*Earthquake Engineering, Tenth World Conference © 1992 Balkema, Rotterdam. ISBN 90 5410 060 5*

# Analysis method of multi-rigid-body model for earthquake responses of shear-type structure

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ABSTRACT: The theory of multi-rigid-body is used in the present study for the earthquake response analysis of shear-type structures, and the multi-rigid-body discrete model for shear-type structures is developed. Dynamic equation for earthquake response of the shear-type structure is derived. It is found that the equation is similar to traditional finite element in formulation, and it can be solved with ordinary numerical methods. However, Wilson-θ method is used in the present study. A computer program for the earthquake response analysis of shear-type structures was developed on the basis of multi-rigid-body model, and it was used to analyse the linear and nonlinear response of two reinforced concrete frames, collapsed in Tangshan Earthquake. The analytical results agree very well with the earthquake damage. Compared with other approaches, the multi-rigid-body model is simple and reliable.

## 1 INTRODUCTION

With the development of modern science and technology, dynamics of multi-rigid-body system has gradually become, to some extent, an independent branch of classical mechanics. It deals with the system consisting of a variety of rigid bodies and its research procedure suits modern computational techniques. In the past two decades, a number of computational methods, completely different in style, have been developed, such as those proposed by Roberson, Wittenberg, Kane, and differential methods, etc., which are suitable for different multi-rigid-body systems. The multi-rigid-body discrete theory has been successfully used in a large variety of fields such as machinery, vehicles, spaceflight, robot, bilogical systems as well as rock and soil engineering. It has exhibted pronounced advantages in its highly formulated computational procedures and nonlinear analysis of a system including material and geometrical nonlinearities, especially in the treatment of large deformations.

People tried long ago to employ the theory of multi-rigid-body to perform structural analyses and the most classical one is the lumped mass method. In order to get adequate accuracy, the method requires sufficient elements and because of the development of finite element in the past few decades, the method seems to be too simple and it has lost its popularity. The finite element method has successfully dealt with linear problems, but it meets with complications in nonlinear analysis. However, the finite element method is approximate unless its shape functions are orthogonal perfect functions of expansion of series such as Taylor's series and it will meet tremendous mathematical difficulties. On the countrary, the theory of multi-rigid-body is precise and there is no need to verify its convergence. So, the multi-rigid-body discrete method is used in the present study for the nonlinear dynamic response of structures.

The multi-rigid-body theory for dynamic response of building structures is a completely new method. Its broad prospects of application are demonstrated through its use in the present study for the earthquake response analysis of shear-type structures.

## 2 GENERAL THEORY OF DYNAMICS OF MULTI-RIGID -BODY SYSTEM

Conventional classical mechanical methods, that is, the vector mechanical approach represented by Newton-Euler equation or the analytical mechnical approach represented by Lagrange equation, can be in principle used to analyse the multi-rigid-body system. Most of the various research approaches stated above were developed on the basis of these two equations. R and W approach uses the concept of graph theory to describe the structure of multi-rigid-body system, considering the relative displacement between neighbouring rigid bodies as generalized coordinates and deriving the motion differential equation of the multi-rigid-body system, which is generally

$$A\ddot{q}=B \tag{1}$$

where q is a column matrix of generalized coordinates. Schiehlen method is used in the study to derive the dynamic equation of multi-rigid-body system.

As for a system, consisting of N rigid bodies which are hinged with each other, the amount of restraint equations is s and the amount of degree of freedom is n=6N-s. Select a column matrix of generalized coordinates $q=[q_1, q_2, \ldots, q_n]^T$, so as to determine the displacement of the system. Then, the sagittal diameter column matrix $r_i=[x_i, y_i, z_i]^T$ and the placement matrix $s_i$ of the center of mass of each rigid body $B_i$ can be expressed with q and time t as follows,

$$\begin{cases} r_i=r_i(q,t) \\ s_i=s_i(q,t) \end{cases} \quad i=1,2,\ldots,N \tag{2}$$

By deriving the above equations for time t, velocity column matrix $V_i$, acceleration column matrix $a_i$ and angular velocity column matrix $\omega_i$ can be obtained as follows,

$$\begin{cases} V_i=H_{Ti}(q,t)\dot{q}+\bar{v}_i(q,t) \\ \omega_i=H_{Ri}(q,t)\dot{q}+\bar{w}_i(q,t) \\ a_i=H_{Ti}(q,t)\ddot{q}+K_{Ti}(q,\dot{q},t)\dot{q}+\dot{\bar{v}}_i(q,t) \\ \dot{\omega}_i=H_{Ri}(q,t)\ddot{q}+K_{Ri}(q,\dot{q},t)\dot{q}+\dot{\bar{w}}_i(q,t) \\ \quad i=1,2,\ldots,N \end{cases} \tag{3}$$

where $H_{Ti}$ and $H_{Ri}$ are partical derivation matries of $r_i$, $s_i$ for q, with 3×n dimensions respectively. $\bar{v}_i$ and $\bar{\omega}_i$ are the derivative column matries of $r_i$, $w_i$ for time t, respectively. Other parameters can be obtained by deriving the first two terms in equation (3) .

Newton-Euler method is used to derive dynamic equation for each rigid body, so

$$\begin{cases} m_i a_i=F_i^a+F_i^c \\ J_i\dot{\omega}_i+\omega_i J_i\omega_i=M_i^a+M_i^c \end{cases} \tag{4}$$

where $m_i$ is mass of rigid body $B_i$, and $J_i$ inertia matrix of $B_i$ relative to center of mass. $F_i^c$ and $M_i^c$ represent the principle vector matrix of restraint resistance to which each restraint applies, and the principle moment column matrix, respectively. $F_i^a$ and $M_i^a$ are a vector column matrix and principle moment column matrix of active forces, respectively.

Combine equation (2) and (4), and write

$$M=\mathrm{diag}[m_1E, m_2E, \ldots, m_NE; J_1, J_2, \ldots, J_N]_{6N\times 6N}$$

$$H=[H_{T1}^T, H_{T2}^T, \ldots, H_{TN}^T; H_{R1}^T, H_{R2}^T, \ldots, H_{RN}^T]^T_{6N\times n}$$

$$Q=[F_1^T, F_2^T, \ldots, F_N^T; M_1^T, M_2^T, \ldots, M_N^T]^T_{6N\times 1}$$

where E represents a unit matrix with 3×3 dimensions, then

$$MH\ddot{q}+K(q,\dot{q},t)=Q^a(q,\dot{q},t)+Q^c(q,\dot{q},t) \tag{5}$$

where $Q^a$, $Q^c$ represent the active force column matrix and the restraint resistance column matrix respectively. According to the principle of victual work,

$$\Sigma(\delta r_i^T F_i^c+\delta\theta_i^T M_i^c)=0 \tag{6}$$

and from equation (3), we obtain

$$\delta r_i=H_{Ti}\delta q,\quad \delta\theta_i=H_{Ri}\delta q \tag{7}$$

Substitute equation(7) into equation(6), noting that $\delta q_i$ is independent, we have

$$H^TQ^c=0 \tag{8}$$

Therefore, when equation (5) is multiplied left hand by $H^T$, the motion differential equation for the multi-rigid-body system is obtained as follow,

$$M^*(q,t)\ddot{q}+K^*(q,\dot{q},t)=Q^*(q,\dot{q},t) \tag{9}$$

Though linearization, equation (9) becomes

$$M(t)\ddot{q}+C(t)\dot{q}+K(t)q=R(t) \tag{10}$$

which is more approperate for most practical engineering problems and according to its format, it can be solved with ordinary numerical approaches.

## 3 DISCRETE TECHNIQUE OF MULTI-RIGID-BODY FOR THE EARTHQUAKE RESPONSE OF SHEAR-TYPE STRUCTURES

Compared with the concept of conventional finite element method, the discrete technique of multi-rigid-body for structural analysis is completely opposite. The latter assumes that structural deformation concentrates totally on

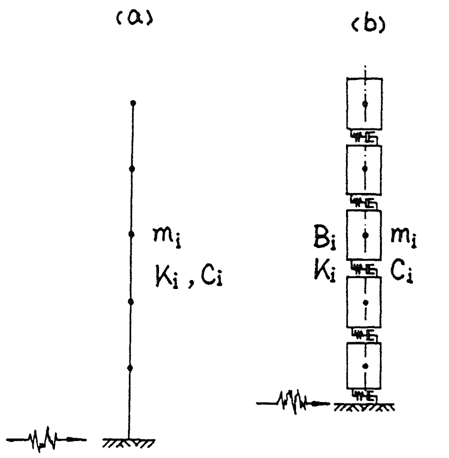


Figure 1. The structural models:(a) shear-type model; (b)multi-rigid-body discrete model

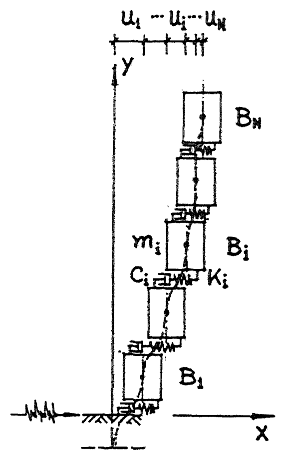


Figure 2. The displacements of shear-type structure discribed by multi-rigid-body model

the nodes of the rigid elements, but the former assumes that the structural deformation distributes within the deformable elements and the displacements of neighbouring elements coordinate at the nodes. The discrete technique of multi-rigid-body for earthquake response of shear-type structures follows the following train of thought.

### 3.1 Discretization of multi-rigid-body of structures

The modelling of engineering strcutures usually involves a great deal of approximation.In modelling a structure as a multi-rigid -body system, the present study takes into account the requirements of simplicity, reliability and engineering accuracy of the model and maintains obvious physical meaning of the model. With respect to ordinary shear-type structures, the effect of axial deformation is usually negnected for practical purpose and the earthquake response of structure is often analysed by using the shear model shown in figure 1(a). Similarly, the present study makes use of the discrete model of multi-rigid-body for the shear-type structure as shown in figure 1(b). According to the number of storeies, the structure is modelled with N elemental rigid bodies, centers of mass of which correspond to each storey height.Each neighbouring element is connected by a slip hinge.A shear spring and a damper are attached to the joint. The stiffness of the spring is just that of the shear stiffness in accordance with each storey. So, the nonlinearities of materials may be incooperated into $K_i$ and this makes the structural analysis, to a large extent, simplified. The damper may be removed if there is no need to account for damping.

### 3.2 Formation of dynamic equation and solution of earthquake response

The planar tree model without a crotch for shear-type structures shown in figure 1(b) , can be mathematically described by utilizing either the link matrix in Wittenberg's metrod or the link array in Huston's method. Only one degree of freedom of translation in a horizontal direction is taken into consideration, and the spring and damper between neighbouring elements may be regarded as force elements, in which the linkage is limited only to force action without any geometrical restraints. Displacement of the system is illustrated in figure 2.

Define generalized coordinates as the relative slip between neighbouring rigid bodies, that is, $q=[u_1, u_2, \ldots, u_N]^T$. Then the dynamic equation for the complete system can be established by using D' Alembert principle for each elemental rigid body.The equation is similar to equation (10) in format and it is not given here. However, response of a structure under earthquake excitation can be solved with commonly-used Wilson-$\theta$ procedure which has adequate precision and staticlity when $\theta \geqslant 1.4$ . The procedure is not stated in the paper.

## 4 COMPUTER PROGRAM AND EXAMPLES OF MULTI-RIGID-BODY DISCRETE ANALYSIS FOR STRUCTURAL EARTHQUAKE RESPONSE

Using the theory of multi-rigid-body system, the authors worked out a computer program, which can be used to analyze the elasto-plastic earthquake responses of shear-type structures under a definite earthquake input. The input can be either a actual record or an artificial earthquake wave produced by using auto-regressive moving

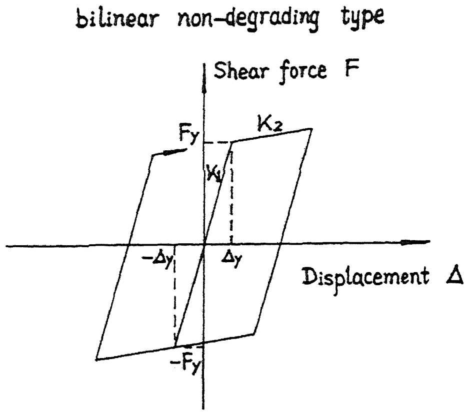


Figure 3. The model of restoring force

average model according to requirement. As the structures are often in a state of elasto-plastic, the program chooses bi-linear type and tri-linear degrading type as the models of restoring force to analyse the nonlinear response.

To inspect the reliability and correctness of the program, the authors calculated a great number of linear and nonlinear time-history responses of evenly-distributed shear-type structures. Compared with the results of other methods, such as finite element method, the multi-rigid-body discrete model is effective. As space is limited, the paper only presents as follows part of results of two actual earthquake disaster examples.

Example 1. The mid-south building of Tianjin No.2 Wollen Mill was a 3-storey cast-in-place reinforced concrete frame. In Tangshan Earthquake with a magnitude of 7.8 on the Richter scale on July 28,1976, the columns in the 2nd floor and the 3rd floor were fissured seriously, and some beam-ends cracked. After being reinforced, it completely collapsed in Ninghe Earthquake with a magnitude of 6.9 on Richter scale on Nov. 15,1976. An overwhelming majority columns of the 1st floor were broken and became s-shaped, and a few were cut off to the ground. In analysis, the input is the record in Tianjin Hospital during Ninghe Earthquake, with a maxium ground acceleration of $1.3472m/s^2$ and with a duration of 6 seconds. The damping ratio is 0.05 . The model of restoring force of the structure is bi-linear nondegrading type,as shown in figure 3 ($k_2$=0). And other primary datas are shown in table 1 .

**Table 1. Basic calculating datas of example 1**

| Floor | 1 | 2 | 3 |
|---|---|---|---|
| Mass (Kg) | 9184 | 9354 | 8462 |
| Stiffness factor K (KN/m) | 6870 | 7412 | 6563 |
| Yielding displacement (m) | 0.0373 | 0.0228 | 0.0142 |

In table 2,parts of analytical results using the multi-rigid-body discrete model ( Model 1) are compared with those using general finite element model ( Model 2).

Table 2.1 Self-oscillatory characteristics using Model 1

| Number of mode | 1 | 2 | 3 |
|---|---|---|---|
| Vibration period | 1.5860 | 0.5795 | 0.3979 |
| 1st storey | -0.113850 | -0.197224 | -0.247288 |
| 2nd storey | -0.233551 | -0.109994 | 0.213577 |
| 3rd storey | -0.203418 | 0.236671 | -0.106811 |

Table 2.2 Self-oscillatory characteristics using Model 2

| Number of mode | 1 | 2 | 3 |
|---|---|---|---|
| Vibration period | 1.5859 | 0.5796 | 0.3979 |
| 1st storey | -0.113838 | -0.197225 | -0.247293 |
| 2nd storey | -0.233554 | -0.109998 | 0.213569 |
| 3rd storey | -0.023417 | 0.236668 | -0.106817 |

Table 2.3 **Muximum** nonlinear responses using Model 1

| Storey | 1 | 2 | 3 |
|---|---|---|---|
| Displacement (m) | 0.09436 | 0.10897 | 0.13210 |
| Storey displacement(m) | 0.09436 | 0.02448 | 0.03124 |
| Storey shear-force(KN) | 256.252 | 168.994 | 93.208 |

Table 2.4 **Muximum** nonlinear responses using Model 2

| Storey | 1 | 2 | 3 |
|---|---|---|---|
| Displacement (m) | 0.05095 | 0.09646 | 0.14060 |
| Storey displacement(m) | 0.05095 | 0.04780 | 0.05570 |
| Storey shear-force(KN) | 256.252 | 168.994 | 93.208 |

And the nonlinear earthquake time-history responses obtained using Model 1 are shown in figure 4. (The numbers on curves in the figure are the orders of storeies.)

From the calculated results we can see that, the collapse of the building was caused by the large displacement due to yielding in the first storey,and the second storey as well as the third storey also yielded. This agrees very well with the phenomenon of earthquake damage.

Example 2. The new-north building of Tianjin Alkali Factory was a twelve-storey cast-in-place reinforced concrete frame structure. The structure collapsed from the seventh storey to the top in Tangshan Earthquake. The input for analysis and the model of restoring force are the same as those of example 1 , and the damping ratio is also 0.05 . Other information is given in table 3 .

## Example 1

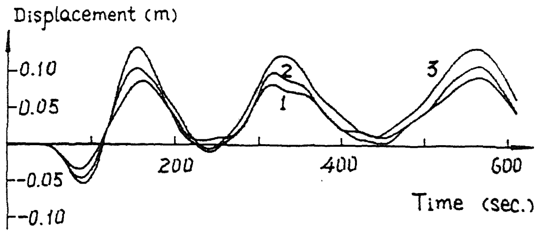


Figure 4.1 Displacement time-histories

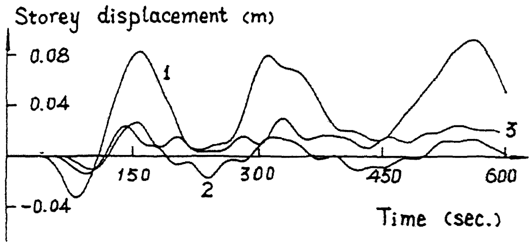


Figure 4.2 Storey displacement time-histories

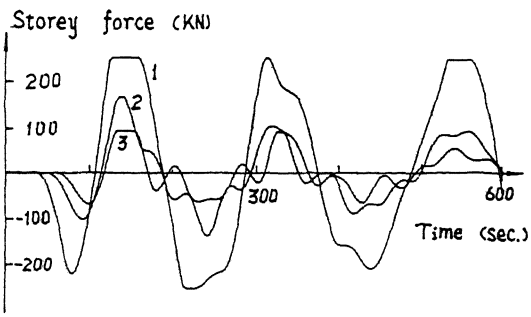


Figure 4.3 Storey shear-force time-histories

Table 3. Basic parameters of example 2

| Storey | Mass (Kg) | Stiffness (KN/m) | Yielding displacement (m) |
|---|---|---|---|
| 1 | 5560 | 425602 | 0.00383 |
| 2 | 6570 | 314475 | 0.00430 |
| 3 | 15500 | 284461 | 0.00449 |
| 4 | 4400 | 169533 | 0.00523 |
| 5 | 4740 | 96469 | 0.00740 |
| 6 | 6030 | 91744 | 0.00632 |
| 7 | 6630 | 65625 | 0.00610 |
| 8 | 7100 | 53771 | 0.00595 |
| 9 | 4580 | 33600 | 0.00714 |
| 10 | 4410 | 20649 | 0.00775 |
| 11 | 4410 | 13834 | 0.00721 |
| 12 | 4620 | 10631 | 0.00468 |

Table 4 gives the maximum nonlinear responses.

## Example 2

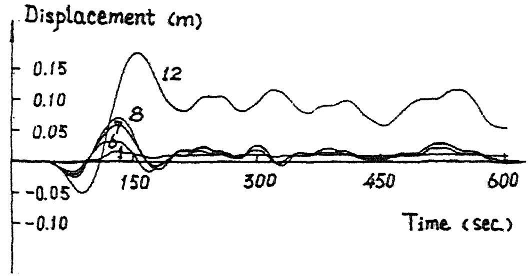


Figure 5.1 Displacement time-histories

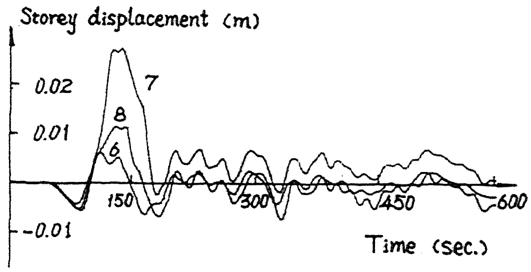


Figure 5.2 Storey displacement time-histories

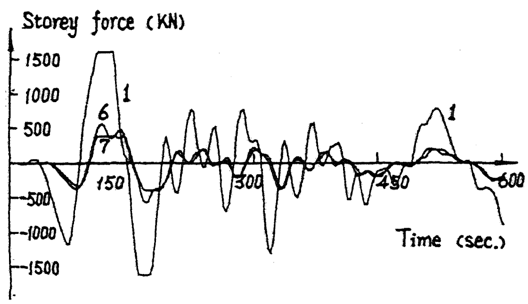


Figure 5.3 Storey shear-force time-histories

Table 4. Maximum nonlinear responses of example 2

| Storey | Displacement (m) | Storey displacement(m) | storey shear force(KN) |
|---|---|---|---|
| 1 | 0.01584 | 0.01584 | 1627.92 |
| 2 | 0.01805 | -0.00259 | -814.60 |
| 3 | 0.02009 | -0.00276 | -785.38 |
| 4 | 0.02230 | 0.00427 | 723.62 |
| 5 | 0.02601 | 0.00711 | 686.88 |
| 6 | 0.03136 | -0.00631 | -579.57 |
| 7 | 0.05743 | 0.02749 | 400.03 |
| 8 | 0.06893 | 0.01161 | 320.00 |
| 9 | 0.07847 | 0.01056 | 240.00 |
| 10 | 0.09737 | 0.02977 | -159.61 |
| 11 | 0.13174 | 0.07536 | -99.79 |
| 12 | 0.17166 | 0.07847 | -49.82 |

Figure 5 shows curves of the structural nonlinear responses. (The numbers on curves in the figure are the orders of storeies.)

According to the results, the seventh storey and the storeies above it yielded, so that the upper part of the structure collapsed. This agrees very well with the earthquake damage.

## 5 CONCLUSIONS

The paper develops a multi-rigid-body theory to analyse the earthquake responses of shear-type structures. By means of theoretical analysis and inspection of calculating examples, the theory has obvious superiority, at least, in three aspects as follows: (a) It is reliable. The methods of multi-rigid-body discrete model has in theory accurate solution and the analytical results of a structure simplified with a model still satisfy the engineering requirement; (b) It is simple and time-saving compared with other models, the geometric description of the model can be conducted by computer, and the majority of calculation is completed in matrix form; and (c)The handling of nonlinearity is easy. The theory incorporates material nonlinearity into the spring factor of joints, and it is not restrained by geometric nonlinearity, with which other methods are beyond comparasion.

It is worth noting that, there is no essential distinction between using the multi-rigid-body and using finite element model to analyse the dynamic responses of shear-type structures. The authors introduced the multi-rigid-body discrete model into shear-type structures with the purpose of looking for a regularity of applying the model to structural dynamic analysis, so as to apply the theory to bending-type structures and shear-bending-type structures.

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