected to monotonic pull-out at one end, monotonic pull-out at one end with simultaneous push-in at the other (called push-pull) and cyclic push-pull.

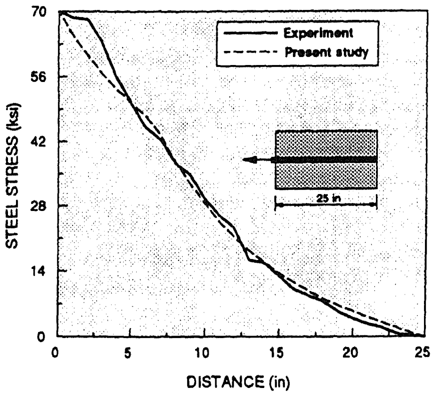

Figure 4. Stress distribution along anchored reinforcing bar for end stress of 70 ksi.

Two specimens are selected for comparison with the proposed reinforcing steel model with bond-slip. The first specimen is an anchored #8 bar in a well confined block of 25 in. width, which corresponds to an anchorage length of 25 bar diameters. This specimen was subjected to a monotonic pull-out under displacement control at one end only. The second specimen also involves a #8 reinforcing bar with identical dimensions which was subjected to a cyclic push-pull loading with gradually increasing end slip value. Both specimens have been the subject of earlier analytical correlation studies (Viwathanatepa et al. 1979b, Ciampi et al. 1982, Yankelevky 1985). The material properties of concrete and reinforcing steel are as follows (Viwathanatepa et al. 1979b): the concrete cylinder strength is 4,700 psi for the specimen under monotonic pull-out and 4,740 psi for the specimen under cyclic push-pull. The yield strength of the reinforcing steel is 68 ksi and the yield strain is 0.23%.

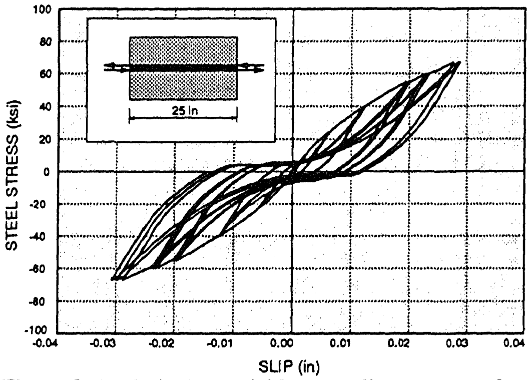

Figure 5. Analytical pre-yield stress-slip response of reinforcing bar under cyclic push-pull loading.

In the monotonic pull-out test a bilinear steel model is used. The strain hardening modulus is equal to 411 ksi, which corresponds to 1.4% of Young's modulus. In the cyclic push-pull test a modified Menegotto-Pinto steel model is used (Filippou et al. 1983). In both specimens the parameters of the bond-slip model in Figure 2 are as follows: $u_1 = 0.028$ in, $u_2 = 0.079$ in, $u_3 = 0.28$ in,

$\tau_1 = 2350$ psi and $\tau_3 = 870$ psi. In a previous study (Ciampi et al. 1982) the bond-slip relation was modified in the outer unconfined portions of the anchorage length. By contrast, in the present study the same bond stress-slip relation is used along the entire anchorage length for the sake of simplicity, since the hysteretic behavior is not the focus of the present study. Under cyclic loading conditions this assumption leads to underestimation of the bond resistance at the push-in end of the reinforcing bar. 25 steel elements of 1 in length each were used in modeling the anchored reinforcing bar.

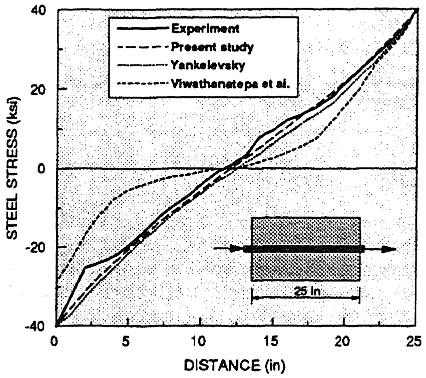

Figure 6. Stress distribution along reinforcing bar at the seventh cyclic push-pull loading cycle.

Figure 4 shows the distribution of steel stresses along the anchorage length of the reinforcing bar under monotonic pull-out for a steel stress of 70 ksi at the loaded end. The analytical results show very good agreement with experimental measurements.

Figure 5 shows the end stress-slip relation of the reinforcing bar under cyclic push-pull loading for load cycles before yielding of the reinforcement. The steel stress distribution along the anchorage length of the reinforcing bar for part of the seventh loading cycle is shown in Figure 6. The experimental results are compared with the results of previous studies (Viwathanatepa et al. 1979b, Yankelevsky 1985) and those of the present study. The results of the proposed model show very good agreement. It should be noted, however, that the reinforcing steel does not yield in the seventh cycle. In latter load cycles the analytical results start to deviate from the measured stress distribution. This is attributed to two factors: (a) the model is tested under load controlled conditions, while the specimen was subjected to displacement controlled testing, and (b) the assumption that the bond stress-slip relation is the same along the anchorage length of the bar is not reasonable for cycles which induce significant bond damage.

3.2 *Reinforced concrete beams*

Three simply supported reinforced concrete beams have been investigated. In these case studies the concrete was modeled by 8-node serendipity plane stress elements with 3x3 Gauss integration and the reinforcement was modeled by the 2-node truss elements. In all studies the bond-slip effect is taken into account with bond link elements, as described in the previous section.

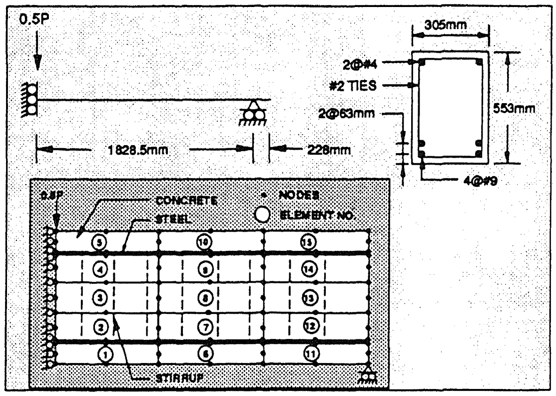

Figure 7. Configuration and finite element idealization of beam A1.

In the following the results for beam A1 (Bresler and Scordelis 1963) are presented. This is a heavily reinforced beam with a longitudinal reinforcing steel ratio of 1.53%. The geometry and cross section dimensions of beam A1 are presented in Figure 7, which also shows the finite element idealization of the beam. The steel material properties of the beam are: (a) for the bottom longitudinal steel (#9 bars) $E_s = 29000$ ksi and $f_y = 80.5$ ksi, (b) for the top longitudinal steel (#4 bars) $E_s = 29200$ ksi and $f_y = 50.1$ ksi and (c) for the transverse reinforcement (#2 bars) $E_s = 27500$ ksi and $f_y = 47.2$ ksi. The concrete material properties of the beam are: $E_c = 3370$ ksi and $f'_c = 3.49$ ksi.

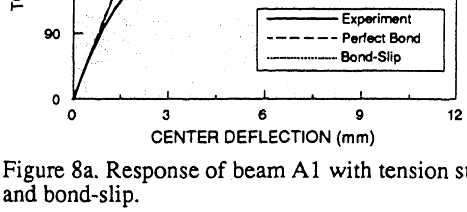

Figure 8a. Response of beam A1 with tension stiffening and bond-slip.

Figure 8a compares the analytical results with the measured load-displacement response of beam A1. Very satisfactory agreement between analysis and experiment is observed.

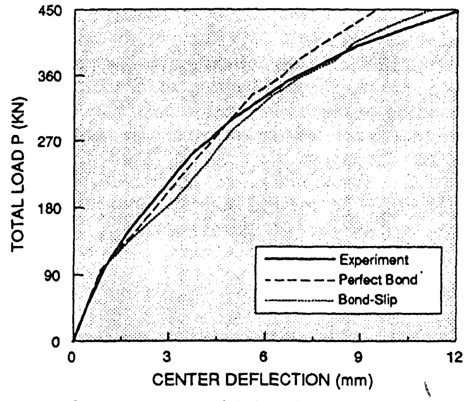

Figure 8b. Response with bond-slip but without tension stiffening.

To identify the relative contribution of tension stiffening and bond-slip four different analyses are performed for this specimen. In Figure 8a the effect of tension stiffening is included in both analytical results. The response depicted by the long-dash line excludes the effect of bond-slip, while the response depicted by the dotted line includes this effect. It is clear from the comparison of the analysis with the experimental data that the inclusion of both effects yields very satisfactory agreement of the model with reality. Figure 8b shows two analytical results which exclude the effect of tension-stiffening. In this case the inclusion of bond-slip (dotted line) produces a slightly more flexible response than the experiment, while the exclusion of bond-slip produces a slightly stiffer response. Figures 8a and 8b show that the contribution of bond-slip to the load-displacement response of the specimen increases with load. Near the ultimate strength of the beam the magnitude of the bond-slip contribution to the load-displacement response is almost twice that of the tension-stiffening effect, which is of opposite sign.

The studies of beam specimen A1 show that the inclusion of, both, tension-stiffening and bond-slip, yields excellent agreement of the analytical results with the experiments. In lightly reinforced beams these two effects are of comparable importance and might cancel each other at certain stages of the load history. This does not happen over the entire response, since gradual bond damage leads to increasing bond-slip contribution with increasing load. The bond-slip effect is more marked in heavily reinforced beams, such as specimen A-1, where it outweighs the effect of tension-stiffening.

3.3 Beam to column joint subassemblage

Experimental studies have shown that bond slip has a pronounced effect on the global deformation of beam-column subassemblages which are subjected to loading simulating the effect of large lateral loads. This happens because the anchorage length of the reinforcing bars in the joint does not suffice to transfer to the concrete the steel forces at the beam-column interfaces of the joint. These forces cause the reinforcing bars to be pulled from one end of the joint and pushed from the other, so that a force equal to almost twice the yield force of the bar needs to be transferred to the concrete within the joint. When lateral load reversals occur, the stress transfer problem is aggravated by bond deterioration under cyclic load reversals.

In order to assess the ability of the proposed finite element model to simulate the behavior of beam-column joint subassemblages specimen BC4 is selected for further study (Viwathanatepa et al. 1979a). The subassemblage consists of two 9 in. by 16 in. girders framing into a 17 in. by 17 in. square column. The column was bolted at the top and bottom to steel clevises for mounting the specimen on the testing frame. The main longitudinal reinforcement of the beams consisted of 4#6 bars at the top and 3#5 bars at the bottom of the section with #2 tied stirrups spaced at 3.5 in. as transverse reinforcement. The longitudinal reinforcement of the column consisted of 12#6 bars with #2 ties spaced 1.6 in. center-to-center along column providing the transverse reinforcement. Seven #2 ties were placed inside the beam-column joint region to satisfy the confinement and shear resistance requirements of the Uniform Building code.

The beam-column subassemblage was subjected to a constant axial load P of 470 kips which simulated the effect of gravity loads and a horizontal load H at the lower column end which was cycled to simulate the effect of lateral loads on the subassemblage. Specimen BC4 was subjected to a very severe, pulse-type loading with a single load reversal in order to study the monotonic behavior of the subassemblage and establish the load-displacement response envelope. Since the load-displacement response of subassemblage BC4 during the first load cycle, which represents a monotonic load test to near failure, was well established during the experiment, it serves as an ideal case study for the proposed model.

The finite element representation of the beam-column subassemblage is shown in Figure 9. Concrete was modeled by 8-node isoparametric elements and the longitudinal and transverse reinforcement was modeled by 2-node truss elements. The bond-slip effect is included in the analysis with bond link elements, as described in the previous section.

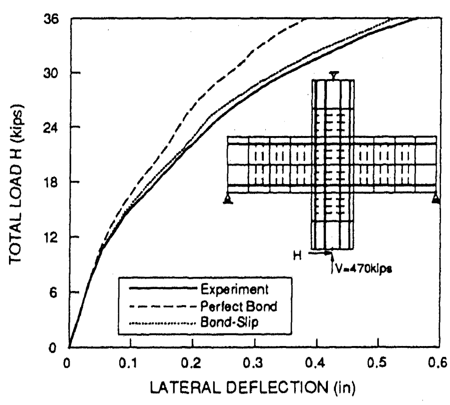

Figure 9. Load-displacement response of beam-column subassemblage with tension stiffening and bond-slip.

Figure 9 compares the analytical results with the measured load-displacement response of the subassemblage. With the effects of tension stiffening and bond-slip the analysis shows excellent agreement with the experimental results. The lateral force of 36 kips closely approximates the ultimate load of the specimen.

Figure 9 shows that the effect of bond slip affects the behavior of the subassemblage much more than tension stiffening. In this case the bond-slip of reinforcing bars in the joint contributes approximately 33% of the total deformation of the subassemblage near the ultimate load of 36 kips. The significance of bond-slip in the response of the beam-column subassemblage agrees with the results of the over-reinforced concrete beam A1, where the interaction of bond and shear plays a significant role in the response. This interaction also has an important effect on the stress transfer in beam-column joints, particularly, when these are subjected to loading simulating the effect of lateral loads. Even though the tension stiffening effect plays a minor role in the response of this beam-column joint, it should always be included in the model, since it helps with numerical instability problems which might arise during crack formation and propagation.

4 CONCLUSIONS

The objective of this study is to develop reliable and computationally efficient finite element models for the analysis of reinforced concrete beams, slabs and beam-column joint subassemblages under monotonic and cyclic loading conditions. Even though the applications in this paper concentrate mostly on the monotonic behavior of reinforced concrete members, the proposed models are general enough to be extended to cyclic loading conditions. A cyclic reinforcing steel model which includes the effect of bond-slip and is embedded in a concrete finite element is presented in this paper.

The correlation studies between analytical and experimental results lead to the following conclusions:

1. The developed finite element model exhibits very satisfactory agreement with experimental results of reinforcing bar anchorages, beams and beam-column joint subassemblages.
2. The effect of tension-stiffening is important in the analysis of monotonic behavior of reinforced concrete beams. Its inclusion is important for the independence of the analytical results from the size of the finite element mesh, but also for avoiding numerical problems in connection with crack formation and propagation.
3. Present smeared crack models are too stiff in connection with large finite elements. A new criterion which limits the effect of tension stiffening to the vicinity of the integration point yields very satisfactory results.
4. The effect of bond-slip is very important in the analysis of reinforced concrete beams and beam-column subassemblages under monotonic, but particularly, under cyclic loads. This effect is more pronounced in heavily reinforced beams and beam-column joints, which are subjected to loads simulating the effect of lateral loads on the structure.
5. Tension-stiffening and bond-slip have opposite effect on the response of reinforced concrete members. While tension-stiffening, which accounts for the concrete tensile stresses between cracks, increases the stiffness of the member, bond-slip leads to a stiffness reduction. In lightly reinforced beams these effects might cancel each other at certain load stages, leading to the erroneous impression that they can be neglected. Since bond-slip increases with loading, while tension-stiffening does not, consistent results can only be obtained when both effects are included. Moreover, the effect of bond-slip clearly outweighs the contribution of tension-stiffening in heavily reinforced beams and beam-column joint subassemblages. In these cases the exclusion of the bond-slip effect can lead to significant overestimation of the stiffness of the structure.

5 ACKNOWLEDGEMENTS

This study was supported by Grant No. ECE-8657525 from the National Science Foundation. This support is gratefully acknowledged. Any opinions expressed in this report are those of the authors and do not reflect the views of the sponsoring agency.

REFERENCES

Bresler, B. and Scordelis, A.C. (1963). Shear strength of reinforced concrete beams. *Journal of ACI*, Vol. 60, No. 1, pp. 51-72.

Ciampi, V., Eligehausen, R., Bertero, V.V. and Popov, E.P. (1982). Analytical model for concrete anchorages of reinforcing bars under generalized excitations. *Report No. EERC 82-23*, University of California, Berkeley.

Darwin, D. and Pecknold, D.A. (1977). Analysis of cyclic loading of plane R/C structures. *Computers & Structures*, Vol. 7, No. 1, pp. 137-147.

Filippou, F.C., Popov, E.P., and Bertero, V.V. (1983). Effects of bond deterioration on hysteretic behavior of reinforced concrete joints. *Report No. UCB/EERC-83/19*, University of California, Berkeley.

Kwak, H.G. and Filippou, F.C., Finite element analysis of reinforced concrete structures under monotonic loads. *Report No. UCB/SEMM-90/14*, University of California, Berkeley.

Ngo, D. and Scordelis, A.C. (1967). Finite element analysis of reinforced concrete beams. *Journal of ACI*, Vol. 64, No. 3, pp. 152-163.

Viwathanatepa, S., Popov, E.P. and Bertero, V.V. (1979a). Seismic behavior of reinforced concrete interior beam-column subassemblages. *Report No. UCB/EERC-79/14*, University of California, Berkeley.

Viwathanatepa, S., Popov, E.P. and Bertero, V.V. (1979b). Effects of generalized loadings on bond of reinforcing bars embedded in confined concrete blocks. *Report No. EERC 79-22*, University of California, Berkeley.

Yankelevsky, D.Z. (1985). New finite element for bond-slip analysis, *Journal of Structural Engineering, ASCE*, Vol. 111, No. 7, pp. 1533-1542.